

# Influences of Thermal Relaxation Times on Generalized Thermoelastic Longitudinal Waves in Circular Cylinder

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**Abstract**—This paper is concerned with propagation of thermoelastic longitudinal vibrations of an infinite circular cylinder, in the context of the linear theory of generalized thermoelasticity with two relaxation time parameters (Green and Lindsay theory). Three displacement potential functions are introduced to uncouple the equations of motion. The frequency equation, by using the traction free boundary conditions, is given in the form of a determinant involving Bessel functions. The roots of the frequency equation give the value of the characteristic circular frequency as function of the wave number. These roots, which correspond to various modes, are numerically computed and presented graphically for different values of the thermal relaxation times. It is found that the influences of the thermal relaxation times on the amplitudes of the elastic and thermal waves are remarkable. Also, it is shown in this study that the propagation of thermoelastic longitudinal vibrations based on the generalized thermoelasticity can differ significantly compared with the results under the classical formulation. A comparison of the results for the case with no thermal effects shows well agreement with some of the corresponding earlier results.

**Keywords**—Wave propagation; longitudinal vibrations; circular cylinder; generalized thermoelasticity; Thermal relaxation times.

## I. INTRODUCTION

THE propagation of waves in thermoelastic circular cylinder bodies with thermal relaxation times based on the generalized theory of thermoelasticity has been the topic for a lot of investigations in recent years. These investigations are considered to be important because of their possible extensive applications in various branches of science and technology. The fields of applications are astrophysics, geophysics, acoustics, plasma physics and seismology.

When an isotropic and homogeneous elastic solid is subjected to a thermal disturbance, the effect is instantaneous at a location distant from the source in the classical linear thermoelastic theory. This means that the thermal wave propagates at infinite velocity which is a physically unreasonable result. Two generalized thermoelastic theories are proposed to eliminate that paradox and correct the classical theory on the assumption that a thermal wave propagates at finite velocity.

These theories are: (i) Lord and Shulman's theory (L-S) [15], which involves one thermal relaxation time and is based on a new law of heat conduction to replace Fourier's law. The

heat equation is replaced by a hyperbolic one which ensures finite speeds of propagation for heat and elastic waves. (ii) the second theory of generalized thermoelasticity with two thermal relaxation times was first introduced by Green and Lindsay (G-L) [12]. In this theory the temperature rates considered among the constitutive variables. This theory also predicts finite speeds of propagation as in (L-S) theory. For a history of the thermodynamic theories on heat equation and a review of generalized thermoelasticity theories, see [3]. The elasto-dynamical analysis of cylinders and cylindrical shells without considering thermal effects is reviewed in [20], while introducing the thermal effects without thermal relaxation times is considered in [4], [8], [16] and [21]. Moreover, several problems reveal interesting phenomena characterizing the generalized thermoelasticity and its applications on a different hypothesis have been considered in [12], [11] and [19].

In this study, we consider the problem of thermoelastic longitudinal vibrations of an infinite circular cylinder. The treatment is in the framework of the generalized thermoelasticity theories with one or two thermal relaxation times. A formulation of generalized thermoelasticity which combines both generalized theories is used. The frequency equation has been derived in the form of a determinant involving Bessel functions. The roots of the frequency equation give the value of the characteristic circular frequency as function of the wave number for different values of the thermal relaxation times. These roots, which correspond to various modes, are numerically calculated and presented graphically. It is found that, due to the thermal relaxation times, the amplitude of both the elastic and thermal waves are higher than that of conventional theories. Moreover, it is noted that if the relaxation times are put equal zero in our results, we arrive at the results of Chadwich [4].

Recently, Abbas et al. [1] illustrated the effects of thermal relaxations times on thermoelastic interactions in an infinite orthotropic elastic medium with a cylindrical cavity. Abbas et al. [2] investigated the thermoelastic interaction in an infinite fibre-reinforced anisotropic plate containing a circular hole using Lord and Shulman model of the generalized thermoelastic theory with one thermal relaxation time. Chitikireddy et al. [7] presented a theoretical study of transient ultrasonic guided waves generated by concentrated heating of the outer surface of an infinite anisotropic hollow circular cylinder. Elhagary [9] considered the problem of a thermoelastic infinitely long hollow cylinder in the context of the theory of generalized thermoelastic diffusion with one

relaxation time. Hosseini et al. [13] are attempted to present an analytical solution to study the thermal and mechanical waves using theory of coupled thermoelasticity without energy dissipation based on Green–Naghdi model. Ponnusamy et al. [18] studied the wave propagation in an infinite, homogeneous, transversely isotropic solid cylinder of arbitrary cross-section Fourier expansion collocation method, within the frame work of linearized, three-dimensional theory of thermoelasticity

## II. BASIC EQUATIONS

The fundamental equations of the generalized thermoelasticity are given by:

\*Equations of motion

$$\sigma_{ij,j} = \rho \ddot{u}_i \quad (1)$$

\*Heat conduction equation

$$\kappa T_{,ii} = \rho c_e (\dot{T} + t_o \ddot{T}) + \gamma T_o (\dot{u}_{k,k} + t_o \delta \ddot{u}_{k,k}) \quad (2)$$

\*Stress-displacement-temperature relations

$$\sigma_{ij} = 2\mu \varepsilon_{ij} + [\lambda \varepsilon_{kk} - \gamma(T + t_1 \dot{T})] \delta_{ij} \quad (3)$$

\* Strain- displacement relations:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (3')$$

where

$\rho$	Constant mass density,	$T$	Absolute temperature,
$\lambda, \mu$	Lamé's constants,	$T_o$	Reference temperature,
$t_o$	First thermal relaxation time,	$C$	Velocity of light,
$t_1$	Second thermal relaxation time,	$\sigma_{ij}$	Stress tensor,
$\chi$	Thermal diffusivity of the material,	$\gamma$	$(3\lambda + 2\mu)\alpha_o$ ,
$\kappa$	Thermal conductivity coefficient,	$t$	Time,
$c_e$	Specific heat at constant temperature,	$\delta_{ij}$	Kroneker delta.
$\alpha_o$	Coefficient of linear thermal expansion,		

A superposed dot denotes differentiation with respect to time and a comma followed by a subscript denoted partial differentiation with respect to the corresponding coordinate.

Each value of the parameter  $\delta$  in the heat conduction equations (2) corresponds to either one of the three different theories:

(i) Classical dynamical coupled theory, (C-D) [11].

$$\text{i.e., } t_o = t_1 = 0, \quad \delta = 0,$$

(ii) Lord and Shulman's theory, (L-S) [15].

$$\text{i.e., } t_o > 0, \quad t_1 = 0, \quad \delta = 1,$$

(iii) Green and Lindsay's theory, (G-L) [12].

$$\text{i.e., } t_1 \geq t_o \geq 0, \quad \delta = 0.$$

Eliminating  $\sigma_{ij}$  from equations (1) and (3), we find the equations of motion presented by the displacement components in the absence of body forces and heat sources for a thermoelastic medium have the form:

$$(\lambda + \mu)u_{k,ki} + \mu \nabla^2 u_i - \gamma(T + t_1 \dot{T})_{,i} = \rho \ddot{u}_i. \quad (4)$$

The displacement components  $u_i$  may be resolved into the sum of an irrotational and a solenoidal parts as follows:

$$u_i = \varphi_{,i} + e_{irs} A_{s,r} \quad i, r, s = 1, 2, 3 \quad (5)$$

where  $\varphi$  is the scalar potential and  $A_i$  is the vector potential.

Substituting from (5) into (4) and (3), we may get the following system of equations:

$$\nabla^2 \phi - \frac{1}{c_1^2} \ddot{\phi} = \frac{\gamma}{\rho} (T + t_1 \dot{T}) \quad (6)$$

$$\kappa \nabla^2 T = \rho c_e (\dot{T} + t_o \ddot{T}) + \gamma T_o (\dot{\phi} + t_o \delta \ddot{\phi}) \quad (7)$$

$$\nabla^2 A_i - \frac{1}{c_2^2} \ddot{A}_i = 0 \quad (8)$$

where  $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$  is the velocity of the propagation of the elastic longitudinal wave,  $c_2 = \sqrt{\mu/\rho}$  is the velocity of the elastic transverse wave and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r} + \frac{\partial^2}{\partial z^2}$$

## III. FORMULATION AND SOLUTION OF THE PROBLEM

Let  $(r, \theta, z)$  be the cylindrical polar coordinates referred to the axis of the cylinder. We make use of the potential function. In the longitudinal disturbances the tangential component of displacement vanishes identically, i.e.,  $u_\theta = 0$ .

The vector potential  $A_i$  has the form  $(0, \psi, 0)$ , and the displacement components are given in terms of the scalar functions  $\phi(r, z, t)$  and  $\psi(r, z, t)$  by:

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial \psi}{\partial z}, \quad u_\theta = 0, \quad u_z = \frac{\partial \phi}{\partial z} + \frac{1}{r} \frac{\partial(r\psi)}{\partial r}. \quad (9)$$

To investigate wave motion in the thermoelastic circular cylinder, we consider solutions of equations (6)-(8) of the form:

$$\begin{pmatrix} \phi(r, z, t) \\ \psi(r, z, t) \\ T(r, z, t) \end{pmatrix} = \begin{pmatrix} \Phi(r) \\ \Psi(r) \\ F(r) \end{pmatrix} \exp[i(qz - \omega t)]. \quad (10)$$

where  $\omega$  is the frequency and  $q$  is the wave number.

Substituting equations (10) into (6)-(8), we obtain:

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r} \frac{d\Phi}{dr} + \alpha_1\Phi = \alpha_2F, \quad (11)$$

$$\frac{d^2F}{dr^2} + \frac{1}{r} \frac{dF}{dr} + \alpha_3F = \alpha_4 \left( \frac{d^2\Phi}{dr^2} + \frac{1}{r} \frac{d\Phi}{dr} - q^2\Phi \right), \quad (12)$$

$$\frac{d^2\Psi}{dr^2} + \frac{1}{r} \frac{d\Psi}{dr} + \left( \alpha_5 - \frac{1}{r^2} \right) \Psi = 0. \quad (13)$$

With  $\Phi, \Psi$  and  $T$  are functions of  $r$  only, and we have set:

$$\begin{aligned} \alpha_1 &= \frac{\omega^2}{c_1^2} - q^2, \quad \alpha_2 = \frac{\gamma}{\rho c_1^2} (1 - i\omega t_1) \\ \alpha_3 &= \frac{i\omega}{\chi} (1 - i\omega t_o) - q^2, \quad \alpha_4 = \frac{-i\omega\gamma T_o}{\kappa} (1 - i\omega\delta t_o), \\ \alpha_5 &= \frac{\omega^2}{c_2^2} - q^2, \quad \chi = \frac{\kappa}{\rho c_e} \end{aligned} \quad (14)$$

The boundary conditions are:

$$[\sigma_{rr}]_{r=a} = 0, \quad [\sigma_{rz}]_{r=a} = 0, \quad \left[ \frac{\partial T}{\partial r} \right]_{r=a} = 0. \quad (15)$$

Introducing the following non-dimensional variables.

$$\begin{aligned} (r^*, u_r^*, u_z^*) &= \frac{c_2}{\chi} (r, u_r, u_z), \quad (t^*, t_o^*, t_1^*) = \frac{c_2^2}{\chi} (t, t_o, t_1), \\ \omega^* &= \frac{\chi}{c_2^2} \omega, \quad q^* = \frac{\chi}{c_2} q, \quad (\phi^*, \psi^*) = \frac{c_2^2}{\chi^2} (\phi, \psi), \quad T^* = \frac{T}{T_o} \end{aligned} \quad (16)$$

One may get the equations (11)-(13) in the following form (dropping the superscript "\*" for convenience)

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r} \frac{d\Phi}{dr} + m_1\Phi = m_2F \quad (17)$$

$$\frac{d^2F}{dr^2} + \frac{1}{r} \frac{dF}{dr} + m_2F = m_4 \left( \frac{d^2\Phi}{dr^2} + \frac{1}{r} \frac{d\Phi}{dr} - q^2\Phi \right) \quad (18)$$

$$\frac{d^2\Psi}{dr^2} + \frac{1}{r} \frac{d\Psi}{dr} + \left( m_3 - \frac{1}{r^2} \right) \Psi = 0 \quad (19)$$

where we have define

$$m_1 = \frac{\omega^2 c_2^2}{c_1^2} - q^2, \quad m_2 = \frac{T_o \gamma}{\rho c_1^2} (1 - i\omega t_1), \quad m_3 = i\omega (1 - i\omega t_o) - q^2,$$

$$m_4 = \frac{-i\omega\gamma}{\kappa} (1 - i\omega\delta t_o), \quad m_5 = \omega^2 - q^2$$

The general solutions of equations (17), (18) and (19) assuming  $\Phi, \Psi$  and  $F$  to be finite as  $r \rightarrow 0$  may be expressed as

$$F = \frac{1}{m_2} [A(m_1 - \zeta_1^2) J_o(\zeta_1 r) + B(m_1 - \zeta_2^2) J_o(\zeta_2 r)] \exp[i(qz - \omega t)] \quad (20)$$

$$\Phi = [A J_o(\zeta_1 r) + B J_o(\zeta_2 r)] \exp[i(qz - \omega t)] \quad (21)$$

$$\Psi = C J_1(\zeta_3 r) \exp[i(qz - \omega t)] \quad (22)$$

where

$$\begin{aligned} \zeta_1^2 &= \frac{1}{2} (s_1 - \sqrt{s_1^2 - 4s_2}), \quad \zeta_2^2 = \frac{1}{2} (s_1 + \sqrt{s_1^2 - 4s_2}), \\ \zeta_3^2 &= \omega^2 - q^2, \quad s_1 = m_1 + m_3 - m_2 m_4, \\ s_2 &= m_1 m_3 + m_2 m_4 q^2 \end{aligned} \quad (23)$$

and  $A, B$  and  $C$  are arbitrary constants and  $J_o, J_1$  are Bessel functions of the orders zero and one respectively. From equations (20), (21) into (9) the displacement components are given by

$$u_r = i [A i \zeta_1 J_1(\zeta_1 r) + B i \zeta_2 J_1(\zeta_2 r) - C q J_1(\zeta_3 r)] \exp[i(qz - \omega t)], \quad (24)$$

$$u_z = [A i q J_o(\zeta_1 r) + B i q J_o(\zeta_2 r) + C \zeta_3 J_o(\zeta_3 r)] \exp[i(qz - \omega t)]. \quad (25)$$

Substituting from equations (24), (25) and (21) into the boundary conditions (15), we get a set of three homogeneous linear equations connecting  $A, B$  and  $C$  as

$$\begin{aligned} A \left\{ \frac{2\zeta_1}{a} J_1(\zeta_1 a) - \left[ \left( \frac{\lambda}{\mu} + 2 \right) \zeta_1^2 + \frac{\lambda}{\mu} q^2 \right. \right. \\ \left. \left. + \frac{\gamma T_o}{m_2 \mu} (1 - i t_1 \omega) (m_1 - \zeta_1^2) \right] J_o(\zeta_1 a) \right\} \\ + B \left\{ \frac{2\zeta_2}{a} J_1(\zeta_2 a) - \left[ \left( \frac{\lambda}{\mu} + 2 \right) \zeta_2^2 + \frac{\lambda}{\mu} q^2 \right. \right. \\ \left. \left. + \frac{\gamma T_o}{m_2 \mu} (1 - i t_1 \omega) (m_1 - \zeta_2^2) \right] J_o(\zeta_2 a) \right\} \\ + C \left\{ 2i q \left[ \frac{1}{a} J_1(\zeta_3 a) - \zeta_3 J_o(\zeta_3 a) \right] \right\} = 0 \end{aligned} \quad (26)$$

$$A [2q \zeta_1 J_1(\zeta_1 a)] + B [2q \zeta_2 J_1(\zeta_2 a)] + C [i(q^2 - \zeta_3^2) J_1(\zeta_3 a)] = 0 \quad (27)$$

$$A \left[ (m_1 - \zeta_1^2) \zeta_1 J_1(\zeta_1 a) \right] + B \left[ (m_1 - \zeta_2^2) \zeta_2 J_1(\zeta_2 a) \right] = 0 \quad (28)$$

Eliminating  $A$ ,  $B$  and  $C$  from equations (26)-(28), we get the determinant of this set must vanish leading to the frequency equation (which is called "dispersion Relation" in the area of physics and engineering) as

$$\Delta = \begin{vmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & 0 \end{vmatrix} = 0 \quad (29)$$

where we have set

$$\begin{aligned} X_{11} &= \frac{2\zeta_1}{a} J_1(\zeta_1 a) - \zeta_1 J_0(\zeta_1 a), \\ X_{12} &= \frac{2\zeta_2}{a} J_1(\zeta_2 a) - \left\{ \left( \frac{\lambda}{\mu} + 2 \right) \zeta_2^2 + \frac{\lambda}{\mu} q^2 \right. \\ &\quad \left. + \frac{\gamma T_o}{m_2 \mu} (1 - it_1 \omega) (m_1 - \zeta_2^2) J_0(\zeta_2 a) \right\}, \\ X_{13} &= 2iq \left[ \frac{1}{a} J_1(\zeta_3 a) - \zeta_3 J_0(\zeta_3 a) \right], \\ X_{21} &= 2q \zeta_1 J_1(\zeta_1 a), \quad X_{22} = 2q \zeta_2 J_1(\zeta_2 a) \\ X_{23} &= i(q^2 - \zeta_3^2) J_1(\zeta_3 a), \quad X_{31} = (m_1 - \zeta_1^2) \zeta_1 J_1(\zeta_1 a), \\ X_{32} &= (m_1 - \zeta_2^2) \zeta_2 J_1(\zeta_2 a) \end{aligned}$$

#### IV. NUMERICAL RESULTS AND DISCUSSIONS

The calculation of the roots of the frequency equation (29) represents a major task and requires a rather extensive effort of numerical computation. Calculations have been carried out in electronic computer for the case of Copper as an example, for which the material constants at 27 °C are as follows [12]:

$$\begin{aligned} \rho &= 8.93 \text{ gm/cm}^3, \quad \lambda = 1.387 \times 10^{12} \text{ dyne/cm}^2, \\ \mu &= 0.448 \times 10^{12} \text{ dyne/cm}^2, \quad c_e = 0.56 \text{ cal/deg}, \\ \kappa &= 0.918 \text{ cal/(s cm deg)}, \quad \alpha_e = 1.67 \times 10^{-8} \text{ 1/deg} \end{aligned}$$

The first three roots of the frequency equation (29) are calculated using bisection method (halving interval). Figs. (1-3) present the first three modes of the real part of the dimensionless frequency  $\Omega$  versus the wave number  $q$  according to the Lord-Shulman (L-S) theory (when  $t_o = 1.0, 1.5, 2.0$ ). Fig. (4-6) show the first three modes of the real part of the dimensionless frequency  $\Omega$  for green-Lindsay (G-L) theory versus different values of the wave number  $q$  when  $t_o = 1.0, t_1 = 2.0, 3.0, 4.0$ . Figs. (7-9) illustrate the first, second and third two modes of the real part of the dimensionless frequency  $\Omega$  versus different values of  $q$  for

(L-S) and (G-L) theories. Figs. (10 and 11) exhibit the first three modes of the real and imaginary parts of the dimensionless frequency  $\Omega$  for (D-L) theory versus different values of the wave number  $q$ . Figs. (12 and 13) display the first three modes of the imaginary parts of the dimensionless frequency  $\Omega$  for (L-S) and (G-L) theories versus different values of the wave number  $q$ .

It is observed that the real part of the frequency  $\text{Re}(\omega)$  increases as the wave number  $q$  increases as well as the thermal relaxation times increase for all modes. Furthermore, it is easy to see that the real part of the frequency in (G-L) theory is always smaller than that of (L-S) theory which is also smaller than that of (C-D) theory. The influence of the second relaxation time is more significant when  $t_1 > t_o$ , but when the two thermal relaxation times are equal, we find that  $\text{Re}(\omega)$  versus  $q$  is almost the same for (L-S) and (G-L) theories. The imaginary part of the frequency  $\text{Im}(\omega)$  is smaller than the real part  $\text{Re}(\omega)$ .

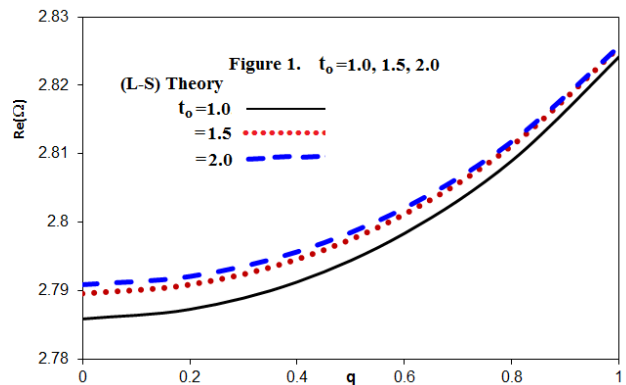


Fig. 1 The first mode of the real part of the dimensionless frequency  $\Omega$  for (L-S) theory versus different values of the wave number  $q$  when  $t_o = 1.0, 1.5, 2.0$

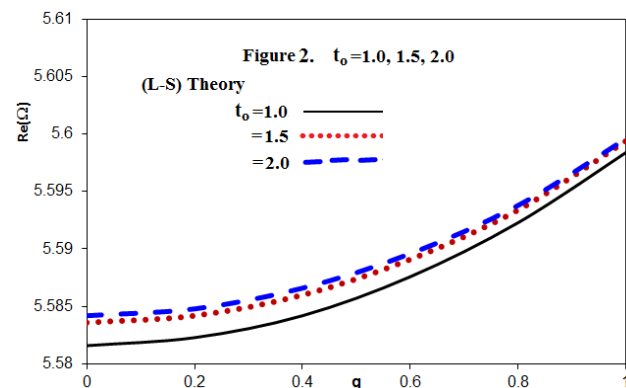


Fig. 2 The second mode of the real part of the dimensionless frequency  $\Omega$  for (L-S) theory versus different values of the wave number  $q$  when  $t_o = 1.0, 1.5, 2.0$

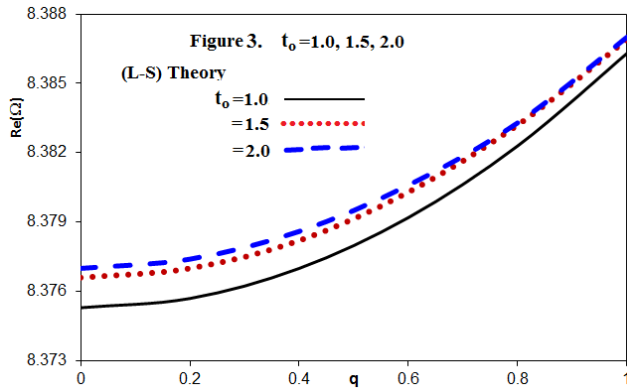


Fig. 3 The third mode of the real part of the dimensionless frequency  $\Omega$  for (L-S) theory versus different values of the wave number  $q$  when  $t_0 = 1.0, 1.5, 2.0$

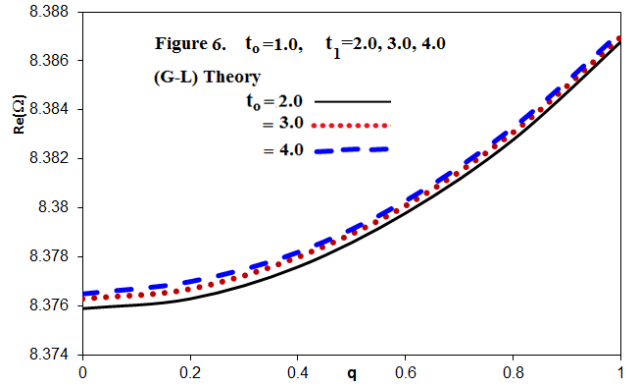


Fig. 6 The third mode of the real part of the dimensionless frequency  $\Omega$  For (G-L) theory versus different values of the wave number  $q$  when  $t_0 = 1.0, t_1 = 2.0, 3.0, 4.0$

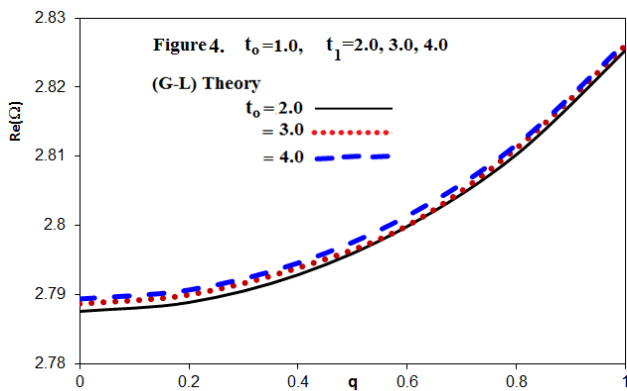


Fig. 4 The first mode of the real part of the dimensionless frequency  $\Omega$  For (G-L) theory versus different values of the wave number  $q$  when  $t_0 = 1.0, t_1 = 2.0, 3.0, 4.0$

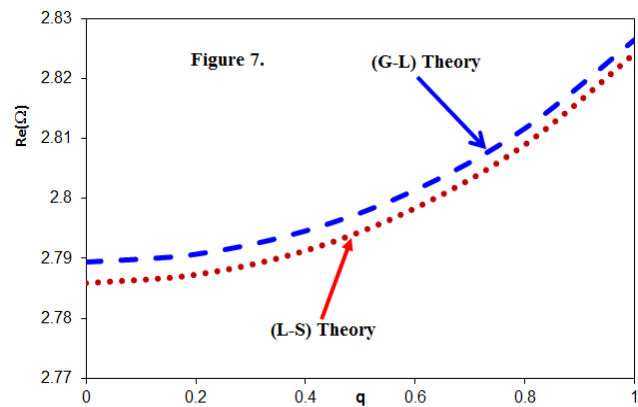


Fig. 7 The first two modes of the real part of the dimensionless frequency  $\Omega$  versus different values of  $q$  for (L-S) and (G-L) theories

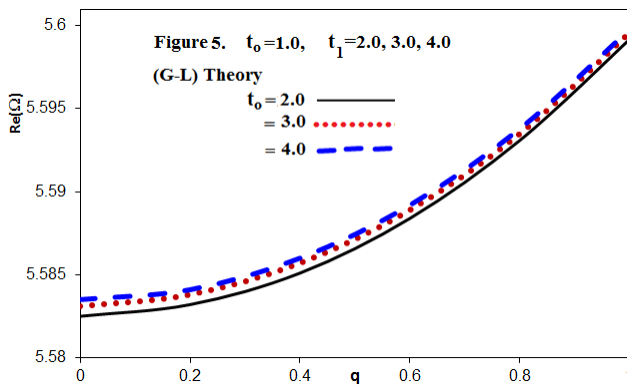


Fig. 5 The second mode of the real part of the dimensionless frequency  $\Omega$  For (G-L) theory versus different values of the wave number  $q$  when  $t_0 = 1.0, t_1 = 2.0, 3.0, 4.0$

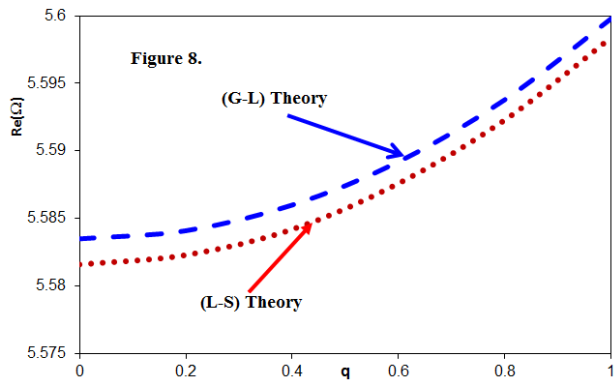


Fig. 8 The second two modes of the real part of the dimensionless frequency  $\Omega$  versus different values of  $q$  for (L-S) and (G-L) theories

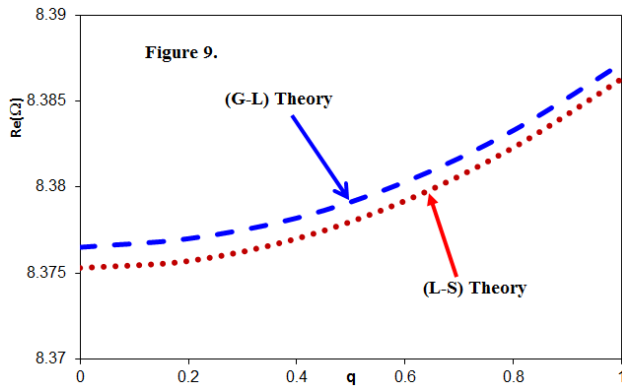


Fig. 9 The third two modes of the real part of the dimensionless frequency  $\Omega$  versus different values of  $q$  for (L-S) and (G-L) theories

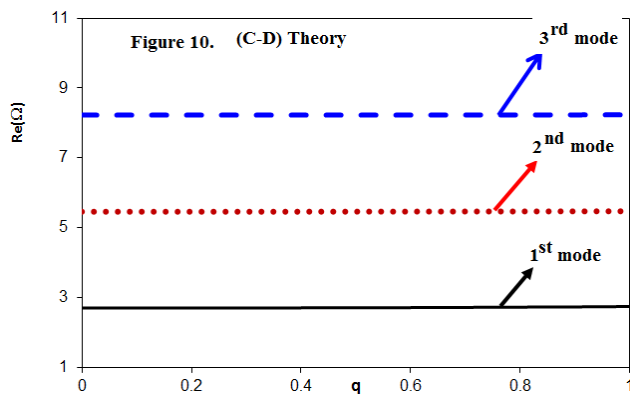


Fig. 10 The first three modes of the real part of the dimensionless frequency  $\Omega$  For (D-L) theory versus different values of the wave number  $q$

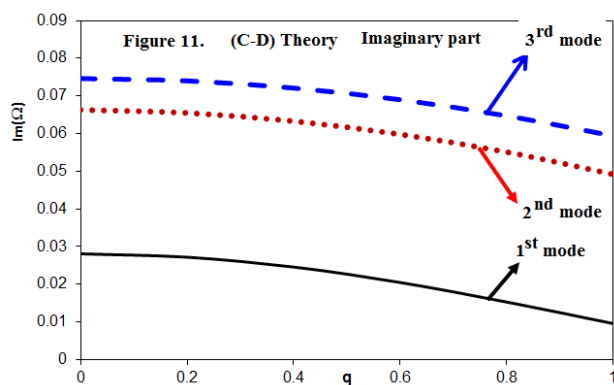


Fig. 11 The first three modes of the imaginary part of the dimensionless frequency  $\Omega$  for (C-D) theory versus different values of the wave number  $q$

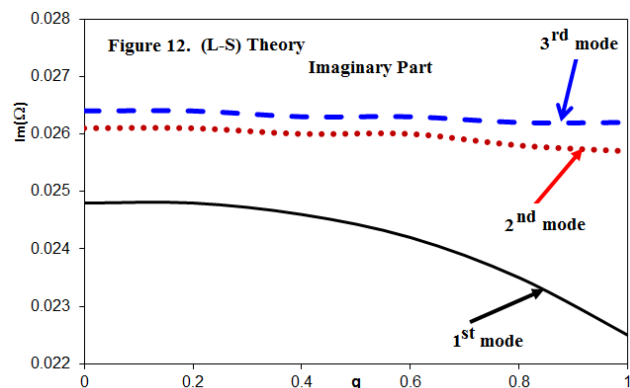


Fig. 12 The first three modes of the imaginary part of the dimensionless frequency  $\Omega$  For (L-S) theory versus different values of the wave number  $q$

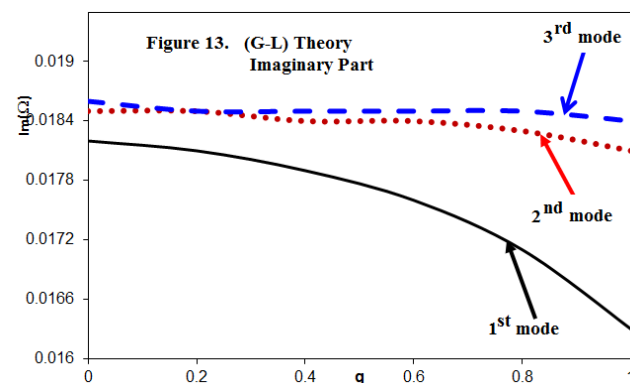


Fig. 13 The first three modes of the imaginary part of the dimensionless frequency  $\Omega$  for (G-L) theory versus different values of the wave number  $q$

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