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Improved robust stability criteria for discrete-time neural networks

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Abstract—In this paper, the robust exponential stability problem of uncertain discrete-time recurrent neural networks with time-varying delay is investigated. By constructing a new augmented Lyapunov-Krasovskii function, some new improved stability criteria are obtained in forms of linear matrix inequality (LMI). Compared with some recent results in literature, the conservatism of the new criteria is reduced notably. Two numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed results.

Keywords—Robust exponential stability, delay-dependent stability, discrete-time neutral networks, time-varying delays.

I. Introduction

RECENTLY, recurrent neural networks (RNNs) have received intensive interest due to their successful applications in various areas including such as pattern recognition, image processing, fixed-point computation, and so on. However, because of the finite switching speed of neurons and amplifiers, time delays, both constant and time-varying, are often unavoidable in various engineering, neural networks, large-scale, biological, and economic systems. Since the occurrence of time delays may cause poor performance or instability. Therefore, stability analysis of neural networks has received much attention. Up to now, various stability conditions have been obtained, and many excellent papers and monographs have been available (see [1]-[7]). On the other hand, when designing the neural networks or in the implementation of neural systems, due to the thermal noise in the electronic devices, modeling error, the deviation of vital data, or the random fluctuations and so on, the convergence of a neural network may often be destroyed. These unavoidable uncertainty can be classified into two types: that is, stochastic disturbances and parameters uncertainties. As pointed out in [8] that, while modeling real nervous systems, both of the stochastic disturbances and parameters uncertainties are probably the main resources of the performance degradations of the implemented neural networks. Therefore, the studies on robust stability of delayed neural network with parameters uncertainty have been a hot reach direction. Many sufficient conditions, either delay-dependent or delay-independent, have been proposed to guarantee the global robust stability for different class of delayed neural networks (see [8]-[18]).

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It should be pointed out that most results concerning dynamics analysis problems for RNNs with mixed time-delays have been on continuous-time models, but few in discrete time. In practice, when implementing and applications of neural networks, discrete-time neural networks play a more important role than their continuous-time counter- parts in today's digital world, such as numerical computation, computer simulation. And they can ideally keep the dynamic characteristics, functional similarity, and even the physical or biological reality of the continuous-time networks under mild restriction. Thus, the stability analysis problems for discrete-time neural networks have received more and more interest, and some stability criteria have been proposed (see [8],[19]-[29]). In [28], Liu and Wang et al., researched a class of discrete-time RNNs with time-varying delay, and proposed a delay-dependent condition guaranteeing the global exponential stability. This result obtained in [28] has been improved by Song and Wang in [21]. And the results obtained in [21] are further improved in [22] by considering some useful terms. Recently, some new improved criteria are derived in [23], [24], [29], respectively.

In this paper, some mew improved delay-dependent stability criteria are obtained via constructing a new augmented Lyapunov-Krasovskii function. These new sufficient conditions are less conservative than those obtained in [8], [21]-[24], [28], [29]. Two numerical examples are provided to illuminate the improvement of the proposed criteria.

Notation: The notations are used in our paper except where otherwise specified. $\|\cdot\|$ denotes a vector or a matrix norm; $\mathbb{R}, \mathbb{R}^n$ are real and n-dimension real number sets, respectively; $\mathbb{N}^+$ is positive integer set. I is identity matrix; $*$ represents the elements below the main diagonal of a symmetric block matrix; Real matrix $P > 0 (< 0)$ denotes P is a positive definite (negative definite) matrix; $\mathbb{N}[a, b] = \{a, a+1, \cdots, b\}$; $\lambda_{min}(\lambda_{max})$ denotes the minimum (maximum) eigenvalue of a real matrix.

II. Preliminaries

Consider a discrete-time recurrent neural network: Σ with time-varying delays described by

$$y(k+1) = C(k)y(k)+A(k)\overline{f}(y(k))+B(k)\overline{g}(y(k-\tau(k)))+J, \quad (1)$$

where $y(k) = [y_1(k), y_2(k), \cdots, y_n(k)]^T \in \mathbb{R}^n$ denotes the neural state vector; $\overline{f}(y(k)) = [\overline{f}_1(y_1(k)), \overline{f}_2(y_2(k)), \cdots, \overline{f}_n(y_n(k))]^T$, $\overline{g}(y(k-\tau(k))) = [\overline{g}_1(y_1(k-\tau(k))), \overline{g}_2(y_2(k-\tau(k))), \cdots, \overline{g}_n(y_n(k-\tau(k)))]^T$ are the neuron activation functions; $J = [J_1, J_2, \cdots, J_n]^T$ is the external input vector; Positive integer $\tau(k)$ represents the transmission delay satisfying $0 < \tau_m \leq \tau(k) \leq \tau_M$, where τ_m, τ_M are known positive integers representing the lower and upper bounds of the delay. $C(k) = C+\triangle C(k)$, $A(k) = A+\triangle A(k)$, $B(k) = B+\triangle B(k)$; $C = diag(c_1, c_2, \cdots, c_n)$ with $|c_i| < 1$ describes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $C, A, B \in \mathbb{R}^{n\times n}$ represent the weighting matrices; $\Delta C(k), \Delta A(k), \Delta B(k)$ denote the time-varying structured uncertainties which are of the following form:

$$[\Delta C(k), \Delta A(k), \Delta B(k)] = KF(k)[E_c \quad E_a \quad E_b],$$

where K, E_c, E_a, E_b are known real constant matrices of appropriate dimensions; $F(k)$ is unknown time-varying matrix function satisfying $F^T(k)F(k) \le I, \forall k \in \mathbb{N}^+$.

The nominal Σ_0 of Σ can be defined as

$$\Sigma_0 : y(k+1) = Cy(k) + A\overline{f}(y(k)) + B\overline{g}(y(k-\tau(k))) + J. \quad (2)$$

For further discussion, we first introduce the following assumption and lemmas.

Assumption 1: For any $x, y \in \mathbb{R}$, $x \neq y$,

$$l_i^- \le \frac{\overline{f}_i(x) - \overline{f}_i(y)}{x-y} \le l_i^+, \sigma_i^- \le \frac{\overline{g}_i(x) - \overline{g}_i(y)}{x-y} \le \sigma_i^+, i \in \mathbb{N}^+ \quad (3)$$

where $l_i^-, l_i^+, \sigma_i^-, \sigma_i^+$ are known constant scalars.

As pointed out in [22] that, under Assumption 1, system (2) has equilibrium points. Assume $y^* = [y_1^*, y_2^*, \cdots, y_n^*]^T$ is an equilibrium point of (2), and let $x_i(k) = y_i(k) - y_i^*$, $f_i(x_i(k)) = \overline{f}_i(x_i(k) + y_i^*) - \overline{f}_i(y_i^*)$, $g_i(x_i(k-\tau(k))) = \overline{g}_i(x_i(k-\tau(k)) + y_i^*) - \overline{g}_i(y_i^*)$. Then, system (2) can be transformed into the following form:

$$x(k+1) = Cx(k) + Af(x(k)) + Bg(x(k-\tau(k))), k \in \mathbb{N}^+, \quad (4)$$

where $x(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T$, $f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \cdots, f_n(x_n(k))]^T$, $g(x(k-\tau(k))) = [g_1(x_1(k-\tau(k))), g_2(x_2(k-\tau(k))), \cdots, g_n(x_n(k-\tau(k)))]^T$. By Assumption 1, for any $x, y \in \mathbb{R}$, $x \neq y$, functions $f_i(\cdot), g_i(\cdot)$ satisfy $l_i^- \le \frac{f_i(x)-f_i(y)}{x-y} \le l_i^+$, $\sigma_i^- \le \frac{g_i(x)-g_i(y)}{x-y} \le \sigma_i^+, i \in \mathbb{N}^+$ and $f_i(0) = 0, g_i(0) = 0$.

Definition 2.1: The delayed discrete-time recurrent neural network in (4) is said to be globally exponentially stable if there exist two positive scalars $\alpha > 0$ and $0 < \beta < 1$ such that

$$\|x(k)\| \le \alpha \cdot \beta^k \sup_{s\in\mathbb{N}[-\tau_M,0]} \|x(s)\|, \forall k \ge 0.$$

Lemma 2.1: [30] (Tchebychev Inequality) For any given vectors $v_i \in \mathbb{R}^n, i \in \mathbb{N}^+$, the following inequality holds:

$$[\sum_{i=1}^n v_i]^T [\sum_{i=1}^n v_i] \le n \sum_{i=1}^n v_i^T v_i.$$

Lemma 2.2: [31] For given matrices $Q = Q^T, H, E$ and $R = R^T > 0$ of appropriate dimensions, then

$$Q + HFE + E^T F^T H^T < 0,$$

for all F satisfying $F^T F \le R$, if and only if there exists an $\varepsilon > 0$, such that

$$Q + \varepsilon^{-1} HH^T + \varepsilon E^T RE < 0.$$

Lemma 2.3: [32] Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1^T = \Sigma_1$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \; or \; \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 2.4: [8] Let N and E be real constant matrices with appropriate dimensions, matrix $F(k)$ satisfying $F^T(k)F(k) \le I$, then, for any $\epsilon > 0$, $EF(k)N + N^T F^T(k) E^T \le \epsilon^{-1} EE^T + \epsilon N^T N$.

III. Main results

Theorem 3.1: For any given positive integers $0 < \tau_m < \tau_M$, then, under Assumption 1, system (4) is globally exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive matrices Q, R, H, γ, positive diagonal matrices $\Lambda_1, \Lambda_2, Z_1, Z_2$, arbitrary matrices C_1, C_2, M_{11}, M_{31} of appropriate dimensions, such that the following LMI holds:

$$\Xi_1 = \begin{bmatrix} \Xi_{11} & 0 & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & \Xi_{19} \\ * & \Xi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & \Xi_{29} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & 0 & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} \\ * & * & * & * & * & * & * & * & * \end{bmatrix} < 0, \quad (5)$$

where $Q = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ * & Q_{22} & Q_{23} \\ * & * & Q_{33} \end{bmatrix} > 0$, $C_1 + C_2 = C$,

$$\Xi_{11} = C_1^T Q_{11} C_1 - Q_{11} - \Lambda_1 L_1 + M_{11}(C_2 - I)$$
$$+ (C_2 - I)^T M_{11}^T + [1 + (\tau_M - \tau_m)^{-1}]R,$$

$$\Xi_{13} = C_1^T (Q_{11} + Q_{12} + Q_{13}) - M_{11} + (C_2 - I)^T M_{31}^T,$$

$$\Xi_{14} = -C_1^T Q_{12}, \; \Xi_{15} = -C_1^T Q_{13}, \; \Xi_{16} = C_1^T Q_{12} - Q_{12},$$

$$\Xi_{17} = C_1^T Q_{13} - Q_{13}, \; \Xi_{18} = M_{11}A + \Lambda_1 L_2, \; \Xi_{19} = M_{11}B,$$

$$\Xi_{22} = -(\tau_M - \tau_m)^{-1} R - \Lambda_2 \Pi_1, \; \Xi_{29} = \Lambda_2 \Pi_2,$$

$$\Xi_{33} = Q_{11} + Q_{22} + Q_{33} + Q_{12} + Q_{13} + Q_{23} + (Q_{12} + Q_{13} + Q_{23})^T$$
$$+ H + \gamma + \tau_m Z_2 + \tau_M Z_1 - M_{31} - M_{31}^T,$$

$$\Xi_{34} = -Q_{12} - Q_{22} - Q_{23}^T, \; \Xi_{35} = -Q_{13} - Q_{23} - Q_{33},$$

$$\Xi_{36} = Q_{12} + Q_{22} + Q_{23}^T, \; \Xi_{37} = Q_{13} + Q_{23} + Q_{33}, \; \Xi_{3,8} = M_{31}A,$$

$$\Xi_{3,9} = M_{31}B, \; \Xi_{44} = Q_{22} - H, \; \Xi_{45} = Q_{23}, \; \Xi_{46} = -Q_{22},$$

$$\Xi_{47} = -Q_{23}, \; \Xi_{55} = Q_{33} - \gamma, \; \Xi_{56} = -Q_{23}^T, \; \Xi_{57} = -Q_{33},$$

$$\Xi_{66} = -\frac{Z_2}{\tau_m}, \; \Xi_{77} = -\frac{Z_1}{\tau_M}, \; \Xi_{88} = -\Lambda_1, \; \Xi_{99} = -\Lambda_2,$$

$$L_1 = diag(l_1^+ l_1^-, \cdots, l_n^+ l_n^-), L_2 = diag(\frac{l_1^+ + l_1^-}{2}, \cdots, \frac{l_n^+ + l_n^-}{2}),$$

$$\Pi_1 = diag(\sigma_1^+ \sigma_1^-, \cdots, \sigma_n^+ \sigma_n^-), \Pi_2 = diag(\frac{\sigma_1^+ + \sigma_1^-}{2}, \cdots, \frac{\sigma_n^+ + \sigma_n^-}{2}).$$

Proof. Constructing a new augmented Lyapunov-Krasovskii function candidate as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k),$$

where

$$V_1(k) = \widehat{X}^T(k) Q \widehat{X}(k),$$

$\widehat{X}^T(k) = [x^T(k), \sum_{i=k-\tau_m}^{k-1} \eta^T(i), \sum_{i=k-\tau_M}^{k-1} \eta^T(i)]$, $\eta(k) = x(k+1) - C_1 x(k)$.

$$V_2(k) = \sum_{i=k-\tau_m}^{k-1} \eta^T(i) H \eta(i) + \sum_{i=k-\tau_M}^{k-1} \eta^T(i) \gamma \eta(i),$$

$$V_3(k) = \sum_{j=-\tau_M}^{-1} \sum_{i=j+k}^{k-1} \eta^T(i) Z_1 \eta(i) + \sum_{j=-\tau_m}^{-1} \sum_{i=k+j}^{k-1} \eta^T(i) Z_2 \eta(i).$$

$$V_4(k) = \frac{1}{\tau_M - \tau_m} \sum_{i=k-\tau(k)}^{k-1} x^T(i) R x(i),$$

$$V_5(k) = \frac{1}{\tau_M - \tau_m} \sum_{j=k+1-\tau_M}^{k-\tau_m} \sum_{i=j}^{k-1} x^T(i)Rx(i).$$

Set $X^T(k) = [x^T(k), x^T(k-\tau(k)), \eta^T(k), \eta^T(k-\tau_m), \eta^T(k-\tau_M), \sum_{i=k-\tau_m}^{k-1}\eta^T(i), \sum_{i=k-\tau_M}^{k-1}\eta^T(i), f^T(x(k)), g^T(x(k-\tau(k)))]$. Define $\Delta V(k) = V(k+1) - V(k)$. Then, along the solution of system (4) we have

$$\begin{aligned} \Delta V_1(k) &= \widehat{X}^T(k+1)Q\widehat{X}(k+1) - \widehat{X}^T(k)Q\widehat{X}(k) \\ &= X^T(k)(\widetilde{I}_1^T Q\widetilde{I}_1 - \widetilde{I}_2^T Q\widetilde{I}_2)X(k), \end{aligned} \tag{6}$$

where

$$\widetilde{I}_1^T = \begin{bmatrix} C_1^T & 0 & 0 \\ 0 & 0 & 0 \\ I & I & I \\ 0 & -I & 0 \\ 0 & 0 & -I \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \widetilde{I}_2^T = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\begin{aligned} \Delta V_2(k) &= \eta^T(k)H\eta(k) - \eta^T(k-\tau_m)H\eta(k-\tau_m) \\ &+ \eta^T(k)\gamma\eta(k) - \eta^T(k-\tau_M)\gamma\eta(k-\tau_M). \end{aligned} \tag{7}$$

From lemma 2.1 we have

$$\begin{aligned} \Delta V_3(k) &= \tau_M\eta^T(k)Z_1\eta(k) - \sum_{i=k-\tau_M}^{k-1}\eta^T(i)Z_1\eta(i) \\ &\quad +\tau_m\eta^T(k)Z_2\eta(k) - \sum_{i=k-\tau_m}^{k-1}\eta^T(i)Z_2\eta(i) \\ &= \tau_M\eta^T(k)Z_1\eta(k) - \sum_{i=k-\tau_M}^{k-1}(\sqrt{Z_1}\eta(i))^T\sqrt{Z_1}\eta(i) \\ &\quad +\tau_m\eta^T(k)Z_2\eta(k) - \sum_{i=k-\tau_m}^{k-1}(\sqrt{Z_2}\eta(i))^T\sqrt{Z_2}\eta(i) \\ &\le \tau_M\eta^T(k)Z_1\eta(k) - (\sum_{i=k-\tau_M}^{k-1}\eta(i))^T\frac{Z_1}{\tau_M}(\sum_{i=k-\tau_M}^{k-1}\eta(i)) \\ &\quad +\tau_m\eta^T(k)Z_2\eta(k) - (\sum_{i=k-\tau_m}^{k-1}\eta(i))^T\frac{Z_2}{\tau_m}(\sum_{i=k-\tau_m}^{k-1}\eta(i)). \end{aligned} \tag{8}$$

$$\begin{aligned} \Delta V_4(k) &= \frac{1}{\tau_M - \tau_m}[x^T(k)Rx(k) - x^T(k-\tau(k))Rx(k-\tau(k))) \\ &\quad + \sum_{i=k+1-\tau(k+1)}^{k-\tau_m} x^T(i)Rx(i) + \sum_{i=k+1-\tau_m}^{k-1} x^T(i)Rx(i) \\ &\quad - \sum_{i=k+1-\tau(k)}^{k-1} x^T(i)Rx(i)] \\ &\le \frac{1}{\tau_M - \tau_m}[x^T(k)Rx(k) - x^T(k-\tau(k))Rx(k-\tau(k))] \\ &\quad + \frac{1}{\tau_M - \tau_m}[\sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i)Rx(i)], \end{aligned} \tag{9}$$

$$\begin{aligned} \Delta V_5(k) &= \frac{1}{\tau_M - \tau_m}[\sum_{j=k+2-\tau_M}^{k+1-\tau_m}\sum_{i=j}^{k} x^T(i)Rx(i) \\ &\quad - \sum_{j=k+1-\tau_M}^{k-\tau_m}\sum_{i=j}^{k-1} x^T(i)Rx(i)] \\ &= x^T(k)Rx(k) - \frac{1}{\tau_M - \tau_m}[\sum_{i=k+1-\tau_M}^{k-\tau_m} x^T(i)Rx(i)]. \end{aligned} \tag{10}$$

For any matrices M_{11}, M_{31} of appropriate dimensions, we have

$$2x^T(k)M_{11}[(C_2 - I)x(k) + Af(x(k)) + Bg(x(k-\tau(k))) - \eta(k)] = 0. \tag{11}$$

$$2\eta^T(k)M_{31}[(C_2 - I)x(k) + Af(x(k)) + Bg(x(k-\tau(k))) - \eta(k)] = 0. \tag{12}$$

From Assumption 1, for any positive-definite diagonal matrices Λ_1, Λ_2 of appropriate dimensions, we have

$$\begin{aligned} &2x^T(k)\Lambda_1L_2f(x(k)) - x^T(k)\Lambda_1L_1x(k) - f^T(x(k))\Lambda_1f(x(k)) \ge 0, \\ &2x^T(k-\tau(k))\Lambda_2\Pi_2g(x(k-\tau(k))) - x^T(k-\tau(k)) \times \\ &\Lambda_2\Pi_1x(k-\tau(k)) - g^T(x(k-\tau(k)))\Lambda_2g(x(k-\tau(k))) \ge 0, \end{aligned} \tag{13}$$

Combining (6)-(13), we get

$$\Delta V(k) \le X^T(k)\Xi_1X(k), \tag{14}$$

If the LMI (5) holds, it follows that there exists a sufficient small positive scalar $\varepsilon > 0$ such that

$$\Delta V(k) \le -\varepsilon\|x(k)\|^2. \tag{15}$$

On the other hand, it can easily to get that

$$V(k) \le \alpha_1\|x(k)\|^2 + \alpha_2\sum_{i=k-\tau_M}^{k-1}\|x(i)\|^2, \tag{16}$$

where $\alpha_1 = \lambda_{max}(Q)(1 + \tau_m + \tau_M) + \lambda_{max}(H) + \lambda_{max}(\gamma) + \tau_M\lambda_{max}(Z_1) + \tau_m\lambda_{max}(Z_2)$, $\alpha_2 = (1 + \|C_1\|^2)\alpha_1 - \|C_1\|^2\lambda_{max}(Q) + (1 + \frac{1}{\tau_M - \tau_m})\lambda_{max}(R)$.

For any $\theta > 1$, it follows from (16) that

$$\begin{aligned} &\theta^{j+1}V(j+1) - \theta^jV(j) = \theta^{j+1}\Delta V(j) + \theta^j(\theta - 1)V(j) \\ &\le \theta^j(-\varepsilon\theta\|x(j)\|^2 + (\theta - 1)\alpha_1\|x(j)\|^2 \\ &+(\theta - 1)\alpha_2\sum_{i=j-\tau_M}^{j-1}\|x(j)\|^2). \end{aligned} \tag{17}$$

Summing up both sides of (17) from 0 to $k-1$ we can obtain

$$\begin{aligned} \theta^kV(k) - V(0) &\le [\alpha_1(\theta - 1) - \varepsilon\theta]\sum_{j=0}^{k-1}\theta^j\|x(j)\|^2 \\ &\quad +\alpha_2(\theta - 1)\sum_{j=0}^{k-1}\sum_{i=j-\tau_M}^{j-1}\theta^j\|x(i)\|^2 \\ &\le \mu_1(\theta)\sup_{j\in\mathbb{N}[-\tau_M,0]}\|x(j)\|^2 + \mu_2(\theta)\sum_{j=0}^{k}\theta^k\|x(j)\|^2, \end{aligned} \tag{18}$$

where $\mu_1(\theta) = \alpha_2(\theta - 1)\tau_M^2\theta^{\tau_M}$, $\mu_2(\theta) = \alpha_2(\theta - 1)\tau_M\theta^{\tau_M} + \alpha_1(\theta - 1) - \varepsilon\theta$. Since $\mu_2(1) = -\varepsilon\theta < 0$, there must exist a positive $\theta_0 > 1$ such that $\mu_2(\theta_0) < 0$. Then we have

$$V(k) \le \mu_1(\theta_0)(\frac{1}{\theta_0})^k\sup_{j\in\mathbb{N}[-\tau_M,0]}\|x(j)\|^2 + (\frac{1}{\theta_0})^kV(0), \tag{19}$$

On the other hand, set $\varpi = \alpha_1 + \tau_M\alpha_2$, we can obtain

$$V(0) \le \varpi\sup_{j\in\mathbb{N}[-\tau_M,0]}\|x(j)\|^2 \; and \; V(k) \ge \lambda_{min}(Q)\|x(k)\|^2. \tag{20}$$

It follows that $\|x(k)\| \le \alpha\cdot\beta^k\sup_{j\in\mathbb{N}[-\tau_M,0]}\|x(j)\|$, where $\beta = (\theta_0)^{-1/2}$, $\alpha = \sqrt{\frac{\mu_1(\theta_0)+\varpi}{\lambda_{min}(Q)}}$. By Definition 1, system (1) is globally exponentially stable, which complete the proof.

Theorem 3.2: For any given positive integers $0 < \tau_m < \tau_M$, then, under Assumption 1, system (4) is globally exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive matrices Q, R, H, γ, positive diagonal matrices $\Lambda_1, \Lambda_2, Z_1, Z_2$, arbitrary matrices M_{11}, M_{31} of appropriate dimensions, such that the following LMI holds:

$$\Xi_2 = \left[\begin{array}{ccccccccc} \Xi'_{11} & 0 & \Xi'_{13} & \Xi'_{14} & \Xi'_{15} & 0 & 0 & \Xi_{18} & \Xi_{19} \\ * & \Xi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & \Xi_{29} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} & \Xi_{37} & \Xi_{38} & \Xi_{39} \\ * & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} & \Xi_{47} & 0 & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} \\ * & * & * & * & * & * & * & * & * \end{array}\right] < 0, \quad (21)$$

where $Q = \left[\begin{array}{ccc} Q_{11} & Q_{12} & Q_{13} \\ * & Q_{22} & Q_{23} \\ * & * & Q_{33} \end{array}\right] > 0,$

$$\Xi'_{11} = -\Lambda_1 L_1 + M_{11}(C-I) + (C-I)^T M_{11}^T$$
$$+(1+(\tau_M-\tau_m)^{-1})R,$$
$$\Xi'_{13} = Q_{11} + Q_{12} + Q_{13} - M_{11} + (C-I)^T M_{31}^T,$$
$$\Xi'_{14} = -Q_{12},\ \Xi'_{15} = -Q_{13},$$

Proof. Similar to the proofs of Theorem 3.1, set $C_1 = I$, one can easily obtain this result, which omitted here.

Theorem 3.3: For any given positive integers $0 < \tau_m < \tau_M$, then, under Assumption 1, system (4) is globally exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive matrices Q, R, H, γ, positive diagonal matrices $\Lambda_1, \Lambda_2, Z_1, Z_2$, symmetric matrix Ψ, arbitrary matrices $M_{11}, M_{31}\, P_1,\ P_2,\ G_1,\ G_2$ of appropriate dimensions, such that the following LMI holds:

$$\Xi_3 = \left[\begin{array}{cc} \Xi_2 & 0 \\ * & 0 \end{array}\right]_{10n\times 10n} + \Psi_{10n\times 10n} < 0, \quad (22)$$

where

$$\Psi_{11} = P_1 + P_1^T + G_1 + G_1^T,\ \Psi_{12} = P_2^T + G_2^T - G_1 - P_1,$$
$$\Psi_{13} = P_1^T,\ \Psi_{1,10} = P_2^T + G_2^T - G_1 - P_1,$$
$$\Psi_{22} = -P_2^T + G_2^T - G_2 - P_2,\ \Psi_{23} = -P_1^T,$$
$$\Psi_{2,10} = -P_2^T - G_2^T - G_2 - P_2,\ \Psi_{3,10} = -P_1$$
$$\Psi_{10,10} = -P_2^T - G_2^T - G_2 - P_2,$$

and other sub-blocks of Ψ are zero matrices.

Proof. Similar to the proof of Theorem 3.1, set $C_1 = I$. Since $x(k) - \sum_{i=k-\tau(k)}^{k-1} \eta(i) - x(k-\tau(k)) = 0$, for arbitrary matrices P_1, P_2, G_1, G_2 of appropriate dimensions, we can obtain that

$$0 = \widetilde{X}_1^T \left[\begin{array}{cc} 0 & P_1 \\ 0 & P_2 \end{array}\right] \widetilde{X}_2,\ 0 = \overline{X}_1^T \left[\begin{array}{cc} 0 & G_1 \\ 0 & G_2 \end{array}\right] \widetilde{X}_2, \quad (23)$$

where $\widetilde{X}_1^T(k) = [\eta^T(k) + x^T(k), \sum_{i=k-\tau(k)}^{k-1} \eta^T(i) + x^T(k-\tau(k))]$, $\widetilde{X}_2^T = [\eta^T(k) + x^T(k), x^T(k) - \sum_{i=k-\tau(k)}^{k-1} \eta^T(i) - x^T(k-\tau(k))]$, $\overline{X}_1^T = [x^T(k), \sum_{i=k-\tau(k)}^{k-1} \eta^T(i) + x^T(k-\tau(k))]$. Combining (6)-(13) and (23), we get

$$\Delta V(k) \le X'^T(k) \Xi X'(k), \quad (24)$$

where $X'^T(k) = [X^T(k), \sum_{i=k-\tau(k)}^{k-1} \eta^T(i)]$.

If the LMI (22) holds, it follows that there exists a sufficient small positive scalar $\varepsilon > 0$ such that

$$\Delta V(k) \le -\varepsilon \|x(k)\|^2. \quad (25)$$

The rest proof is the same as in Theorem 3.1, which omitted here.

Remark 1. In Theorem 3.1, we proposed V_1 which takes $x^T(k), \sum_{i=k-\tau_m}^{k-1} \eta^T(i), \sum_{i=k-\tau_M}^{k-1} \eta^T(i)$ as augmented state. The proposed augmented Lyapunov function V_1 does not considered in the existing works and may reduce the conservatism of the delay-dependent result. Moreover, the decomposition of matrix $C = C_1 + C_2$ makes the conservatism of the stability criterion reduced further, since the elements of matrices C_1, C_2 are not restricted into $(-1, 1)$ any more.

Remark 2. Zero equations (23) provides a new method to introduce free-weighting matrix, which do not considered in existing works. And free-weighting matrices P_1, P_2, G_1, G_2 make an important role in the reducing of conservatism of a criterion (details see example 2).

Remark 3. It is worth pointing out that the criteria obtained in above Theorems can be easily extended to robust exponential stability condition. As for the robust stability of system (1), according to Lemma 2.2, we can obtain the following results.

Corollary 3.1: For any given positive integers $0 < \tau_m < \tau_M$, then, under Assumption 1, system (1) is globally robustly and exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive matrices Q, R, H, γ, positive diagonal matrices $\Lambda_1, \Lambda_2, Z_1, Z_2$, arbitrary matrices M_{11}, M_{31} of appropriate dimensions, and $\epsilon > 0$, such that the following LMI holds:

$$\Xi_4 \triangleq \left[\begin{array}{ccc} \Xi_1 & \xi_1 & \epsilon\xi_2^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{array}\right] < 0, \quad (26)$$

where $\xi_1^T = [K^T M_{11}^T, 0, 0, K^T M_{31}^T, 0, 0, 0, 0, 0]$, $\xi_2 = [E_c, 0, 0, 0, 0, 0, E_a, E_b, 0]$.

Proof. Replacing $A,\ B,\ C_2$ in inequality (5) with $A + KF(t)E_a$, $B + KF(t)E_b$ and $C_2 + KF(t)E_c$, respectively. Inequality (26) for system (1) is equivalent to $\Xi_1 + \xi_1 F(t)\xi_2 + \xi_2^T F^T(t)\xi_1^T < 0$. From lemma 2.2, lemma 2.3 and lemma 2.4, we can easily obtain this result, which complete the proof.

Corollary 3.2: For any given positive integers $0 < \tau_m < \tau_M$, then, under Assumption 1, system (1) is globally robustly and exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive matrices Q, R, H, γ, positive diagonal matrices $\Lambda_1, \Lambda_2, Z_1, Z_2$, arbitrary matrices M_{11}, M_{31} of appropriate dimensions, and $\epsilon > 0$, such that the following LMI holds:

$$\Xi_5 \triangleq \left[\begin{array}{ccc} \Xi_2 & \xi'_1 & \epsilon\xi_2'^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{array}\right] < 0, \quad (27)$$

where $\xi_1'^T = [K^T M_{11}^T, 0, 0, K^T M_{31}^T, 0, 0, 0, 0, 0]$, $\xi'_2 = [E_c, 0, 0, 0, 0, 0, 0, E_a, E_b]$.

Corollary 3.3: For any given positive integers $0 < \tau_m < \tau_M$, then, under Assumption 1, system (1) is globally robustly and exponentially stable for any time-varying delay $\tau(k)$ satisfying $\tau_m \le \tau(k) \le \tau_M$, if there exist positive matrices Q, R, H, γ, positive diagonal matrices $\Lambda_1, \Lambda_2, Z_1, Z_2$, arbitrary matrices $M_{11}, M_{31}\, P_1,\ P_2,\ G_1,\ G_2$ with appropriate dimensions, and $\epsilon > 0$, such that the following LMI holds:

$$\Xi_6 \triangleq \left[\begin{array}{ccc} \Xi_3 & \widetilde{\xi}_1 & \epsilon\widetilde{\xi}_2^T \\ * & -\epsilon I & 0 \\ * & * & -\epsilon I \end{array}\right] < 0, \quad (28)$$

where $\widetilde{\xi}_1^T = [K^T M_{11}^T, 0, 0, K^T M_{31}^T, 0, 0, 0, 0, 0, 0]$, $\widetilde{\xi}_2 = [E_c, 0, 0, 0, 0, 0, 0, E_a, E_b, 0]$.

TABLE I
ALLOWABLE UPPER BOUNDS τ_M FOR GIVEN τ_m (EXAMPLE 1)

Cases	$\tau_m = 1$	$\tau_m = 4$	$\tau_m = 8$	$\tau_m = 15$	$\tau_m = 25$
By [8], [28]	3	6	10	17	27
By [21]	12	14	16	21	29
By [22]	12	14	18	25	35
By [29]	14	17	19	26	36
By [23]	14	17	21	28	38
By [24]	20	22	26	33	43
By Theorem 3.1,3.2, 3.3	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$

TABLE II
ALLOWABLE UPPER BOUNDS τ_M FOR GIVEN τ_m (EXAMPLE 2)

Cases	$\tau_m = 2$	$\tau_m = 4$	$\tau_m = 6$	$\tau_m = 8$	$\tau_m = 10$
By [21]	failed	failed	failed	failed	failed
By [22]	failed	failed	failed	failed	failed
By [28]	failed	failed	failed	failed	failed
By Corollary 3.1	failed	failed	failed	failed	failed
By Corollary 3.2	failed	failed	failed	failed	failed
By [8]	18	20	22	24	26
By [24]	24	26	28	30	34
By Corollary 3.3	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$	$\tau_M > 0$

IV. NUMERICAL EXAMPLES

In this section, two numerical examples will be presented to show the improvement and effectiveness of the main results derived above.

Example 1. For the convenience of comparison, consider a delayed discrete-time recurrent neural network in (4) with parameters given by

$$C = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.7 \end{bmatrix}, A = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.005 \end{bmatrix}, B = \begin{bmatrix} -0.1 & 0.01 \\ -0.2 & -0.1 \end{bmatrix}.$$

The activation functions are assumed to be $f_i(s) = g_i(s) = 0.5 * (|s+1| - |s-1|)$. Obviously, $l_1^- = \sigma_1^- = -1$, $l_2^+ = \sigma_2^+ = 1$. It can be verified that the LMI (5), (21), (22) are feasible. For $\tau_m = 1, 4, 8, 15, 25$, Table 1 gives out the allowable upper bound τ_M of the time-varying delay for given τ_m, respectively, which shows that, for this example, the delay-dependent exponential stability result proposed in Theorem 3.1, Theorem 3.2, and Theorem 3.3 are less conservative than these previous results.

Example 2. For the convenience of comparison, consider a delayed discrete-time recurrent neural network in (1) with parameters given by

$$C = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, A = \begin{bmatrix} 0.3 & -0.1 & 0.2 \\ 0 & -0.3 & 0.2 \\ -0.1 & -0.1 & -0.2 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ -0.2 & 0.3 & 0.1 \\ 0.1 & -0.2 & 0.3 \end{bmatrix}, K = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$E_c = E_a = E_b = K, J = [0, 0, 0]^T,$$

$$f_1(s) = \tanh(0.2s), f_2(s) = \tanh(0.4s), f_3(s) = \tanh(0.2s),$$

$$g_1(s) = \tanh(0.12s), g_2(s) = \tanh(0.2s), g_3(s) = \tanh(0.4s).$$

It can be verified that $L_1 = \Pi_1 = 0$, $L_2 = diag(0.1, 0.2, 0.1)$, $\Pi_2 = diag(0.06, 0.1, 0.2)$, and the LMI (28) is feasible. For $\tau_m = 2, 4, 6, 8, 10$, Table 2 gives out the allowable upper bound τ_M of the time-varying delay for given τ_m, respectively, which implies that, for this example, the delay-dependent exponential stability result proposed in Corollary 3.3 in this paper provides less conservatism than those obtained in [8], [21], [22], [24], [28].

When $\tau_m = 2, \tau_M = 100$, by the Matlab LMI Toolbox, a feasible solution to the LMI (28) is obtained as follows:

$$Q_{11} = \begin{bmatrix} 142.1658 & -17.6253 & -1.1583 \\ -17.6253 & 135.1619 & 8.5093 \\ -1.1583 & 8.5093 & 140.1468 \end{bmatrix},$$

$$Q_{12} = \begin{bmatrix} -3.6440 & 0.3497 & 0.3433 \\ 0.1807 & -6.0973 & -0.8800 \\ 0.2302 & -0.3793 & -4.4553 \end{bmatrix},$$

$$Q_{13} = \begin{bmatrix} -0.0502 & 0.0074 & 0.0019 \\ 0.0081 & -0.0560 & -0.0009 \\ 0.0022 & -0.0044 & -0.0439 \end{bmatrix},$$

$$Q_{22} = \begin{bmatrix} 4.6496 & -0.9163 & -0.2124 \\ -0.9163 & 5.0049 & 0.5412 \\ -0.2124 & 0.5412 & 3.8276 \end{bmatrix},$$

$$Q_{23} = \begin{bmatrix} -0.0139 & 0.0041 & 0.0007 \\ 0.0027 & -0.0118 & -0.0029 \\ -0.0002 & -0.0004 & -0.0111 \end{bmatrix},$$

$$Q_{33} = \begin{bmatrix} 0.0962 & -0.0157 & -0.0041 \\ -0.0157 & 0.1030 & 0.0076 \\ -0.0041 & 0.0076 & 0.0863 \end{bmatrix},$$

$$R = \begin{bmatrix} 8.6947 & -2.4727 & -0.5482 \\ -2.4727 & 10.0667 & 1.5172 \\ -0.5482 & 1.5172 & 7.3986 \end{bmatrix},$$

$$H = \begin{bmatrix} 46.5167 & -2.5896 & -0.8314 \\ -2.5896 & 62.7329 & 3.2843 \\ -0.8314 & 3.2843 & 50.2522 \end{bmatrix},$$

$$\gamma = \begin{bmatrix} 16.4910 & -4.8464 & -1.1441 \\ -4.8464 & 18.7861 & 3.1320 \\ -1.1441 & 3.1320 & 14.2678 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.1168 & 0 & 0 \\ 0 & 0.1463 & 0 \\ 0 & 0 & 0.0961 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 13.7752 & 0 & 0 \\ 0 & 15.9935 & 0 \\ 0 & 0 & 12.9945 \end{bmatrix},$$

$$\Lambda_2 = \begin{bmatrix} 107.3772 & 0 & 0 \\ 0 & 123.4630 & 0 \\ 0 & 0 & 95.9376 \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} 106.8048 & 0 & 0 \\ 0 & 86.8649 & 0 \\ 0 & 0 & 114.8134 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} -52.1439 & 9.9421 & 2.6629 \\ -1.0630 & -52.4450 & -8.7828 \\ 0.7615 & 0.8028 & -54.3665 \end{bmatrix},$$

$$M_{31} = \begin{bmatrix} 124.5100 & -21.5080 & -3.4123 \\ -8.3535 & 128.0636 & 12.5404 \\ 0.5521 & 3.3418 & 119.2432 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 6.7570 & -1.3781 & -0.7266 \\ -0.9612 & 5.2725 & -0.8791 \\ 0.3573 & 1.1824 & 6.9043 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 230.1390 & -20.6622 & 5.1184 \\ -4.8568 & 199.3365 & -27.2650 \\ 3.6819 & 2.2878 & -24.6629 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 2.7491 & 1.0033 & 1.4947 \\ 1.7218 & 6.9440 & -2.2799 \\ 0.8398 & -0.6938 & 12.1390 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 215.2528 & -20.1392 & 4.9019 \\ -3.9677 & 180.2401 & -28.0725 \\ 5.7910 & 3.0529 & -53.0478 \end{bmatrix},$$

$\epsilon = 57.6638.$

V. Conclusions

Combined with linear matrix inequality (LMI) technique, a new augmented Lyapunov-Krasovskii functional is constructed, and some new improved sufficient conditions ensuring globally exponential stability or robust exponential stability are obtained. Numerical examples show that the new results are less conservative than some recent results obtained in literature cited therein.

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