

# Instability of Ties in Compression

T. Cornelius

**Abstract**—Masonry cavity walls are loaded by wind pressure and vertical load from upper floors. These loads results in bending moments and compression forces in the ties connecting the outer and the inner wall in a cavity wall. Large cavity walls are furthermore loaded by differential movements from the temperature gradient between the outer and the inner wall, which results in critical increase of the bending moments in the ties. Since the ties are loaded by combined compression and moment forces, the loadbearing capacity is derived from instability equilibrium equations. Most of them are iterative, since exact instability solutions are complex to derive, not to mention the extra complexity introducing dimensional instability from the temperature gradients. Using an inverse variable substitution and comparing an exact theory with an *analytical* instability solution a method to design tie-connectors in cavity walls was developed. The method takes into account constraint conditions limiting the free length of the wall tie, and the instability in case of pure compression which gives an optimal load bearing capacity. The model is illustrated with examples from praxis.

**Keywords**—Masonry, tie connectors, cavity wall, instability, differential movements, combined bending and compression.

## I. INTRODUCTION

DESIGN capacity of tie connectors in masonry cavity walls is determined by four basic failure modes: tension failure in steel, anchorage failure in the mortar joint, instability in compression (column buckling) and bending failure due to constraint condition. The first two failure criteria appear in pure tension and are not within the scope of this paper, but can be studied in for instance [1]. The last failure criterion, the bending criterion, gives a minimum length limitation of the wall tie, which is explained by the fact that the differential movement between the outer and inner wall is constant, which leads to a higher moment, when the free binder length decreases. An exact iterative solution of this phenomenon is derived in [2] to be presented and used as comparison with the model derived in this paper. The instability failure due to second order deformation appears when the free tie length reaches the critical column length, mainly determined by the stiffness and slenderness of the tie-connector. This failure mode may be determined from instability column equations. For the analytical model developed, the method of Euler/Ostenfeld is used [3].

In addition to the above mentioned failure modes, it should be mentioned that several types of tie connectors are built by a combination of ties easing the mounting of the tie connectors and to some extend to optimize the load bearing

capacity, see for instance [4]. The connections in combined tie connectors also may lead to failure; therefore the connection also needs to be designed. The model introduced also considers that combined tie connectors can be developed to obtain reduced effect from differential movements by establishing a charnier between the main tie connector and the constraint in the wall. Hence the moment in the tie connector is reduced. This may be accomplished by U-formed connectors or other system, see [1] and [5] and Fig. 1.

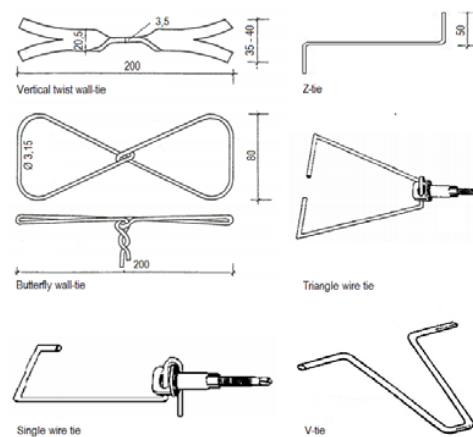


Fig. 1 Typical tie connectors in masonry cavity walls

Earlier test on cavity walls mainly examines the loadbearing capacity of the masonry used, see for instance [1] and [7], therefore the main verification is based on comparison with exact solutions. Nevertheless some test may be found in [6] and [7], which, however, may be used for verifying the level of the load capacity, but not for a detailed parameter study.

## II. STATE OF THE ART THEORIES

### A. Column Instability

Instability failure is determined by the magnitude of deformation, and by having high amount of deformation, the second order effect will be dominating the failure, and the failure will approach Euler failure, i.e. stiffness properties are decisive, while lower deformation relatively to the strength parameters, the failure will be determined by the material strength parameter i.e. yield strength. The level of deformation depends on the curvature  $\kappa$  of the loaded column, i.e. the tie connector, by (1):

$$\kappa = \frac{0.8f_y}{4\pi E} \quad (1)$$

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where  $f_y$  is the yield strength of the tie connector and  $E$  is the Modulus of Elasticity, and the stiffness parameter (or slenderness ratio) given by (2):

$$s = \frac{l_s}{i} \quad (2)$$

where  $l_s$  is the free column length having  $l_s = kl$  where  $l$  is the free tie length, i.e. opening between outer and inner wall and  $k$  being the column effect depending on the constraint condition, and  $i$  is the radius of inertia.

In the case of  $\kappa s \leq 0.5$  the failure is dominated by strength parameters, hence the decisive failure, i.e. the critical normal force  $N_{cr}$ , may be determined by Ostenfeld column equation, see [3], as given in (3):

$$N_{cr} = 0.8 f_y A \left( 1 - \kappa \left( \frac{l_s}{i} \right)^2 \right) \quad (3)$$

where  $A$  is the area of the cross section.

When  $\kappa s \geq 0.5$  the failure is dominated by the stiffness of the tie connector and the critical failure load may be determined by Euler given in (4):

$$N_{cr} = \frac{\pi^2 EI}{l_s^2} \quad (4)$$

where  $I$  is the area moment of inertia.

#### B. Differential Movements

Observing the tie connector in compression and pre-deformed by influence of temperature movements between outer and inner wall, the second failure type may be determined by the solution derived in [2], see (5):

$$N \left( \frac{1}{A} + \phi \frac{\Delta r}{2W} \right) \leq f_y \quad (5)$$

where  $N$  is the actual normal force acting on the tie,  $\Delta r$  being the maximal differential movement of the tie,  $W$  the section modulus as a function of the diameter  $d$  and  $\phi$  the column factor. An exact solution for ties is derived in [2] determined by (6):

$$\phi = \frac{\sinh bl_s}{bl_s \cosh bl_s - \sinh bl_s} \quad (6)$$

where

$$b = \sqrt{\frac{N}{EI}} \quad (7)$$

We observe that the solution for the column factor depends on  $N$ , which results in a solution that is to be solved by

iteration.

### III. THEORETICAL APPROACH

The theory for the failure caused by differential movements determines the minimum length of the tie connection. The theory should to be solved by iteration, but it is possible to present the result visually by using variable substitution by defining the variable  $u$  in (8):

$$u = bl_s \quad (8)$$

Then  $\phi$  is determined using (6) as a function of  $u$ , and  $b$  is determined by (7) inserting  $N$  from (5) rewritten as given in (9):

$$N = f_y / \left( \frac{1}{A} + \phi \frac{\Delta r}{2W} \right) \quad (9)$$

The corresponding free tie length is finally determined by (10):

$$l = \frac{l_s}{k} = \frac{b}{ku} \quad (10)$$

The result of the theories presented is illustrated in Fig. 2 for a typical Z-tie connection fixed in both the outer and inner wall corresponding to a column effect factor  $k=0.5$  and the material and geometrical properties given in Table I.

TABLE I  
MATERIAL AND GEOMETRICAL PROPERTIES

d	$f_y$	E	$\Delta r$
mm	MPa	MPa	mm
4	600	210.000	2.0

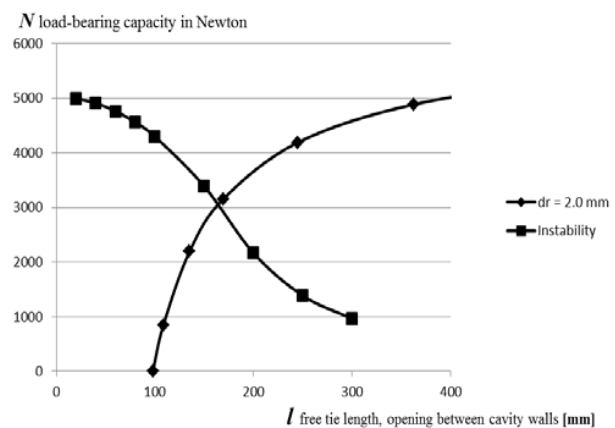


Fig. 2 Example of exact solution for instability and differential movements using parameters from Table I

The column instability theory takes both failure due to the strength and failure due to instability into consideration. It is

therefore assumed, that if a combined failure criteria for the differential movement failure and the instability failure are introduced in the general form in (11), this may lead to an analytical solution, which can give comparable results.

$$\left(\frac{N}{N_{cr}}\right)^{\eta} + \frac{M}{M_u} \leq 1 \quad (1)$$

where the power  $\eta$  is a correlation factor reducing the second order effect when moment from differential movements dominate the failure,  $N_{cr}$  is determined by the instability failure using (3) and (4) and the ultimate bending capacity  $M_u$  determined by (12):

$$M_u = Wf_y \quad (12)$$

and  $M$  is the additional torque contribution from the differential movements determine from (13):

$$M = \Delta r \cdot \alpha EI / l^2 \quad (13)$$

where  $\alpha$  is the factor taking the constraint condition into account, i.e. having  $\alpha = 6$  for a tie fixed at both ends.

Since  $N$  in (11) is the only unknown variable,  $N$  may be determined analytically from any free tie length by the reformulated (14):

$$N = N_{cr} e^{\beta/\eta} \quad (14)$$

defining  $\beta$  in (15) as:

$$\beta = \left( \ln \left( 1 - \frac{M}{M_u} \right) \right) \quad (15)$$

#### IV. ANALYSIS

Solving (14) using an elastic stress distribution, having the Section Modulus  $W = \pi l^3/32$ , and keeping  $\eta = 1.0$  for simplicity, we get the conservative result presented in Fig. 3.

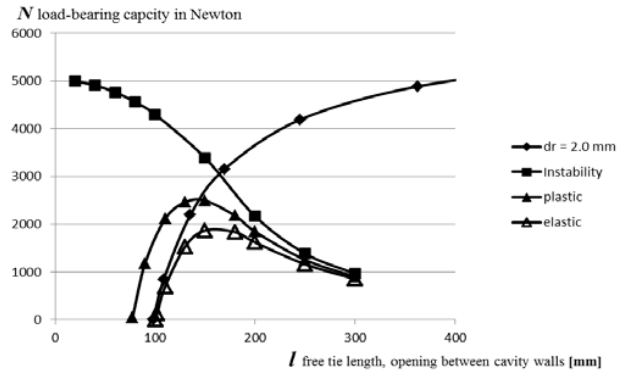


Fig. 3 Elastic and plastic solution new theory compared to exact solution

Similarly the solution using the plastic section Modulus  $W = d^3/6$  is shown in Fig. 3. This may be defended assuming compressions in cross section. As observed, to some extent, the solution gives better results compared to instability failure, but gives results on the unsafe side, when considering differential movements or failure purely dominated by bending failure. It may be argued that a plastic solution is acceptable, but due to the fact that the slenderness ratio  $l/d$  is large, this needs to be confirmed by test data.

A tie connector structure where the moment from differential movements is small compared to the critical normal force,  $N_{cr}$ ,  $\eta$  will decrease approaching 1. In the case of relatively high moments,  $\eta$  increases resulting in relatively higher capacity. This can be approximated by the semi-empirical (16):

$$\eta = a \left( \frac{\Delta r \cdot N_{cr} - M}{N_{cr}} \right)^b \quad (16)$$

Using (16) by optimizing on the empirical parameters  $a$  and  $b$ , we find using elastic section modulus a very fine comparison with the exact solution having  $a = 1$  and  $b = 1.3$  as shown in Fig. 4 denoted 'modified elastic'.

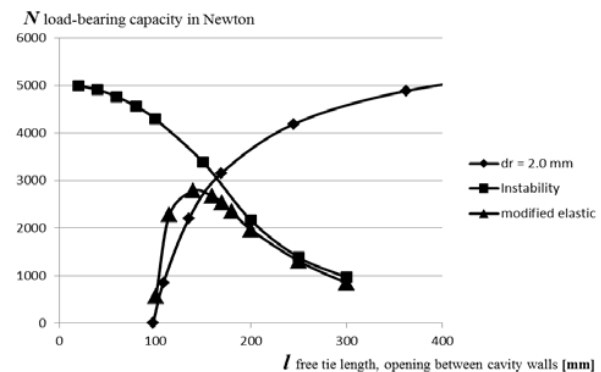


Fig. 4 Modified theory using empirical values for the power  $\eta$

Nevertheless, a more simple solution even not that theoretically founded, may be found by having a constant value of  $\eta = 3,0$  as shown in Fig. 5 denoted 'elastic  $\eta = 3$ '.

We observe that using the simple empirical fitting it tends to give results a somewhat on the unsafe side. Nevertheless this might be accepted using the same argument that failure is dominated by bending and compression. Further contrary to the plastic solution, this method approaches the elastic solution, when normal force approaches zero.

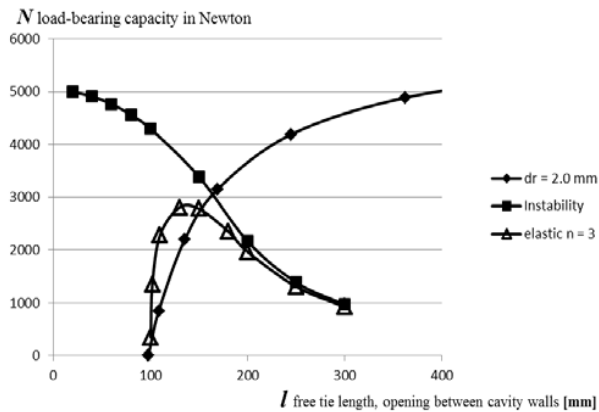


Fig. 5 Elastic solution using constant  $\eta$  compared to exact solution

Further we need to analyze solutions for various temperature gradients.  $\Delta r$  is normally in the range of 0.5 to 8.0 mm since  $\Delta r$  may be determined by (17):

$$\Delta r = \Delta \cdot \sqrt{\left(\frac{w}{2}\right)^2 + h^2} \quad (17)$$

$\Delta = \Delta T \cdot c$  being the initial differential movement,  $w$  being the width of the wall and  $h$  the height. For an ordinary brick masonry wall, we have a typically temperature gradient  $c$  of  $6 \cdot 10^{-6}$  mm/C° and a maximum temperature difference  $\Delta T = 35$  C° and for a wall with a width of 20 m and height 15m we obtain  $\Delta r = 4.5$ mm.

If we use the same properties from Table I and then let  $\Delta r = 4.5$  mm and calculate the results using  $\eta$  from (16) and for a constant  $\eta = 3$ , we see from Fig. 6 that both calculations are stable and in compliance with the exact solution.

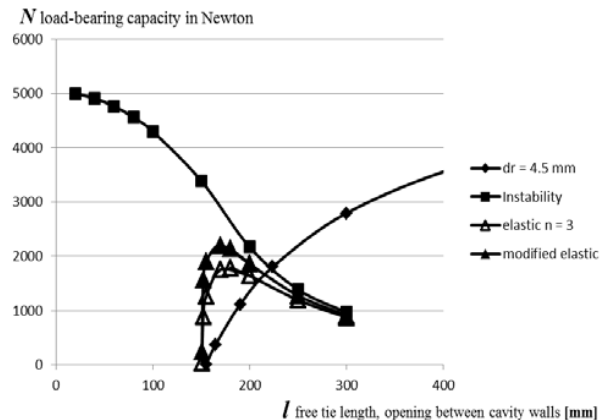


Fig. 6 Elastic solution using constant  $\eta$  compared to exact solution with differential movement  $\Delta r = 4.5$  mm

## V. DISCUSSION

As we did observe the semi-empirical method given in (16) did not optimize or make the solution more stable even though more accurate data were achieved, it is recommended for simplicity's sake to use the elastic solution with a constant correction factor  $\eta = 3,0$  in praxis. This gives a little higher bending resistance with increasing normal force, which is reasonable to assume tending the plastic solution, when the stress distribution are mainly in compression.

It is not reasonable to assume fully plastic behavior in the case of pure bending, due to the slenderness ratio of general used tie, at least special conditions has to be established taken into consideration the slenderness ratio.

## VI. CONCLUSION

An analytically theoretical model to design cavity wall tie connectors was developed. The model shows good correlation with the exact solutions and is easier to use in design.

The theory may be extended to be used for plastic solution, but tests are recommended before such a procedure is introduced for design.

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