Influence of Fiber Packing on Transverse Plastic Properties of Metal Matrix Composites

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***Abstract*—**The present paper concerns with the influence of fiber packing on the transverse plastic properties of metal matrix composites. A micromechanical modeling procedure is used to predict the effective mechanical properties of composite materials at large tensile and compressive deformations. Microstructure is represented by a repeating unit cell (RUC). Two fiber arrays are considered including ideal square fiber packing and random fiber packing defined by random sequential algorithm. The micromechanical modeling procedure is implemented for graphite/aluminum metal matrix composite in which the reinforcement behaves as elastic, isotropic solids and the matrix is modeled as an isotropic elastic-plastic solid following the von Mises criterion with isotropic hardening and the Ramberg-Osgood relationship between equivalent true stress and logarithmic strain. The deformation is increased to a considerable value to evaluate both elastic and plastic behaviors of metal matrix composites. The yields strength and true elastic-plastic stress are determined for graphite/aluminum composites.

***Keywords*—** Fiber packing, metal matrix composites, micromechanics, plastic deformation, random

I. Introduction

METAL matrix composites have found many applications as constructional and functional materials in different industries. The presence of reinforcement in metal matrix materials improves the properties such as the tensile strength, creep resistance, fatigue strength, thermal shock resistance, and corrosion resistance. To design a metal matrix composite for desired working conditions, a model is required to relate the macroscopic response of such heterogeneous materials to the arrangement of the reinforcements in the microstructure and the properties of constituents and interaction between them. The micromechanical model provides an efficient procedure to determine properties of composite materials.

Initially, Adams [1] studied the transverse mechanical behavior of a unidirectional continuous fiber-reinforced composite with fibers of circular cross section by adopting finite element cell models under plane strain conditions. A simple geometrical cell composed of matrix and inclusion material is repeated by appropriate boundary conditions to represent a composite with a periodic microstructure. Good agreement was achieved between calculated and experimental stress-strain curves for a rectangular fiber arrangement. Sun and Chen [2] developed a simple micromechanical model to describe the elastic-plastic behavior of fibrous composites. A square cross section for fibers and plane strain conditions are assumed to simplify the micromechanical model. Ability to predict composite properties from those of the constituents is always an attractive idea. However, the micromechanical approach is often hindered by simplifying assumptions in geometrical representation of microstructures. Experimental observations have shown that the fiber strings are distributed in the random pattern. Hence, a model is required to analysis the large deformation of three-dimensional RUC with random fiber packing to determine the effective properties of metal matrix composites.

The present research works determines the influence of fiber packing on the plastic properties of metal matrix composites. The micromechanical modeling procedure is implemented to evaluate the response of unidirectional continuous fiber composites subjected to finite axial deformation. The microstructure of the metal matrix materials is represented by a RUC. Two fiber arrangements are considered including ideal square fiber packing and random fiber packing defined by random sequential algorithm. RUC subjected to tensile and compressive uniaxial deformation to determine the effective properties of metal matrix composite considering the periodicity conditions on the deformation of RUC boundary surfaces. The Volume averaging scheme is implemented to apply the local macroscopic deformation gradient tensor to the RUC assigned to the microstructure. The micromechanical modeling procedure is implemented for graphite/aluminum metal matrix composite in which the reinforcement behaves as elastic, isotropic solids and the matrix was modeled as an isotropic elastic-plastic solid following the von Mises criterion with isotropic hardening and the Ramberg-Osgood relationship is assumed between equivalent true stress and logarithmic strain. RUC is subjected to uniaxial large deformation increased to a considerable value to evaluate both elastic and plastic behaviors of metal matrix composites. The yields strength and true elastic-plastic stress are determined for graphite/aluminum composites.

II. Microstructure

The microstructures of unidirectional fiber reinforced composites are commonly described by three fiber arrangement including square, hexahedral and random fiber-packing patterns. The micromechanical results for linear anisotropic elastic materials revealed that the calculated axial and shear elastic modulus are dependent on the fiber packing [3]. Since the microstructures with square and hexahedral fiber-packing patterns are idealized geometrical representation for fiber arrangement, the microstructure with random fiber packing yields more accurate results. At large plastic deformation of anisotropic materials, the results highly

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depends on the fiber packing and for some fiber arrangement, the deformation locking may be observed at lower strain.

Since the heterogeneities are orders of magnitude smaller than the total body, the deformation field in the vicinity of one inclusion is approximately the same as the deformation field near neighboring inclusions [4]. Experimental observations [5-7] have shown that deformation field in the vicinity of a subvolume is approximately the same as deformation field of the near neighboring subvolumes. The size of subvolume is small enough compared to the total microstructure size so that the effective properties computed from the subvolume are independent of its size and position within the microstructure. Therefore, the microstructure is represented by a periodic unit cell that deforms in a repetitive way. The periodic modeling can be quite useful, because it provides rigorous estimations with a priori prescribed accuracy for various material properties [8-10].

Microstructure shown in Fig. 1 is considered for the unidirectional continuous fiber composites. The circular fibers with identical radius are dispersed in the microstructure in a random and isotropic manner. It is assumed that the composite material has a periodic microstructure which can be obtained by translating RUC along three orthogonal axes. The fiber distribution in the unit cell is generated using the random sequential adsorption algorithm [11] which ensures a random, isotropic and homogeneous distribution for the fibers within the RUC. The random coordinates in the cross-section of microstructure are generated for the center of circular fibers with specific diameter, denoted by *d*. When a fiber intersects the boundaries of unit cell, another fiber is generated on the neighboring unit cell in order to obtain periodic unit cell. The new fiber is added to the microstructure when the distance between the centers of a given fiber and the closest fibers previously generated is greater than a minimum values (1.1*d*). Such condition prevents overlapping fibers as well as ensuring adequate mesh geometry for the matrix material located between fibers. To prevent element distortion during mesh generation, the fiber surface should not be too close (greater than 0.1*d*) to the boundary surfaces of the RUC. When such conditions are satisfied, the fiber is added to the unit cell at the generated random coordinates. The procedure is repeated until the fiber volume fraction reaches close to a pre-defined value. The square cross section is considered for unit cell ($b_2 = b_3$) and the ratio of fiber diameter to unit cell dimension (d / $2b_2$) is set to 0.05.

Aluminum alloy reinforced with stiff graphite fibers is considered. The fibers behaved as elastic, isotropic solids characterized by the elastic modulus E_f = 250 GPa and the Poisson's ratio υ_f = 0.2. The matrix is modeled as an isotropic elastic-plastic solid following the von Mises criterion with isotropic hardening. The matrix elastic constants are E_m = 70 GPa and υ_m = 0.33, and the Ramberg-Osgood relationship is assumed between equivalent true stress, σ_m^{eq}, and logarithmic strain, ε_m, i.e.,

$$\varepsilon_m = \left(\frac{\sigma_m^{eq}}{K} \right)^{1/n} \tag{1}$$

Where K = 400 MPa is the strength coefficient and n = 0.1 is the matrix strain hardening exponent [12]. Regarding these data, an initial yield stress of 225.3 MPa is obtained. The aluminum material is reinforced with 0.4 fiber volume fraction.

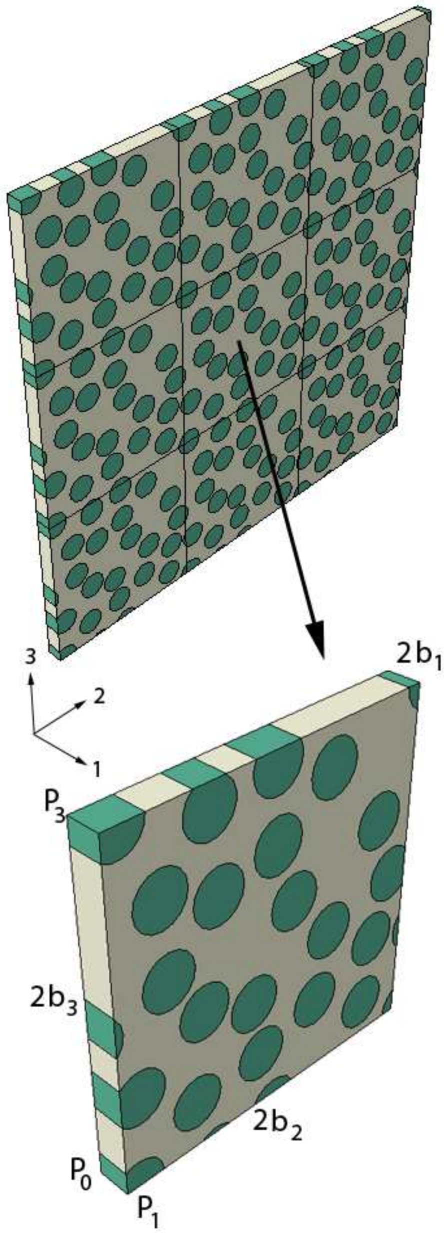


Fig. 1 Microstructure considered for metal matrix composites having random fiber packing pattern

III. Micromechanical Model

Micromechanical model provides efficient tool to characterize composite materials from known properties of their constituents and the distribution of the reinforcement in the microstructure through the analysis of a RUC. The essence in micromechanical approach is that the heterogeneous structure of the composite is replaced by a homogeneous medium with anisotropic properties.

A Lagrangian viewpoint is used to describe the material motion and the components of vectors and tensors are described in a fixed rectangular coordinate system. In the reference configuration of RUC, the position of a typical material particle is expressed with vector **X** (components X_i). In the deformed configuration at instance t, the particle moves to a position described with vector $\mathbf{x}_{(\mathbf{X},t)}$ (components x_i) corresponding to the displacement vector $\mathbf{u}_{(\mathbf{X},t)}$ (components u_i). The deformation is typically described using the deformation gradient tensor, designated by **F**, whose components are given by;

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \tag{1}$$

The reference geometry of RUC is assumed to be a rectangular prismatic volume whose surfaces are parallel to the surfaces defined in a fixed Cartesian coordinate system with origin located at the centre of RUC. As shown in Fig. 1, the initial dimension of RUC is $2b_1 \times 2b_2 \times 2b_3$. The boundary surfaces of reference geometry perpendicular to i-axis are designated with S_i^+ and S_i^- intersecting i-axis at $X_i = +b_i$ and $X_i = -b_i$, respectively. The displacement of the points located on each boundary surface is measured respect to corner points labeled as points P_0, P_1, P_2 and P_3 in Fig. 1. Such points are called reference points. The current position of points located on surface S_i^- is measured respect to point P_0, while the points located on S_1^+, S_2^+ and S_3^+ are measured respect to points P_1, P_2 and P_3, respectively. To enforce the periodicity constraint, the current position of boundary surface is described by [13]:

$$x_{i(-b_1,X_2,X_3,t)} - x_{i(t)}^{(0)} = x_{i(b_1,X_2,X_3,t)} - x_{i(t)}^{(1)} \quad i = \{1,2,3\} \tag{2a}$$

$$x_{i(X_1,-b_2,X_3,t)} - x_{i(t)}^{(0)} = x_{i(X_1,b_2,X_3,t)} - x_{i(t)}^{(2)} \quad i = \{1,2,3\} \tag{2b}$$

$$x_{i(X_1,X_2,-b_3,t)} - x_{i(t)}^{(0)} = x_{i(X_1,X_2,b_3,t)} - x_{i(t)}^{(3)} \quad i = \{1,2,3\} \tag{2c}$$

Where $x_{i(t)}^{(j)}$ is the components of current position vector of corner point P_j.

To relate the macrostructure deformation to the microstructure deformation, it is assumed that the local macroscopic deformation gradient tensor at a given point to be equal the volume averaged deformation gradient tensor of RUC assigned at that point. Using the periodicity constraining equations (1), it can be shown [13] that the macroscopic deformation gradient tensor is a function of current position of corner points P_0, P_1, P_2 and P_3 as follows:

$$F_{ij} = \frac{x_i^{(j)} - x_i^{(0)}}{2b_j} = \frac{u_i^{(j)} - u_i^{(0)}}{2b_j} + \delta_{ij} \tag{3}$$

It should be noted that no summation is considered on j superscript in Eq. (3).

An energy balance is considered to relate stress tensor in the macroscopic and microscopic scales. The internal power at macroscopic level at a given point is set equal to the internal power in RUC assigned at the corresponding point in a given deformed configuration. It was shown [13] that the energy balance results in

$$\sum_{j=1}^{3}\sum_{i=1}^{3} \dot{F}_{ji} P_{ij} = \sum_{j=1}^{3}\sum_{i=1}^{3} \frac{1}{S_i^+} \dot{F}_{ji} \int_{s_i^+} t_j ds \tag{4}$$

Where the dot superscript denotes to the time derivative, P_{ij} are the components of nominal stress tensor defined in macroscopic levels, t_j are the components of traction force and s_i^+ is the deformed geometry of boundary surface Si^+.

IV. Plastic Behavior of MMC

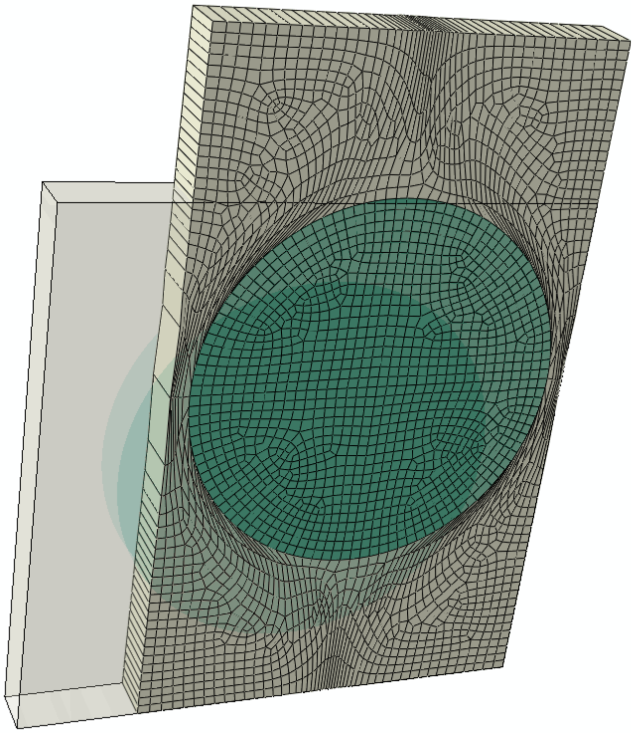
The finite element analysis is used to determine the response of RUC subjected to large deformations. Since the periodicity constraint enforces that the opposite faces deform identically, the geometry of RUC is meshed so that the number and distribution of nodes on opposite faces are identical. The RUC is meshed by eight-node linear brick elements using sweep technique along 1-axis.

The initial and maximum allowable increment sizes are set to 0.001 and 0.025 of total increment size, respectively. The increment size is automatically modified based on the convergence rate. The small value for initial increment size causes that several initial increments concern with elastic behavior and prevent the abrupt transition from elastic to plastic behaviors. Therefore, the yield strength is calculated with reasonable accuracy.

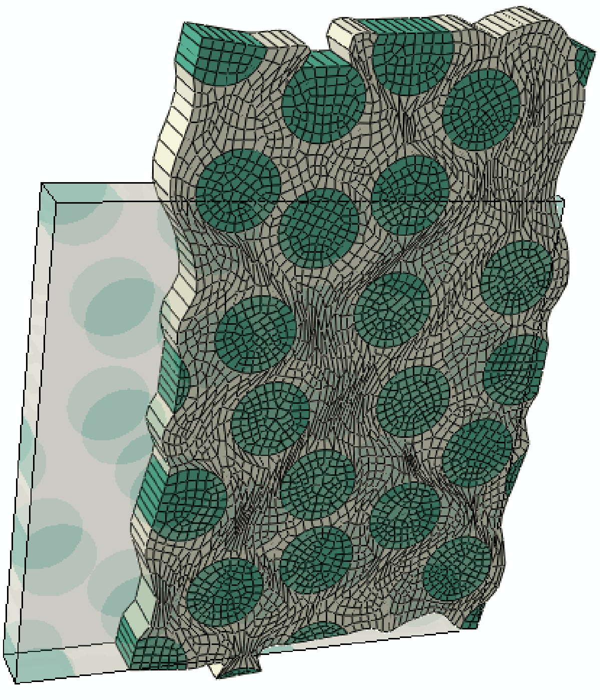
The numerical procedure is used to determine the effective macroscopic mechanical response of metal matrix composite in transverse tensile and compressive deformation mechanisms. Two fiber arrangements are considered including ideal square fiber packing and random fiber packing.

A. Tension normal to fiber direction

The RUC is subjected to a specific axial tensile deformation along 2-axis normal to the fiber direction, while the RUC is free to deform along two other axes. The displacement of reference points is described by Eq. (3), in which the value of F_{22} is increased from a unit value to a specific value, while F_{12}, F_{13}, F_{21}, F_{23}, F_{31} are F_{32} are set to zero to prevent shear deformation. The values of F_{11} and F_{33} are calculated in micromechanical modeling. Fig. 2 depicts the deformed geometry of RUC subjected to F_{22} = 1.3. Since the microstructure is extruded uniformly along fiber direction and there is no gradient on geometry, material properties and loading conditions, single raw of elements is considered along 1-axis. As shown in Fig. 2a, the boundary surfaces of RUC with square fiber packing pattern remain flat and orthogonal at the deformed status. It was verified [14] that the axial deformation causes no distortion on the boundary surfaces of RUCs having three orthogonal reflectional symmetric planes and the initial flat surfaces remain flat without any rotations.



(a)



(b)

Fig. 2 Initial and deformed geometries subjected to tensile axial deformation normal to fiber direction in the RUS of graphite/aluminum composite having 0.4 fiber volume fraction and microstructures with a) square fiber packing b) random fiber packing

Fig. 2b shows the deformed geometries for RUC with random fiber packing meshed. There is considerable displacement in the center of fibers, while negligible deformation is observed in fibers because of their high stiffness compared to matrix material. The initial boundary flat surfaces normal to the 2 and 3 axes are disported in the RUC because of non-uniform fiber distribution on the cross section. It should be noted that the plane normal to 1-axis remains flat in both RUCs due to reflectional symmetric plane.

Fig. 3 illustrates the equivalent stress in matrix material subjected to transverse stretch ratio 1.3 in graphite/aluminum composite. As shown in Fig. 3a, The von Mises stress reaches to maximum value at the mid-distant between fibers at the symmetric planes normal to fiber direction as well as the surface in the fiber/matrix interaction. Fig. 3b shows that more volume of matrix materials reaches to maximum stress between fibers compared to fiber/matrix interface in microstructure with random fiber packing pattern. As depicts in Fig 3, the more equivalent stress is observed in the microstructure with random fiber packing due to local severely deformation.

The nominal stress P_{22} is calculated using the resultant forces applied to boundary surfaces and the stretch ratio, namely,

$$P_{22} = \frac{1}{S_1^+}\int_{s_1^+} t_1 ds + \frac{1}{S_2^+}\frac{\dot{F}_{22}}{\dot{F}_{11}}\int_{s_2^+} t_2 ds + \frac{1}{S_3^+}\frac{\dot{F}_{33}}{\dot{F}_{11}}\int_{s_3^+} t_3 ds \quad (5)$$

Based on deformation gradient and nominal stress tensor, the Cauchy stress applied along tension direction is determined as,

$$\sigma_{22} = J^{-1}F_{22}P_{22} = \frac{P_{22}}{F_{11}F_{33}} \quad (6)$$

Fig. 4 depicts the variation of calculated Cauchy stress σ_{22} as the deformation proceeds for aluminum material and graphite/aluminum composites with random and square fiber-packing patterns. To verify numerical procedure used for micromechanical analysis, the properties of fiber material are set the same as matrix material in the microstructure with random fiber packing pattern and the calculated effective properties are compared to net aluminum material. The yield strength and stress in elastic and plastic regions correlate well with the properties of aluminum materials. The micromechanical model evaluates the same yield strength for composite for both microstructures. The yield strength of metal matrix composite with 0.4 fiber volume fraction has little increase respect to net aluminum materials, because some regions of matrix material experience plastic deformation at low level deformation due to local plastic deformation. Since the matrix material has more freedom to flow between fibers in random fiber packing, less stress is required to apply plastic deformation compared to microstructure with square fiber packing.

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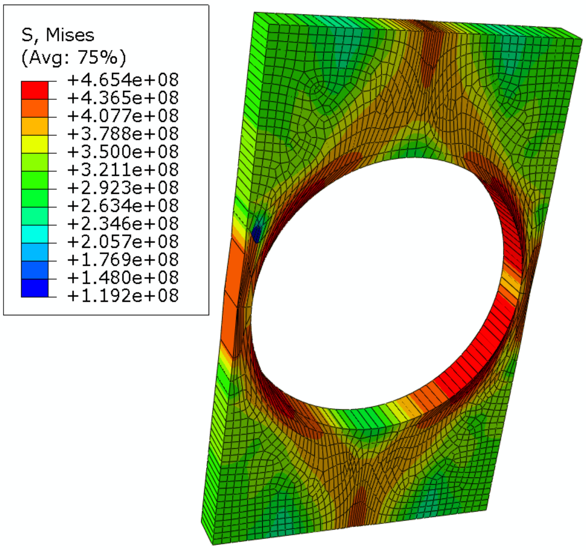
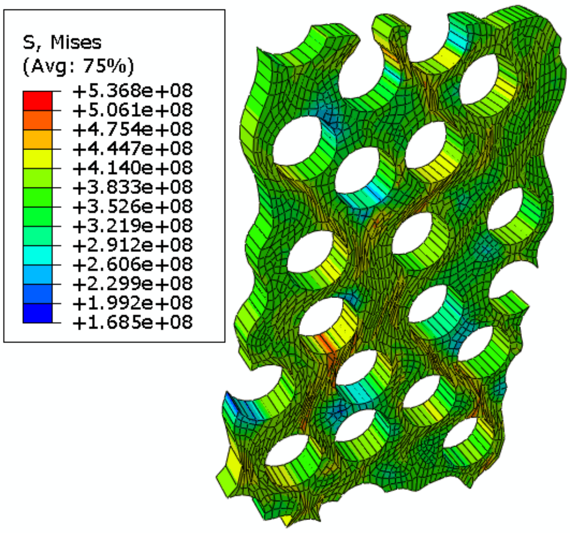


(a)



(b)

Fig. 3 Von Mises stress in matrix material subjected to transverse stretch ratio 1.3 in graphite/aluminum composite having 0.4 fiber volume fraction and microstructure with a) square fiber packing b) random fiber packing

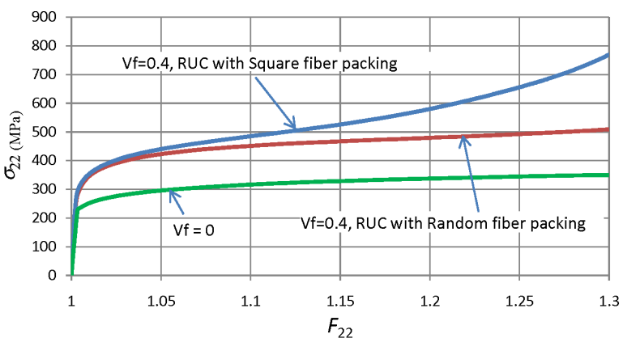


Fig. 4 Cauchy stress required to applied elastic-plastic tensile transverse deformation to net matrix material and composites with different microstructures

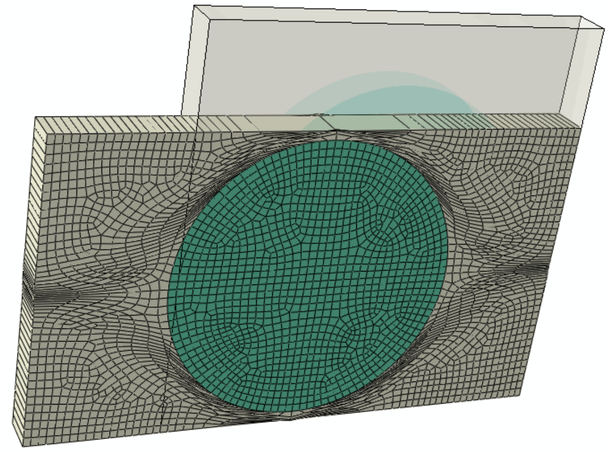
B. Compression normal to fiber direction

The micromechanical modeling procedure is used to determine the elastic-plastic transverse properties of metal matrix composites in the compressive loading conditions. The RUC is subjected to a specific compressive axial deformation along 2-axis normal to the fiber direction, while the RUC is free to deform along two other axes. The displacement of reference points is described by Eq. (3), in which the value of F_{22} is reduced from unit value up to 0.75, while F_{12}, F_{13}, F_{21}, F_{23}, F_{31} are F_{32} are set to zero to prevent shear deformation. The values of F_{11} and F_{33} are calculated in micromechanical modeling procedure.

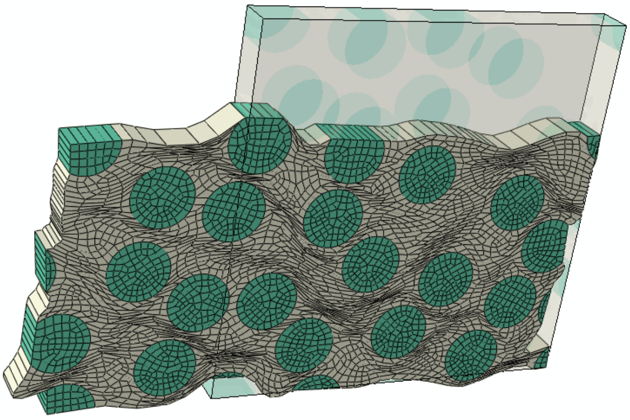
Fig. 5 depicts the deformed geometry of RUC subjected to compressive deformation. Similar to tensile deformation, the boundary surfaces of RUC with square fiber packing pattern remain flat, as shown in Fig. 5a. The compression of RUC along 2-axis makes a considerable increase in RUC dimension along 3-axis normal to fiber direction, while there is negligible dimension change of RUC along fiber direction. The high stiffness fibers make more severely deformation in matrix material when F_{22} is reduces more than 0.75 and there is high distortion in the elements considered for matrix material. Therefore, the analysis stops when F_{22} reaches to 0.75. As shown in Fig. 5b, the microstructure with random fiber packing has more flexibility because the fiber strings can move between each other and F_{22} is reduced to 0.7. Since the fibers are distributed randomly and there is no symmetric plane, the boundary surfaces normal to fiber direction are distorted from initial flat surfaces.

Fig. 6 illustrates the variation of compressive Cauchy stress as the deformation applies to aluminum material and graphite/aluminum composites with random and square fiber-packing patterns. There is considerable increase in yield strength in metal matrix composites with 0.4 fiber volume fraction compared to net matrix material. Both microstructures have the same yield strength. Similar to tensile deformation, the microstructure with random fiber packing requires lower stress to apply plastic deformation compared to microstructure with square fiber packing, because fibers in random pattern can move between each other and lower stress is observed in matrix material located between fibers. As shown in Fig. 6, there is considerable stress rise in microstructures with square fiber packing as the compressive plastic deformation applies, because of fiber distant decrease.

The logarithmic strain is used to describe the large deformation in plastic deformation and It is defined as the logarithm of the ratio of current length to initial length. Fig. 7 shows the graph of true (Catchy) stress-logarithmic strain for net matrix material and graphite/aluminum composites with microstructures having fiber arrangement in square and random packing patterns. The microstructure with square fiber-packing patterns has similar plastic properties in tension and compression, while the microstructures with random fiber-packing patterns require more plastic stress in compressive plastic deformation than tensile plastic deformation. Table 1 lists the true stress values required to apply the same plastic strain in tension and compression. The difference of true stress is a reasonable value for microstructures having random fiber-packing patterns.



(a)



(b)

Fig. 5 Initial and deformed geometries subjected to compressive axial deformation normal to fiber direction in the RUC of graphite/aluminum composite having 0.4 fiber volume fraction and microstructure with a) square fiber packing b) random fiber packing

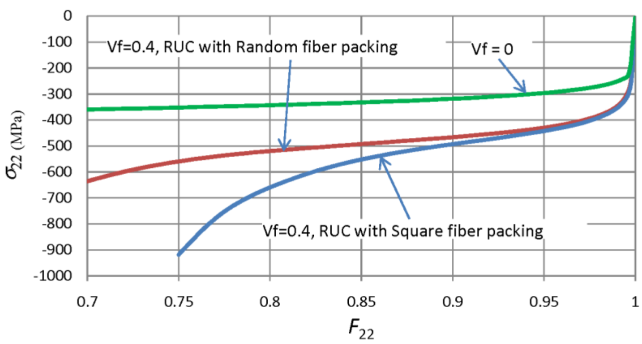


Fig. 6 Cauchy stress required to applied elastic-plastic compressive deformation to net matrix material and composites with different microstructures

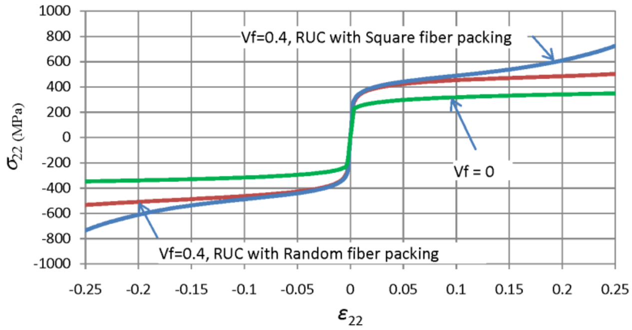


Fig. 7 True stress – logarithmic plastic strain graph for aluminum material and graphite/aluminum composite having 0.4 fiber volume fraction and different microstructures

TABLE I
TRUE STRESS VALUES REQUIRED TO APPLY THE SAME PLASTIC STRAIN TO THE MICROSTRUCTURES WITH FIBER ARRANGEMENT IN SQUARE AND RANDOM PACKING PATTERNS

Fiber packing	Logarithmic plastic strain	True stress (MPa)	True stress difference (MPa)
Random	-0.1 +0.1	-463.48 +453.36	+10.12
Square	-0.1 +0.1	-488.37 +488.74	0.37

V. CONCLUSIONS

The micromechanical technique provides an efficient tool to characterize transverse plastic properties of metal matrix composites at tensile and compressive large deformations. The present procedure is useful to develop or verify the finite strain constitutive laws for metal matrix composites based on the distribution of the reinforcement in the microstructure and the properties of constituents and interaction between them. The composite microstructure is described by RUC with two fiber distributions including ideal square and random fiber-packing patterns. Both microstructures predict the same yield strength for composite materials. However, as the plastic strain applies to microstructures, it is shown that different stress requires applying tensile or compressive deformation and stress difference becomes considerable value for more plastic strain. Since the fibers can move between each other in axial deformation of the microstructure with random fiber packing, lower stress requires applying plastic strain compared to microstructure with square fiber-packing patterns. The microstructure with square fiber packing has similar plastic properties in tension and compression, while the microstructures with random fiber-packing patterns requires more plastic stress in compressive plastic deformation than tensile plastic deformation.

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