

# Intelligent Fuzzy Input Estimator for the Input Force on the Rigid Bar Structure System

Ming-Hui Lee\*, Tsung-Chien Chen and Yuh-Shiou Tai

**Abstract**—The intelligent fuzzy input estimator is used to estimate the input force of the rigid bar structural system in this study. The fuzzy Kalman filter without the input term and the fuzzy weighting recursive least square estimator are two main portions of this method. The practicability and accuracy of the proposed method were verified with numerical simulations from which the input forces of a rigid bar structural system were estimated from the output responses. In order to examine the accuracy of the proposed method, a rigid bar structural system is subjected to periodic sinusoidal dynamic loading. The excellent performance of this estimator is demonstrated by comparing it with the use of difference weighting function and improper the initial process noise covariance. The estimated results have a good agreement with the true values in all cases tested.

**Keywords**—Fuzzy Input Estimator, Kalman Filter, Recursive Least Square Estimator.

## I. INTRODUCTION

**I**N the course of the fatigue analysis, the anti-vibration design, and the reliability assessment of the structure system, the most important and necessary procedure is to obtain the values of the true loads to the system. However, for some physical systems, direct measurements of excitation loads are difficult to be realized because of very large magnitudes of loads. Besides, there are always difficulties in installing the load transducers used to measure the active loads to the structure system. One of the methods is to estimate the input forces by applying the measured dynamic responses by an inverse technique.

The inverse technique general has been applied to both structural dynamic and heat transfer problems. Hollandsworth and Busby [1] used the modal methods to analyze the structure and dynamic programming. Bushy and Trujillo [2] used the inverse technique to analyze an inverse heat conduction problem. Inverse problems usually tend to be ill-posed, in the sense that infinitesimally small variations in the input data can cause large variations in the results, and tend to be ill-conditioned, namely, a small noise in measurements results in erroneous estimations of the forces. In order to overcome these difficulties, Wang [3] used the weighted total acceleration

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method to detect the vibration force acting on the concentrated -massed nonlinear beam. Inoue et al. [4] proposed the least square method, which is based on the wiener filtering theory, the mean square error, and the singular value decomposition (SVD), to improve the estimation precision and to obtain the optimal estimates. Haung [5] adopted the conjugate gradient method (CGM) to estimate the unknown time-dependent external forces in a multiple-degree-of-freedom damped system. Doyle showed that an objective function based on the correlation of forces could be used to evaluate different guessed for the case of a simple beam [6]. Doyle's work has been further developed and extended to include frame structures [7]. The estimation algorithms of the above references are all in the batch forms. This kind of method is not an on-line procedure of the unknown input estimation. This method is time-consuming and is not efficient to process the measurement data.

In order to promote the estimation efficiency, some recent studies [8-9] use the input estimation method to inversely solve the 1-D and 2-D heat conduction problems. Lee et al. [10] utilized the adaptive weighted input estimation method to inversely solve the burst load of the truss structure system. Chen et al. [11-12] investigated the adaptive input estimation method applied to the inverse estimation of load input in the multi-layer shearing stress structure and the identification of moving load in the bridge structure system. As opposed to the batch process, the input estimation method is using the recursive form to process the data when dealing with more complex systems. There is no need to store all the data to implement the process, and the quantity of memory used can be reduced. The disadvantage was that not only has higher effectiveness but also the magnitude of unknown could be estimated in time.

In this study, an intelligent input estimator to estimate the periodic sinusoidal dynamic loading of a rigid bar structural system is presented. The efficient estimator are accelerated and weighted by the fuzzy accelerating and weighting factor proposed based on the fuzzy logic inference system. By comparing the results with the adaptive and constant weighted factor input estimation methods, the efficiency, adaptively and robustness of the proposed method can be demonstrated.

## II. PROBLEM FORMULATION

The geometry and coordinates of a rigid bar structural system are shown in figure 1(a). The displacement and joined forces of the rigid bar structural system is shown in figure 1(b). All forces of the bar  $A_1 - B_1$  and  $A_2 - B_2$  are performed by the

moment summation acting at point  $A_1$  and  $B_2$ , respectively, i.e.

$$k_1 \left( \frac{2}{3}Y - Y_1 \right) 3a = 2aF_0 \sin \bar{\omega}t \quad (1)$$

$$\frac{I_0}{3a} \ddot{Y} + \frac{3}{4}m\ddot{Y} + \frac{a}{3}c\dot{Y} + k_1 \left( \frac{2}{3}Y - Y_1 \right) 2a + 3ak_2Y = 0 \quad (2)$$

Substituting  $Y_1$  of equation (1) into equation (2), the motion equation can be established by variable  $Y(t)$  of the generalized coordinate leads to

$$M\ddot{Y}(t) + C\dot{Y}(t) + KY(t) = F(t) \quad (3)$$

in which  $M = \frac{I_0}{3a^2} + \frac{3m}{4}$  is the generalized mass matrix,

$C = \frac{c}{3}$  is the generalized damping matrix

$K = 3k_2$  is the generalized stiffness vector and

$F(t) = -\frac{4}{3}F_0 \sin \bar{\omega}t$  is the generalized input force.

$I_0$  is the mass moment of inertia.

$m$  is the total mass.

The input estimation algorithm is a calculation method using the state space. Therefore, the state equation and the measurement equation have to be constructed before applying this method. In order to satisfy this situation, the equality,

$X(t) = [Y \quad \dot{Y}]^T$  is used to transfer the movement equation to the state space form. The continuous-time state equation and measurement equation of the structure system can be presented as follows:

$$\dot{X}(t) = AX(t) + BF(t), \quad (4)$$

$$Z(t) = HX(t), \quad (5)$$

in which

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix},$$

$$H = [1 \quad 0]$$

$A$  and  $B$  are both constant matrices composed of the  $n$ th natural frequency and the inertia moment of the structure system.  $X(t)$  is the modal state vector.  $F(t)$  is the input dynamic loading.  $Z(t)$  is the observation vector, and  $H$  is the measurement matrix. Generally speaking, there always exists the noise turbulence in the practical engineering environment. Nevertheless, equations (4) and (5) do not take the noise turbulence into account. In order to construct the statistic model of the system state characteristics, a noise disturbance term, which can reflect these statistical characteristics of the state, will need to be added into these two equations. For this reason, the continuous-time state equation (4) can be sampled with the sampling interval,  $\Delta t$ , to obtain the discrete-time statistic model of the state equation shown as the following [13]:

$$X(k+1) = \Phi X(k) + \Gamma[G(k) + w(k)] \quad (6)$$

where

$$X(k) = [Y(k) \quad \dot{Y}(k)]^T$$

$$\Phi = \exp(A\Delta t)$$

$$\Gamma = \int_{k\Delta t}^{(k+1)\Delta t} \exp\{A[(k+1)\Delta t - \tau]\} B d\tau$$

$\Phi$  is the state transition matrix.  $X(k)$  is the discrete state vector.  $\Gamma$  is the input matrix.  $\Delta t$  is the sampling interval.  $G(k)$  is the sequence of deterministic acceleration input, and  $w(k)$  is the processing error vector, which is assumed as the Gaussian white noise. In the equation (6), when describing the active characteristics of the structure system, the additional term,  $w(k)$ , can be used to represent the uncertainty in a numerical manner. The uncertainty could be the random disturbance, the uncertain parameters, or the error due to the over-simplified assumption of numerical models. Note that  $E\{w(k)w^T(k)\} = Q\delta_{kj}$ ,  $Q = Q_w \times I_{2n \times 2n}$ ,  $Q$  is the discrete-time processing noise covariance matrix.  $\delta_{kj}$  is the Kronecker delta function.

In order to additionally consider the measurement noise, equation (3) is then expressed as

$$Z(k) = HX(k) + v(k) \quad (7)$$

$Z(k)$  is the discrete observation vector.  $v(k)$  represents the measurement noise vector and is assumed as the Gaussian white noise with zero mean and the variance,  $E\{v(k)v^T(k)\} = R\delta_{kj}$ ,  $R = R_v \times I_{2n \times 2n}$ ,  $R$  is the discrete-time measurement noise covariance matrix.

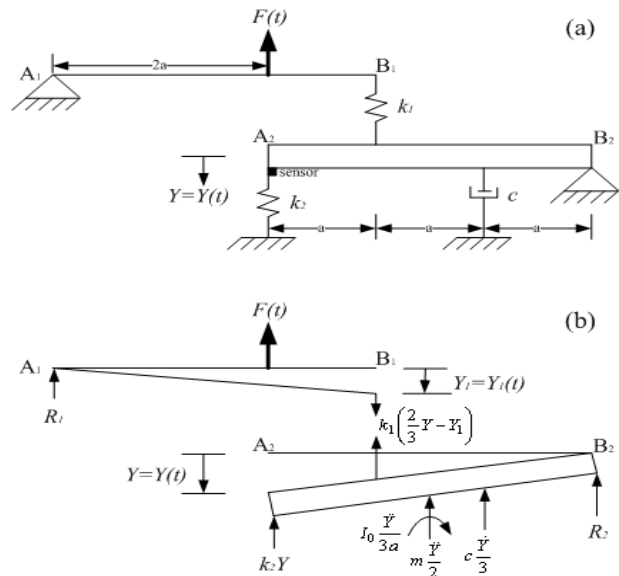


Fig. 1:(a) Considered rigid bar structural system. (b) Proposed displacement and joined forces of the rigid bar structural system [14].

### III. FUZZY INPUT ESTIMATION METHOD

The fuzzy estimator are accelerated and weighted by the fuzzy accelerating factor of the processing noise covariance matrix and weighting factor of the input estimation method proposed based on the fuzzy logic inference system. The presented method can inversely estimate the unknown inputs by applying the active reaction of the structure system. This method is composed of the fuzzy Kalman filter without the input term and the fuzzy weighted recursive least square estimator. The fuzzy Kalman filter can produce the residual innovation sequence, which contains the bias or systematic error caused by the unknown time-varying inputs, and the variance or random error caused by the measurement error. Therefore, the estimator utilizes the innovation sequence to estimate the inputs over time by adopting the fuzzy weighted recursive least square method. The Kalman filter without the input term is shown as follows [13]:

The state prediction is

$$\bar{X}(k/k-1) = \Phi\bar{X}(k-1/k-1) \quad (8)$$

The prediction error covariance matrix is

$$P(k/k-1) = \Phi P(k-1/k-1)\Phi^T + \Gamma Q \Gamma^T \quad (9)$$

The covariance of residual is  $S(k)$

$$S(k) = H P(k/k-1) H^T + R \quad (10)$$

The Kalman gain is

$$K(k) = P(k/k-1) H^T S^{-1}(k) \quad (11)$$

The filter error covariance matrix is expressed by

$$P(k/k) = [I - K(k)H] P(k/k-1) \quad (12)$$

The bias innovation produced by the measurement noise and input disturbance is expressed by

$$\bar{Z}(k) = Z(k) - H\bar{X}(k/k-1) \quad (13)$$

And the state filter value is expressed as

$$\bar{X}(k/k) = \bar{X}(k/k-1) + K(k)\bar{Z}(k) \quad (14)$$

The equations of the recursive least square estimator are as follows:

The sensitivity matrices are  $B(k)$  and  $M(k)$

$$B(k) = H[\Phi M(k-1) + I]\Gamma \quad (15)$$

$$M(k) = [I - K(k)H][\Phi M(k-1) + I] \quad (16)$$

The correction gain is expressed as

$$K_b(k) = \gamma^{-1} P_b(k-1) B^T(k) [B(k) \gamma^{-1} P_b(k-1) B^T(k) + S(k)]^{-1} \quad (17)$$

where  $\gamma$  is the weighting factor. The error covariance of the input estimation process is

$$P_b(k) = [I - K_b(k)B(k)] \gamma^{-1} P_b(k-1) \quad (18)$$

The estimated input force is

$$\hat{F}(k) = \hat{F}(k-1) + K_b(k) [\bar{Z}(k) - B(k) \hat{F}(k-1)] \quad (19)$$

Above equations (12) to (18), a superscript ‘-’ indicates filter estimation.  $\bar{X}(k/k-1)$  is the state estimation,  $P(k/k-1)$  is the state estimation error covariance,  $\bar{Z}(k)$  is the residual of predictor,  $S(k)$  is the innovation covariance,

$K(k)$  is the Kalman gain,  $\bar{X}(k/k)$  is the state filter,  $P(k/k)$  is the state filter error covariance. Where  $\bar{Z}(k)$  is the bias innovation produced by the measurement noise and input disturbance,  $K_b(k)$  is the correction gain. Besides,  $B(k)$  and  $M(k)$  are the sensitivity matrices.  $\gamma$  is the weighting factor.  $P_b(k)$  is the error covariance of the input estimation process and  $\hat{F}(k)$  is the estimated dynamic inputs.

Some parameters of filter must be obtained before filtering process. Such as, the state transition matrix of the structure system,  $\Phi$ , the measurement matrix,  $H$ , the discrete-time processing noise covariance matrix,  $Q$  and the discrete-time measurement noise covariance matrix,  $R$ . The on-line state estimation,  $\bar{X}(k/k-1)$  and state estimation error covariance,  $P(k/k-1)$  of the filter will be acquired when the observation vector is unceasingly input immediately after the initial conditions  $X_0$  and  $P_0$  are drawn into the estimator.  $K(k)$  gets smaller as the processing noise covariance matrix,  $Q$  and the state filter error covariance get smaller according to equations (9) and (11), indicate that the new measurement is mitigating to the state predicted correction.  $K(k)$  gets smaller as the measurement noise covariance matrix,  $R$  get larger according to equations (10) and (11), that is to say, the measurement error is mitigating to the state estimation of the estimator. In other words, the Kalman gain  $K(k)$  depends on the  $R_v$  and  $Q_w$ . The above-mentioned is an important principle and a key problem that the appropriate  $R_v$  and  $Q_w$  can be chosen in accordance with the system property and the magnitude of noise interference in the estimation process.  $R_v$  can be chosen in accordance with the precision of the measurement instrument.  $Q_w$  can be chosen in accordance with the modular error of the system. The Kalman gain can be slightly corrected with the higher precision of the measurement instrument, this is to say, the modular error of the system change from big to small. For this reason, the processing noise covariance can be defined as following:

$$Q_w(k+1) = Q_w(k) \times 10^{\alpha(k)} \quad (20)$$

where  $\alpha(k)$  is the fuzzy accelerating factor, which is chosen in the interval,  $[-1, 1]$ . The estimation precision gets better as the  $\alpha(k)$  get smaller. On the contrary, the estimation precision gets worse as the  $\alpha(k)$  get larger.

The weighting factor  $\gamma(k)$  is important another parameter which affecting the estimation precision in the estimation process. It also plays the role as an adjustable parameter to control the bandwidth of estimator or the gain magnitude of recursive least square estimator. It can operate at each step based on the innovation produced by the Kalman filter. Furthermore, the weighting factor  $\gamma(k)$  is employed to compromise between the tracking capability and the loss of estimation precision. The fuzzy estimator in this paper is proposed based on the fuzzy logic inference system. The processing noise covariance and the weighting factor can be adjusted by means of the each step innovation produced by the Kalman filter. The fuzzy logic system includes four basic

components, which are the fuzzy rule base, fuzzy inference engine, fuzzifier, and defuzzifier. The value of fuzzy logic system input,  $\theta(k)$ , may be chosen within the interval,  $[0,1]$ . The Pythagorean theorem with the transverse axle (time,  $t$ ) and the vertical axle (residual of predictor,  $\bar{z}$ ) can be used to solve the length of the hypotenuse. In other words, the length of the hypotenuse is the variation rate of the residual in the sampling interval. The dimensionless input variable is defined as the following:

$$\theta(k) = \frac{\left| \frac{\Delta \bar{z}(k)}{\bar{z}(k)} \right|}{\sqrt{\left( \frac{\Delta \bar{z}(k)}{\bar{z}(k)} \right)^2 + \left( \frac{\Delta t}{t_f} \right)^2}} \quad (21)$$

where  $\Delta \bar{z}(k) = \bar{z}(k) - \bar{z}(k-1)$ ,  $\Delta t$  is the sampling interval, let  $t_f = 1$ . The fuzzy sets for  $\theta(k)$ ,  $\alpha(k)$  and  $\gamma(k)$  are labeled in the linguistic terms of EP (extremely large positive), VP (very large positive), LP (large positive), MP (medium positive), SP (small positive), VS (very small positive), and ES (extremely small). A fuzzy rule base is a collection of fuzzy IF-THEN rules which are shown in Table 1.

TABLE I THE FUZZY RULE BASE

		Input variable, $\theta(k)$						
		ES	VS	SV	MV	LV	VL	EP
Output variable	$\alpha(k)$	EP	VL	LV	MV	SV	VS	ES
	$\gamma(k)$	EP	VL	LV	MV	SV	VS	ES

where  $\theta(k)$  is input variable,  $\alpha(k)$  and  $\gamma(k)$  are the output variables of the fuzzy logic system, respectively. The fuzzifier maps a crisp point  $\theta(k)$  into a fuzzy set  $A$ . Therefore, the nonsingleton fuzzifier can be expressed as the following [15].

$$\mu_A(\theta(k)) = \exp\left(-\frac{(\theta(k) - \bar{x}_i^l)^2}{2(\sigma_i^l)^2}\right) \quad (22)$$

$\mu_A(\theta(k))$  decreases from 1 as  $\theta(k)$  moves away from  $\bar{x}_i^l$ .  $(\sigma_i^l)^2$  is a parameter characterizing the shape of  $\mu_A(\theta(k))$ .

The Mamdani maximum-minimum inference engine is used in this paper. The max-min-operation rule of fuzzy implication of the output variable,  $\alpha(k)$  is shown as the following [15].

$$\mu_B(\alpha(k)) = \max_{j=1}^c \left\{ \min_{i=1}^d \left[ \mu_{A_i}(\theta(k)), \mu_{A_i \rightarrow B^j}(\theta(k), \alpha(k)) \right] \right\} \quad (23)$$

The output variable,  $\gamma(k)$  can be similarly shown as following [15]:

$$\mu_B(\gamma(k)) = \max_{j=1}^c \left\{ \min_{i=1}^d \left[ \mu_{A_i}(\theta(k)), \mu_{A_i \rightarrow B^j}(\theta(k), \gamma(k)) \right] \right\} \quad (24)$$

where  $c$  is the fuzzy rule, and  $d$  is the dimension of input variables.

The defuzzifier maps a fuzzy set  $B$  to a crisp point  $\alpha \in V$ . The fuzzy logic system with the center of gravity is defined as the following [15].

$$\alpha(k) = \frac{\sum_{l=1}^n \bar{y}^l \mu_B(\alpha^l(k))}{\sum_{l=1}^n \mu_B(\alpha^l(k))} \quad (25)$$

The defuzzifier of the output variable,  $\gamma(k)$  can be similarly shown as following:

$$\gamma^*(k) = \frac{\sum_{l=1}^n \bar{y}^l \mu_B(\gamma^l(k))}{\sum_{l=1}^n \mu_B(\gamma^l(k))} \quad (26)$$

$n$  is the number of outputs.  $\bar{y}^l$  is the value of the  $l$ th output.  $\mu_B(\alpha^l(k))$  and  $\mu_B(\gamma^l(k))$  represent the membership of  $\alpha^l(k)$  and  $\gamma^l(k)$  in the fuzzy set  $B$ , respectively. Substituting  $\alpha(k)$  of equation (25) in equation (20) and  $\gamma^*(k)$  of equation (26) in equations (17) and (18) allows us to configure the fuzzy estimator. A flow chart of the computation for the application of the proposed input estimation algorithm is shown in Figure 2.

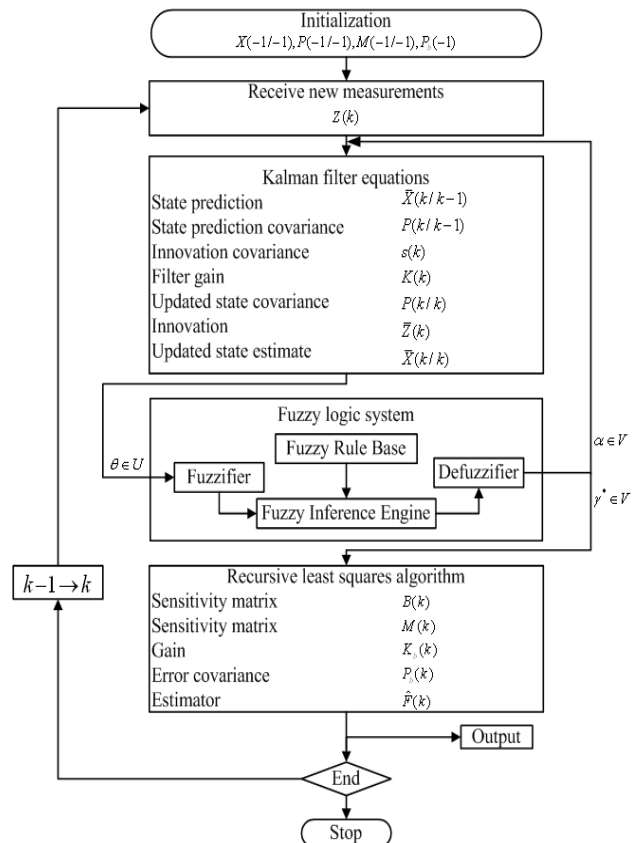


Fig. 2: Flowchart of the intelligent fuzzy input estimation algorithm.

## IV. RESULTS AND DISCUSSION

In order to demonstrate the accuracy and efficiency of the presented method in estimating the unknown input force, several numerical simulations are investigated. A rigid bar structural system is shown in figure 1(a). The displacement and joined forces of the rigid bar structural system is shown in figure 1(b). Assuming the bar  $A_1 - B_1$  is massless rigid bar with a total span,  $L = 0.6m$ . The bar  $A_2 - B_2$  is uniform mass rigid bar with a total mass,  $m=10kg$ . The stiffness,  $k$ , against the summation of the vertical stresses is  $10^6 N/m$ . The damping coefficient,  $c = 10 N s/m$ . The estimation algorithm includes the fuzzy Kalman filter technique and the fuzzy weighted recursive least square method. The initial conditions and other parameters of simulation are shown as follows:  $P(0/0) = diag[10^4]$ ,  $\hat{F}(0) = 0$ ,  $P_b(0) = 10^4$ , and  $M(0)$  is assumed to be a zero matrix. The sampling interval,  $\Delta t = 0.001$  and the total simulation time,  $t_f = 0.5s$ . The weighting factor,  $\gamma$ , is adaptive weighted, fuzzy weighting and constant weighted function respectively

*Example: Periodic sinusoidal dynamic loading*

The periodic sinusoidal dynamic loading is applied on the rigid bar structural system. These sinusoidal dynamic loading is shown as follows:

$$F(t) = -\frac{4}{3}F_0 \sin \bar{\omega}t \quad (27)$$

where  $F_0 = 40N$ , and  $\bar{\omega} = 10Hz$ . The periodic sinusoidal dynamic loading of the structure system is determined by using the presented approach when considering the influence due to the initial processing noise and the measurement noise of the system. The initial processing noise variance,  $Q_w(0) = 10^8$ . The measurement noise variance,  $R_v = \sigma^2 = 10^{-18}$ . By applying the active dynamic reaction which contains noise to the presented algorithm, the estimation result of the periodic sinusoidal dynamic loading can be obtained and plotted in Fig. 3. The coarse estimation result in the initial response on account of the larger initial processing noise variance is shown in Fig. 3. The presented estimator has the property of faster convergence with the regulated processing noise variance in the estimation process. Fig. 4 shows that the estimator has great tracking performance for what the larger output variable,  $\alpha(k)$ , can be chosen to generate the larger processing noise variance,  $Q_w(k)$ , according to equation (20). The estimator has great in reducing the effect of noise for what the smaller output variable,  $\alpha(k)$ , can be chosen to generate the smaller processing noise variance,  $Q_w(k)$ , when the unknown is steady input system. Fig. 5 shows that the smaller weighting factor can be chosen in the fuzzy recursive least square method when the larger unknown input the system. It should be noted that the faster the forgetting effect is, the lower the smoothing effect will be, that is, it introduces oscillation. The fuzzy weighting factor  $\gamma(k)$  is employed to compromise between the

upgrade of tracking capability and the loss of estimation precision.

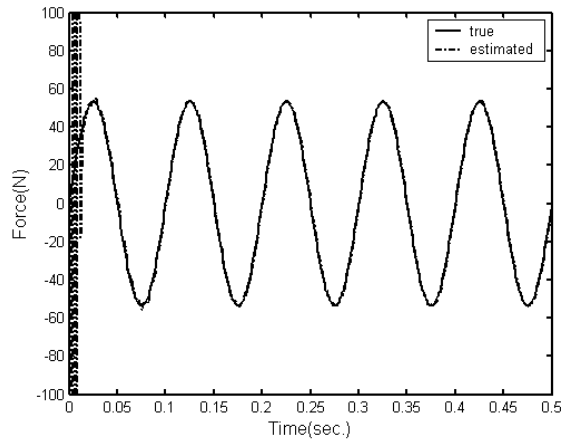


Fig. 3: Estimation results for the periodic sinusoidal dynamic loading ( $Q_w(0) = 10^8$ ,  $R_v = \sigma^2 = 10^{-18}$ ,  $\bar{\omega} = 10Hz$ ).

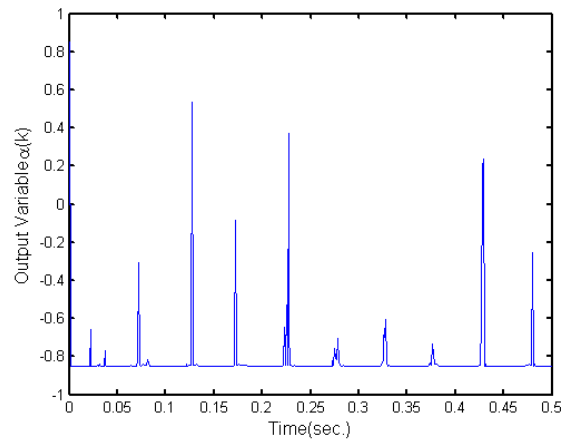


Fig. 4: The varied value of output variable,  $\alpha(k)$ .

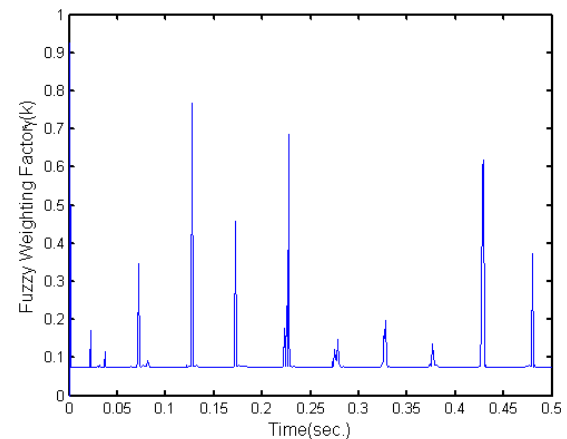


Fig. 5: The varied value of fuzzy weighting factor,  $\gamma(k)$ .

The estimates of  $F(t)$  using the adaptive weighted factor are plotted in Fig. 6. The estimates of  $F(t)$  using the constant

weighting factor,  $\gamma = 0.1$  and  $0.95$ , are plotted in Fig. 7 and 8 respectively. The estimation results show that the tracking performance of estimators is not good enough, and they are not suitable in reducing the effect of the noise. Moreover, the rapider input frequency ( $20\text{Hz}$ ) was taken into account during the simulation process. Figure 9 demonstrates that the presented estimator has good estimation resolution under larger initial process noise variance and more rapid input frequency.

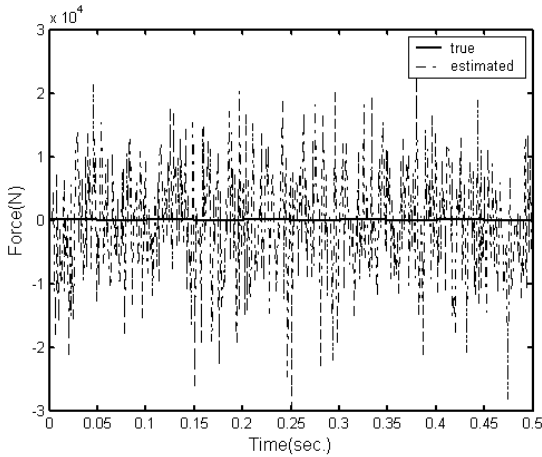


Fig. 6: Estimation result of the periodic sinusoidal dynamic loading using the adaptive weighted factor,  $\gamma$  ( $Q_w(0) = 10^8$ ,  $R_v = \sigma^2 = 10^{-18}$ ).

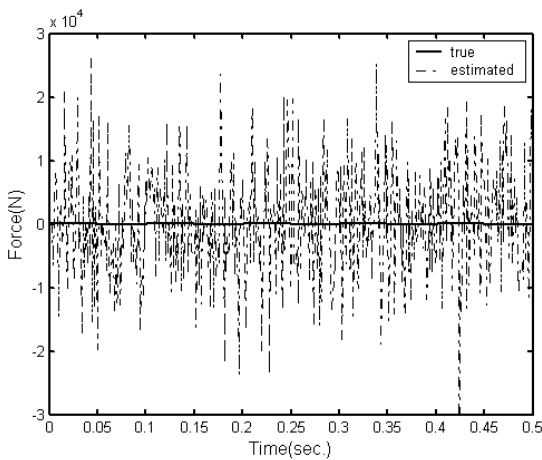


Fig. 7: Estimation result of the periodic sinusoidal dynamic loading using the constant weighted factor,  $\gamma = 0.1$  ( $Q_w(0) = 10^8$ ,  $R_v = \sigma^2 = 10^{-18}$ ).

### V. CONCLUSIONS

This study proposed the intelligent fuzzy input estimator combing the fuzzy Kalman filter technology with the fuzzy weighting recursive least square method to estimate the periodic sinusoidal dynamic loading of a rigid bar structural system under improper initial modeling and measurement noise conditions. In this study, the suitable values of the

measurement noise covariance and the fuzzy weighting factor can be chosen to cope with the uncertain restricted conditions, such as the precision of actual measuring equipments, and the simplified or imprecise mathematical model, and to enhance the estimation performance.

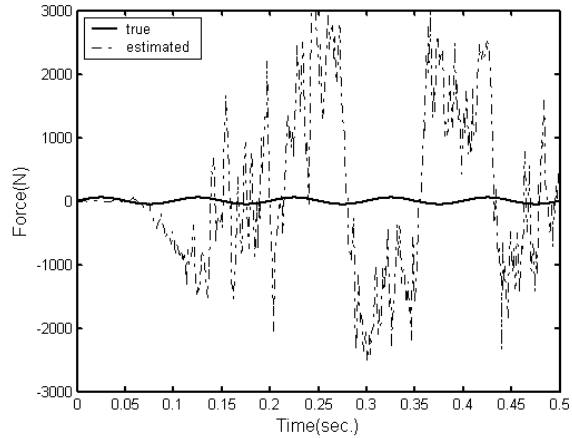


Fig. 8: Estimation result of the periodic sinusoidal dynamic loading using the constant weighted factor,  $\gamma = 0.95$  ( $Q_w(0) = 10^8$ ,  $R_v = \sigma^2 = 10^{-18}$ ).

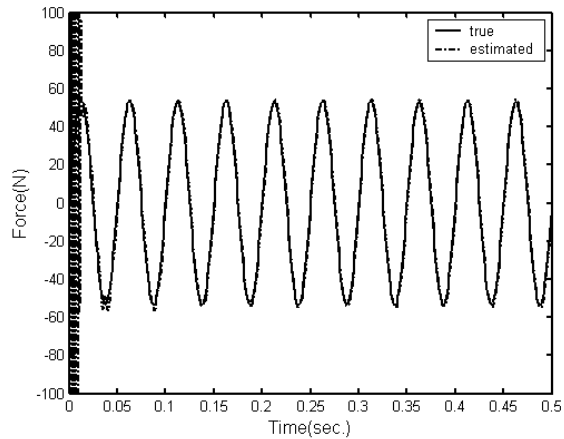


Fig. 9: Estimation results for the periodic sinusoidal dynamic loading ( $Q_w(0) = 10^8$ ,  $R_v = \sigma^2 = 10^{-18}$ ,  $\bar{\omega} = 20\text{Hz}$ ).

According to the results of the simulation and the computation, the fuzzy estimator has the properties of fast tracking and efficiency against noise because it is accelerated and weighted by the accelerating factor,  $\alpha(k)$  and weighting factor,  $\gamma(k)$  of the proposed method based on the fuzzy logic inference system. The superior estimation capability of the proposed method was shown by comparing it with the adaptive weighting function and the constant weighting factor input estimation method. Results also demonstrate that this method has the properties of better target tracking capability, more efficient noise and measurement bias reduction and faster convergence in the initial response. Future study includes the extension of this study to non-uniform loading distributing using the analytical method.

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## REFERENCES

- [1] P. E. Hollandsworth, and H. R. Busby, "Impact force identification using the general inverse technique," *International Journal of Impact Engineering*, vol. 8, pp. 315-322, 1989.
- [2] H. R. Busby, and D. M. Trujillo, "Numerical solution to a two-dimensional heat conduction problem," *International Journal for Numerical Methods in Engineering*, vol. 21, pp. 349-359, 1985.
- [3] M. L. Wang, T. J. Kreitinger, "Identification of force from response data of a nonlinear system," *Soil Dynamics and Earthquake Eng.*, vol. 13, pp. 267-280, 1994.
- [4] H. Inoue, N. Ikeda, K. Kishimoto, T. Shibuya, and T. Koizumi, "Inverse analysis of the magnitude and direction of impact force," *JSME Int. J., Series A*, vol. 38, pp.84-91, 1995.
- [5] C. H. Huang, "A Generalized Inverse Force Vibration Problem for simultaneously estimating the time-dependent external forces," *Applied Mathematical Modelling*, vol.29, pp. 1022-1039, 2005.
- [6] J. F. Doyle, "A genetic algorithm for determining the location of structural impacts", *Exp. Mech.*, vol. 34, no. 1, pp. 37-44, 1994.
- [7] M. T. Martin, and J. F. Doyle, (1996), "Impact force location in frame structures", *Int. J. of Impact Eng.*, vol. 18, no. 1, pp. 79-97, 1996.
- [8] P. C. Tuan, S. C. Lee, and W. T. Hou, "An efficient on-line thermal input estimation method using Kalman Filter and recursive least square algorithm," *Inverse Problem Eng.*, vol. 5, pp. 309-333, 1997.
- [9] P. C. Tuan and W. T. Hou, "Adaptive robust weighting input estimation method for the 1-D inverse heat conduction problem," *Numer. Heat Transfer*, vol. 34, pp. 439-456, 1998.
- [10] M. H. Lee, and T. C. Chen, "Blast load input estimation of the Medium Girder Bridge using inverse method," *Defence Science Journal*, Vol. 58, no. 1, pp. 46-56, 2008.
- [11] T. C. Chen, and M. H. Lee, "Inverse active wind load inputs estimation of the multilayer shearing stress structure," *Wind and Structures, An International Journal* vol. 11, no. 1, pp.19-33, 2008.
- [12] T. C. Chen, and M. H. Lee, "Research on moving force estimation of the bridge structure using the adaptive input estimation method," *Electronic Journal of Structural Engineering* vol. 8, pp.20-28, 2008.
- [13] P. C. Tuan, L. W. Fong, and W. T. Huang, "Analysis of On-Line Inverse Heat Conduction Problems," *Journal of Chung Cheng Institute of Technology*, vol. 25, no. 1, pp. 59 -73, 1996.
- [14] P. Mario, *Dynamics of Structures*, Hsiao-Yuan Publication Company Limited; 1986.
- [15] L. X. Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1994.

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