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Intuitionistic T-S Fuzzy Subalgebras and Ideals in BCI-algebras

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***Abstract*—**The aim of this paper is to introduce the notions of intuitionistic T-S fuzzy subalgebras and intuitionistic T-S fuzzy ideals in BCI-algebras, and then to investigate their basic properties.

***Keywords*—**BCI-algebra, intuitionistic T-S fuzzy subalgebra, intuitionistic T-S fuzzy ideal, norm intersection, direct product, epimorphism, isomorphism.

I. Introduction

A BCI-algebra is an important class of logical algebras, the theory of BCI-algebras introduced by Iséki [1] has been studied deep by several researchers so far. Xi [2] introduced the concepts of fuzzy subalgebras and ideas in BCI-algebras and discussed some properties of them. On the other hand, triangular norm is a powerful tool in the theory research and application development of fuzzy sets. On the basis of the definition of the intuitionistic fuzzy groups, Li [4], [5] generalized the operators "$\wedge$" and "$\vee$" to T-norm and S-norm and defined the intuitionistic fuzzy groups of T-S norms. In this paper, the concepts of intuitionistic T-S fuzzy subalgebras and intuitionistic T-S fuzzy ideas are introduced in BCI-algebras. Some properties are discussed. We prove that the norm intersection and direct product of two intuitionistic T-S fuzzy subalgebras (ideals) are also intuitionistic T-S fuzzy subalgebras (ideals) in BCI-algebras.

II. Preliminaries

An algebra $(X;*,0)$ of type $(2,0)$ is called a BCI-algebra if it satisfies the following axioms:

(BCI-1) $((x*y)*(x*z))*(z*y)=0,$

(BCI-2) $(x*(x*y))*y=0,$

(BCI-3) $x*x=0,$

(BCI-4) $x*y=0$ and $y*x=0$ imply $x=y,$

for all $x,y,z\in X$. In a BCI-algebra X, we can define a partial ordering $\le$ by putting $x\le y$ if and only if $x*y=0$.

In this paper, X always means a BCI-algebra unless otherwise specified.

Definition 1 [3] Let S be any set. An intuitionistic fuzzy subset A of S is an object of the following form

$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in S\}$, where $\mu_A:S\to[0,1]$ and $\nu_A:S\to[0,1]$ define the degree of membership and the degree of non-membership of the element $x\in S$ respectively and for every $x\in S$, $0\le\mu_A(x)+\nu_A(x)\le 1$.

The all intuitionistic fuzzy subsets of S be denote $IFS[S]$.

Definition 2 [4] Mapping $T:[0,1]\times[0,1]\to[0,1]$ is called as triangular norm, if T satisfies:

(T-1) $T(0,0)=0$, $T(1,1)=1$;

(T-2) $T(a,b)\ge T(c,d)$ *if* $a\ge c, b\ge d$;

(T-3) $T(a,b)=T(b,a)$;

(T-4) $T(a,T(b,c))=T(T(a,b),c)$.

When the triangular T satisfies $T(a,1)=a,$ it is called as T-norm; When T satisfies $T(a,0)=a,$ it is called as S-norm.

Specifically, let $T_0=x\wedge y$ and $S_0=x\vee y$ for all $x,y\in[0,1]$, we have $T\le T_0\le S_0\le S$.

Definition 3 [4] Let $x^c=1-x$ for all $x\in[0,1]$, we say that x^c is a complement of x. Given T-norm $T(x,y)$, let $S(x,y)=1-T(1-x,1-y)=\left(T\left(x^c,y^c\right)\right)^c$, obviously, $S(x,y)$ is an S-norm. We say that S and T are dual.

Definition 4 [7] Let X,Y be two nonempty classical sets and $A\in IFS[X], B\in IFS[Y]$. Let the mapping $f:X\to Y,$ then the mapping f can induce a mapping F_f from $IFS[X]$ to $IFS[Y]$ and F_f^{-1} from $IFS[Y]$ to $IFS[X]$,

$$F_f(A)=\left\{\left\langle y,\mu_{F_f(A)}(y),\nu_{F_f(A)}(y)\right\rangle : y\in Y\right\},$$

$$F_f^{-1}(B)=\left\{\left\langle x,\mu_{F_f^{-1}(B)}(x),\nu_{F_f^{-1}(B)}(x)\right\rangle : x\in X\right\},$$

where

$$\mu_{F_f(A)}(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)}\mu_A(x), & f^{-1}(y)\ne\varnothing\\ 0, & f^{-1}(y)=\varnothing\end{cases},$$

$$\nu_{F_f(A)}(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\nu_A(x), & f^{-1}(y)\ne\varnothing\\ 1, & f^{-1}(y)=\varnothing\end{cases},$$

$$\mu_{F_f^{-1}(B)}(x)=\mu_B(f(x)),\ \nu_{F_f^{-1}(B)}(x)=\nu_B(f(x)).$$

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Proposition 1 [4] Let T be a T-norm and S be an S-norm, for all T and S, if $a,b,c,d \in [0,1]$, then

(1) $T\left(T(a,b),T(c,d)\right)=T\left(T(a,c),T(b,d)\right)$
$$=T\left(T(a,d),T(b,c)\right);$$

(2) $S\left(S(a,b),S(c,d)\right)=S\left(S(a,c),S(b,d)\right)$
$$=S\left(S(a,d),S(b,c)\right).$$

III. Intuitionistic T-S Fuzzy Subalgebras in BCI-Algebras

Definition 5 Let T be a T-norm, S be an S-norm and S and T be dual norm. An intuitionistic fuzzy set A in X is called an intuitionistic T-S fuzzy subalgebra of X if the following are satisfied:

$$\mu_A(x*y)\ge T\left(\mu_A(x),\mu_A(y)\right),$$
$$\nu_A(x*y)\le S\left(\nu_A(x),\nu_A(y)\right),$$

for all $x, y\in X$.

Proposition 2 Let T be a T-norm, S be an S-norm and S and T be dual norm. Let $A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle : x\in X\right\}$ be an intuitionistic T-S fuzzy subalgebra of X, then

$$\Box A=\left\{\left\langle x,\mu_A(x),1-\mu_A(x)\right\rangle : x\in X\right\},$$
$$\Diamond A=\left\{\left\langle x,1-\nu_A(x),\nu_A(x)\right\rangle : x\in X\right\}$$

are both intuitionistic T-S fuzzy subalgebras of X.

Proof. Denote $\omega_A(x)=1-\mu_A(x)$, for all $x,y\in X$, we have

$$\begin{aligned}\omega_A(x*y)&=1-\mu_A(x*y)\\ &\le 1-T\left(\mu_A(x),\mu_A(y)\right)\\ &=\left(T\left(\mu_A(x),\mu_A(y)\right)\right)^c\\ &=S\left(\left(\mu_A(x)\right)^c,\left(\mu_A(y)\right)^c\right)\\ &=S\left(1-\mu_A(x),1-\mu_A(y)\right)\\ &=S\left(\omega_A(x),\omega_A(y)\right).\end{aligned}$$

Thus

$\Box A=\left\{\left\langle x,\mu_A(x),1-\mu_A(x)\right\rangle : x\in X\right\}$ is an intuitionistic T-S fuzzy subalgebra of X.

Denote $\gamma_A(x)=1-\nu_A(x)$, for all $x,y\in X$, we have

$$\begin{aligned}\gamma_A(x*y)&=1-\nu_A(x*y)\\ &\ge 1-S\left(\nu_A(x),\nu_A(y)\right)\\ &=\left(S\left(\nu_A(x),\nu_A(y)\right)\right)^c\\ &=T\left(\left(\nu_A(x)\right)^c,\left(\nu_A(y)\right)^c\right)\\ &=T\left(1-\nu_A(x),1-\nu_A(y)\right)\\ &=T\left(\gamma_A(x),\gamma_A(y)\right).\end{aligned}$$

Thus

$\Diamond A=\left\{\left\langle x,1-\nu_A(x),\nu_A(x)\right\rangle : x\in X\right\}$ is also an intuitionistic T-S fuzzy subalgebra of X.

Definition 6 [8] Let X be any set, $A\in IFS[X]$, for all $\lambda\in[0,1]$, $\lambda A=\left\{\left\langle x,\lambda\mu_A(x),\lambda\nu_A(x)\right\rangle : x\in X\right\}$,

where

$$\lambda\mu_A(x)=\begin{cases}\mu_A(x),\lambda\ge\mu_A(x)\\ \lambda,\lambda<\mu_A(x)\end{cases},\ \lambda\nu_A(x)=\begin{cases}\nu_A(x),\lambda\ge\nu_A(x)\\ \lambda,\lambda<\nu_A(x)\end{cases},$$

λA is called as cutproduct of λ and A.

Proposition 3 Let A be an intuitionistic T-S fuzzy subalgebra of X, then for any $\lambda\in[0,1]$, λA is also an intuitionistic T-S fuzzy subalgebra of X.

Proof. It is clear that $\lambda A\in IFS[X]$. In the following we need to verify that λA satisfies the conditions of Definition 5.

If $\lambda\ge\mu_A(x*y)$, then

$$\begin{aligned}\lambda\mu_A(x*y)&=\mu_A(x*y)\\ &\ge T\left(\mu_A(x),\mu_A(y)\right)\\ &\ge T\left(\lambda\mu_A(x),\lambda\mu_A(y)\right);\end{aligned}$$

If $\lambda<\mu_A(x*y)$, then

$$\lambda\mu_A(x*y)=\lambda=T(\lambda,1)\ge T\left(\lambda\mu_A(x),\lambda\mu_A(y)\right);$$

hence we can obtain $\lambda\mu_A(x*y)\ge T\left(\lambda\mu_A(x),\lambda\mu_A(y)\right)$ for any $\lambda\in[0,1]$.

If $\lambda\ge\nu_A(x*y)$ and $\lambda\ge\max\left\{\nu_A(x),\nu_A(y)\right\}$, then

$$\begin{aligned}\lambda\nu_A(x*y)&=\nu_A(x*y)\\ &\le S\left(\nu_A(x),\nu_A(y)\right)\\ &=S\left(\lambda\nu_A(x),\lambda\nu_A(y)\right);\end{aligned}$$

If $\lambda\ge\nu_A(x*y)$ and $\lambda\le\max\left\{\nu_A(x),\nu_A(y)\right\}$, then

$$S\left(\lambda\nu_A(x),\lambda\nu_A(y)\right)\ge S(\lambda,0)=S(0,\lambda)=\lambda,$$
$$\lambda\nu_A(x*y)=\nu_A(x*y)\le\lambda\le S\left(\lambda\nu_A(x),\lambda\nu_A(y)\right);$$

If $\lambda<\nu_A(x*y)$ and $\lambda\ge\max\left\{\nu_A(x),\nu_A(y)\right\}$, then

$$\begin{aligned}\lambda\nu_A(x*y)&=\lambda\\ &<\nu_A(x*y)\\ &\le S\left(\nu_A(x),\nu_A(y)\right)\\ &=S\left(\lambda\nu_A(x),\lambda\nu_A(y)\right);\end{aligned}$$

If $\lambda<\nu_A(x*y)$ and $\lambda\le\max\left\{\nu_A(x),\nu_A(y)\right\}$, then

$$S\left(\lambda\nu_A(x),\lambda\nu_A(y)\right)\ge S(\lambda,0)=S(0,\lambda)=\lambda,$$
$$\lambda\nu_A(x*y)=\lambda\le S\left(\lambda\nu_A(x),\lambda\nu_A(y)\right);$$

Summarizing the above arguments, λA is an intuitionistic T-S fuzzy subalgebra of X.

Definition 7 [6] If $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in S\}$ and $B=\{\langle x,\mu_B(x),\nu_B(x)\rangle : x\in S\}$ be any two intuitionistic fuzzy subsets of a set S, then

$$A\cap B=\{\langle x,\mu_{A\cap B}(x),\nu_{A\cap B}(x)\rangle : x\in S\}$$
$$=\{\langle x,T(\mu_A(x),\mu_B(x)),S(\nu_A(x),\nu_B(x))\rangle : x\in S\}.$$

Proposition 4 Let A and B be two intuitionistic T-S fuzzy subalgebras of X, then $A\cap B$ is also an intuitionistic T-S fuzzy subalgebra of X.

Proof. For all $x,y\in X$. By Definition 5, Definition 7 and Proposition 1, we have

$$\begin{aligned}\mu_{A\cap B}(x*y)&=T(\mu_A(x*y),\mu_B(x*y))\\&\geq T(T(\mu_A(x),\mu_A(y)),T(\mu_B(x),\mu_B(y)))\\&=T(T(\mu_A(x),\mu_B(x)),T(\mu_A(y),\mu_B(y)))\\&=T(\mu_{A\cap B}(x),\mu_{A\cap B}(y)),\end{aligned}$$

$$\begin{aligned}\nu_{A\cap B}(x*y)&=S(\nu_A(x*y),\nu_B(x*y))\\&\leq S(S(\nu_A(x),\nu_A(y)),S(\nu_B(x),\nu_B(y)))\\&=S(S(\nu_A(x),\nu_B(x)),S(\nu_A(y),\nu_B(y)))\\&=S(\nu_{A\cap B}(x),\nu_{A\cap B}(y)).\end{aligned}$$

Hence $A\cap B$ is an intuitionistic T-S fuzzy subalgebra of X.

Definition 8 [6] Let A and B be two intuitionistic fuzzy sets of a set X.

The Cartesian product of A and B is defined by

$$A\times B=\{\langle x,\mu_{A\times B}(x),\nu_{A\times B}(x)\rangle : x=(x_1,x_2)\in X\times X\},$$

where

$$\mu_{A\times B}(x)=T(\mu_A(x_1),\mu_B(x_2)),\ \nu_{A\times B}(x)=S(\nu_A(x_1),\nu_B(x_2)).$$

Proposition 5 Let A and B be two intuitionistic T-S fuzzy subalgebras of X, then $A\times B$ is also an intuitionistic T-S fuzzy subalgebra of $X\times X$.

Proof. For all $x=(x_1,x_2), y=(y_1,y_2)\in X\times X$, by Definition 5, Definition 8 and Proposition 1, we get

$$\begin{aligned}\mu_{A\times B}(x*y)&=\mu_{A\times B}((x_1,x_2)*(y_1,y_2))\\&=\mu_{A\times B}((x_1*y_1),(x_2*y_2))\\&=T(\mu_A(x_1*y_1),\mu_B(x_2*y_2))\\&\geq T(T(\mu_A(x_1),\mu_A(y_1)),T(\mu_B(x_2),\mu_B(y_2)))\\&=T(T(\mu_A(x_1),\mu_B(x_2)),T(\mu_A(y_1),\mu_B(y_2)))\\&=T(\mu_{A\times B}(x),\mu_{A\times B}(y)),\end{aligned}$$

$$\begin{aligned}\nu_{A\times B}(x*y)&=\nu_{A\times B}((x_1,x_2)*(y_1,y_2))\\&=\nu_{A\times B}((x_1*y_1),(x_2*y_2))\\&=S(\nu_A(x_1*y_1),\nu_B(x_2*y_2))\\&\leq S(S(\nu_A(x_1),\nu_A(y_1)),S(\nu_B(x_2),\nu_B(y_2)))\\&=S(S(\nu_A(x_1),\nu_B(x_2)),S(\nu_A(y_1),\nu_B(y_2)))\\&=S(\nu_{A\times B}(x),\nu_{A\times B}(y)).\end{aligned}$$

Hence $A\times B$ is an intuitionistic T-S fuzzy subalgebra of $X\times X$.

IV. Intuitionistic T-S Fuzzy Ideals in BCI-Algebras

Definition 9 Let T be a T-norm, S be an S-norm and S and T be dual norm. An intuitionistic fuzzy set A in X is called an intuitionistic T-S fuzzy ideal of X if the following are satisfied:

(F_1) $\mu_A(0)\geq\mu_A(x)$,

(F_2) $\mu_A(x)\geq T(\mu_A(x*y),\mu_A(y))$,

(F_3) $\nu_A(0)\leq\nu_A(x)$,

(F_4) $\nu_A(x)\leq S(\nu_A(x*y),\nu_A(y))$,

for all $x,y\in X$.

Proposition 6 Let A be an intuitionistic T-S fuzzy ideal of X and $\mu_A(0)=1$, $\nu_A(0)=0$. If $x\leq y$ holds in X, then $\mu_A(x)\geq\mu_A(y)$, $\nu_A(x)\leq\nu_A(y)$.

Proof. For all $x,y\in X$, if $x\leq y$, then $x*y=0$, so by Definition 9,

$$\begin{aligned}\mu_A(x)&\geq T(\mu_A(x*y),\mu_A(y))\\&=T(\mu_A(0),\mu_A(y))\\&=\mu_A(y),\end{aligned}$$

$$\begin{aligned}\nu_A(x)&\leq S(\nu_A(x*y),\nu_A(y))\\&=S(\nu_A(0),\nu_A(y))\\&=\nu_A(y).\end{aligned}$$

Proposition 7 Let A be an intuitionistic T-S fuzzy ideal of X, $\mu_A(0)=1$ and $\nu_A(0)=0$. If the inequality $x*y\leq z$ holds in X, then

$$\mu_A(x)\geq T(\mu_A(y),\mu_A(z)),\ \ \nu_A(x)\leq S(\nu_A(y),\nu_A(z)).$$

Proof. For all $x,y,z\in X$, if $x*y\leq z$, then

$$\begin{aligned}\mu_A(x)&\geq T(\mu_A(x*y),\mu_A(y))\\&\geq T(\mu_A(y),\mu_A(z)),\end{aligned}$$

$$\begin{aligned}\nu_A(x)&\leq S(\nu_A(x*y),\nu_A(y))\\&\leq S(\nu_A(y),\nu_A(z)).\end{aligned}$$

Proposition 8 Let $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ be an intuitionistic T-S fuzzy ideal of X, then

$$\Box A=\{\langle x,\mu_A(x),1-\mu_A(x)\rangle : x\in X\},$$

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$\Diamond A=\left\{\left\langle x,1-\nu_A(x),\nu_A(x)\right\rangle : x\in X\right\}$

are both intuitionistic T-S fuzzy ideals of X.

Proof. Denote $\omega_A(x)=1-\mu_A(x)$, for all $x,y\in X$, we have

$$
\begin{aligned}
\omega_A(0)&=1-\mu_A(0)\\
&\le 1-\mu_A(x)\\
&=\omega_A(x),\\
\omega_A(x)&=1-\mu_A(x)\\
&\le 1-T\left(\mu_A(x*y),\mu_A(y)\right)\\
&=\left(T\left(\mu_A(x*y),\mu_A(y)\right)\right)^c\\
&=S\left(\left(\mu_A(x*y)\right)^c,\left(\mu_A(y)\right)^c\right)\\
&=S\left(1-\mu_A(x*y),1-\mu_A(y)\right)\\
&=S\left(\omega_A(x*y),\omega_A(y)\right).
\end{aligned}
$$

Thus

$\Box A=\left\{\left\langle x,\mu_A(x),1-\mu_A(x)\right\rangle : x\in X\right\}$ is an intuitionistic T-S fuzzy ideal of X.

Denote $\gamma_A(x)=1-\nu_A(x)$, for all $x,y\in X$, we have

$$
\begin{aligned}
\gamma_A(0)&=1-\nu_A(0)\\
&\ge 1-\nu_A(x)\\
&=\gamma_A(x),\\
\gamma_A(x)&=1-\nu_A(x)\\
&\ge 1-S\left(\nu_A(x*y),\nu_A(y)\right)\\
&=\left(S\left(\nu_A(x*y),\nu_A(y)\right)\right)^c\\
&=T\left(\left(\nu_A(x*y)\right)^c,\left(\nu_A(y)\right)^c\right)\\
&=T\left(1-\nu_A(x*y),1-\nu_A(y)\right)\\
&=T\left(\gamma_A(x*y),\gamma_A(y)\right).
\end{aligned}
$$

Thus

$\Diamond A=\left\{\left\langle x,1-\nu_A(x),\nu_A(x)\right\rangle : x\in X\right\}$

is also an intuitionistic T-S fuzzy ideal of X.

Proposition 9 Let A and B be two intuitionistic T-S fuzzy ideals of X, then $A\cap B$ is also an intuitionistic T-S fuzzy ideal of X.

Proof. For all $x,y\in X$. By Definition 7, Definition 9 and Proposition 1, we have

$$
\begin{aligned}
\mu_{A\cap B}(0)&=T\left(\mu_A(0),\mu_B(0)\right)\\
&\ge T\left(\mu_A(x),\mu_B(x)\right)\\
&=\mu_{A\cap B}(x),\\
\nu_{A\cap B}(0)&=S\left(\nu_A(0),\nu_B(0)\right)\\
&\le S\left(\nu_A(x),\nu_B(x)\right)\\
&=\nu_{A\cap B}(x),\\
\mu_{A\cap B}(x)&=T\left(\mu_A(x),\mu_B(x)\right)\\
&\ge T\left(T\left(\mu_A(x*y),\mu_A(y)\right),T\left(\mu_B(x*y),\mu_B(y)\right)\right)\\
&=T\left(T\left(\mu_A(x*y),\mu_B(x*y)\right),T\left(\mu_A(y),\mu_B(y)\right)\right)\\
&=T\left(\mu_{A\cap B}(x*y),\mu_{A\cap B}(y)\right),\\
\nu_{A\cap B}(x)&=S\left(\nu_A(x),\nu_B(x)\right)\\
&\le S\left(S\left(\nu_A(x*y),\nu_A(y)\right),S\left(\nu_B(x*y),\nu_B(y)\right)\right)\\
&=S\left(S\left(\nu_A(x*y),\nu_B(x*y)\right),S\left(\nu_A(y),\nu_B(y)\right)\right)\\
&=S\left(\nu_{A\cap B}(x*y),\nu_{A\cap B}(y)\right).
\end{aligned}
$$

Hence $A\cap B$ is also an intuitionistic T-S fuzzy ideal of X.

Proposition 10 Let A and B be two intuitionistic T-S fuzzy ideals of X, then $A\times B$ is also an intuitionistic T-S fuzzy ideal of $X\times X$.

Proof. For all $x=(x_1,x_2)$, $y=(y_1,y_2)\in X\times X$,

by Definition 8, Definition 9 and Proposition 1, we get

$$
\begin{aligned}
\mu_{A\times B}(0)&=T\left(\mu_A(0),\mu_B(0)\right)\\
&\ge T\left(\mu_A(x_1),\mu_B(x_2)\right)\\
&=\mu_{A\times B}(x),\\
\nu_{A\times B}(0)&=S\left(\nu_A(0),\nu_B(0)\right)\\
&\le S\left(\nu_A(x_1),\nu_B(x_2)\right)\\
&=\nu_{A\times B}(x),\\
\mu_{A\times B}(x)&=T\left(\mu_A(x_1),\mu_B(x_2)\right)\\
&\ge T\left(T\left(\mu_A(x_1*y_1),\mu_A(y_1)\right),T\left(\mu_B(x_2*y_2),\mu_B(y_2)\right)\right)\\
&=T\left(T\left(\mu_A(x_1*y_1),\mu_B(x_2*y_2)\right),T\left(\mu_A(y_1),\mu_B(y_2)\right)\right)\\
&=T\left(\mu_{A\times B}(x*y),\mu_{A\times B}(y)\right),\\
\nu_{A\times B}(x)&=S\left(\nu_A(x_1),\nu_B(x_2)\right)\\
&\le S\left(S\left(\nu_A(x_1*y_1),\nu_A(y_1)\right),S\left(\nu_B(x_2*y_2),\nu_B(y_2)\right)\right)\\
&=S\left(S\left(\nu_A(x_1*y_1),\nu_B(x_2*y_2)\right),S\left(\nu_A(y_1),\nu_B(y_2)\right)\right)\\
&=S\left(\nu_{A\times B}(x*y),\nu_{A\times B}(y)\right).
\end{aligned}
$$

Hence $A\times B$ is an intuitionistic T-S fuzzy ideal of $X\times X$.

Proposition 11 If $f:X\to X'$ is an epimorphism from X to X' and B is an intuitionistic T-S fuzzy ideal of X', then $F_f^{-1}(B)$ is an intuitionistic T-S fuzzy ideal of X.

Proof. Let $B=\left\{\left\langle x',\mu_B(x'),\nu_B(x')\right\rangle : x'\in X'\right\}$ be an intuitionistic T-S fuzzy ideal of X', then

$$F_f^{-1}(B)=\left\{\left\langle x,\mu_{F_f^{-1}(B)}(x),\nu_{F_f^{-1}(B)}(x)\right\rangle : x\in X\right\}.$$

Since $f: X \to X'$ is an epimorphism from X to X', therefore $F_f(0) = 0'$. Let $x', y' \in X'$, then exist $x, y \in X$, such that $x' = F_f(x), y' = F_f(y)$.

Let $F_f^{-1}(x') * F_f^{-1}(y') = \{x * y : x \in F_f^{-1}(x'), y \in F_f^{-1}(y')\}$,

then $F_f^{-1}(x') * F_f^{-1}(y') \subseteq F_f^{-1}(x' * y')$,

we get $F_f(x * y) = x' * y'$.

$$\begin{aligned}\mu_{F_f^{-1}(B)}(0) &= \mu_B(F_f(0)) \\ &= \mu_B(0') \\ &\geq \mu_B(x') \\ &= \mu_{F_f^{-1}(B)}(x),\end{aligned}$$

$$\begin{aligned}\nu_{F_f^{-1}(B)}(0) &= \nu_B(F_f(0)) \\ &= \nu_B(0') \\ &\leq \nu_B(x') \\ &= \nu_{F_f^{-1}(B)}(x),\end{aligned}$$

$$\begin{aligned}\mu_{F_f^{-1}(B)}(x) &= \mu_B(F_f(x)) \\ &= \mu_B(x') \\ &\geq T(\mu_B(x' * y'), \mu_B(y')) \\ &= T\left(\mu_{F_f^{-1}(B)}(x * y), \mu_{F_f^{-1}(B)}(y)\right),\end{aligned}$$

$$\begin{aligned}\nu_{F_f^{-1}(B)}(x) &= \nu_B(F_f(x)) \\ &= \nu_B(x') \\ &\leq S(\nu_B(x' * y'), \nu_B(y')) \\ &= S\left(\nu_{F_f^{-1}(B)}(x * y), \nu_{F_f^{-1}(B)}(y)\right).\end{aligned}$$

Hence $F_f^{-1}(B)$ is an intuitionistic T-S fuzzy ideal of X.

Proposition 12 If $f: X \to Y$ is an isomorphism mapping from X to Y and A, B are two intuitionistic T-S fuzzy ideals of X and Y respectively then

(1) $F_f^{-1}(F_f(A)) = A$;

(2) $F_f(F_f^{-1}(B)) = B$.

Proof. (1) Since f is an isomorphism mapping from X to Y, for all $x \in X$, let $y = f(x)$, then $f^{-1}(y) = \{x\}$ and

$$\mu_{F_f^{-1}(F_f(A))}(x) = \mu_{F_f(A)} f(x) = \mu_{F_f(A)}(y)$$
$$= \sup_{x \in f^{-1}(y)} \mu_A(x) = \mu_A(x).$$

By the similar proof ways, we can have

$\nu_{F_f^{-1}(F_f(A))}(x) = \nu_A(x)$, for every $x \in X$.

This implies that $F_f^{-1}(F_f(A)) = A$.

(2) It is clear that $F_f(F_f^{-1}(B))$ is an intuitionistic T-S fuzzy ideal of Y, for all $y \in Y$, we have

$\mu_{F_f(F_f^{-1}(B))}(y) = \sup_{x \in f^{-1}(y)} \mu_{F_f^{-1}(B)}(x) = \sup_{f(x)=y} \mu_B(f(x)) = \mu_B(y)$. By the similar proof ways, we can have

$\nu_{F_f(F_f^{-1}(B))}(y) = \nu_B(y)$, for every $x \in X$.

This implies that $F_f(F_f^{-1}(B)) = B$.

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