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Improved Neutron Leakage Treatment on Nodal Expansion Method for PWR Reactors

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***Abstract*—**For a quick and accurate calculation of spatial neutron distribution in nuclear power reactors 3D nodal codes are usually used aiming at solving the neutron diffusion equation for a given reactor core geometry and material composition. These codes use a second order polynomial to represent the transverse leakage term. In this work, a nodal method based on the well known nodal expansion method (NEM), developed at COPPE, making use of this polynomial expansion was modified to treat the transverse leakage term for the external surfaces of peripheral reflector nodes.

The proposed method was implemented into a computational system which, besides solving the diffusion equation, also solves the burnup equations governing the gradual changes in material compositions of the core due to fuel depletion. Results confirm the effectiveness of this modified treatment of peripheral nodes for practical purposes in PWR reactors.

***Keywords*—**Transverse leakage, nodal expansion method, power density, PWR reactors

I. Introduction

THE neutron distribution has to be frequently calculated in PWR reactors. This calculation is validated with measured values of reactor parameters. Even the neutron population varies along the reactor operation period depending on time variation of the nuclide distribution, which in turn varies with the spatial distribution of the depletion rate, one is fully justified to use a quasi-static approximation [1], in which the reactor cycle is divided into certain time intervals, during which the neutron fluxes are held constant. Therefore, in this case, the steady state diffusion equation and the depletion equations are solved, for each of the defined time intervals for the reactor operation period.There are a lot of methods to solve the neutron diffusion equation for a given reactor core, but determining factors that affect the quality of a system for calculating the neutronic parameters of the reactor core are the accuracy and speed with which the operational performance of the reactor is predicted. For power reactors, the most popular method is the nodal expansion method (NEM) [2]. An example of application of this method is the CNFR code (Portuguese acronym for National Reactor Physics Code) [3] that consists of three main modules, which generate nuclear data for fuel elements, 3D nuclear power distributions and characteristic parameter calculations of the nuclear reactor. The majority of nodal diffusion codes employ the diffusion equation integrated in a transverse area for given direction. From this integration, the transverse leakage is obtained.

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These codes use a second order polynomial to represent the transverse leakage term. CNFR code has also made use of this polynomial expansion for all nodes so far. But there was a motivation to better represent the transverse leakage for the external surface of nodes in the reflector periphery, so that this paper presents a modified treatment of the transverse leakage for these nodes. In section 2, a brief comment on the nodal expansion method (NEM) is presented. Section 3 presents the modified procedure. Section 4 shows results obtained with this modification. Conclusions are presented in section 5.

II. Nodal Expansion Method

The reactor core is divided, in space, into contiguous parallelepipeds called nodes. Since NEM requires that the nodes be homogeneous, special models are adopted to deal with cross-sections that are no longer uniform inside the node, due to burnup and control rod motion [4]. With these special models, nodes remain homogeneous and core nodalization, which was previously established, is maintained.

The NEM uses partial interface currents and has as its starting point the neutron continuity equation and Fick's Law. The nodal balance equation, from which one obtains average nodal fluxes $\bar{\phi}_g^n(t_\ell)$ for each time t_ℓ, results from integration of the continuity equation in the volume $V_n = a_x^n a_y^n a_z^n$ of a node n, shown in Figure 1, and subsequent division of the integrated equation by this volume, i.e.

$$\sum_{u=x,y,z} \frac{1}{a_u^n}[\bar{J}_{gur}^n(t_\ell) - \bar{J}_{gul}^n(t_\ell)] + \Sigma_{tg}^n(t_\ell)\bar{\phi}_g^n(t_\ell) =$$

$$\sum_{g'=1}^{2} \{\frac{\chi_g}{k_{eff}} \nu\Sigma_{fg'}^n(t_\ell) + \bar{\Sigma}_{gg'}^n(t_\ell)\}\bar{\phi}_{g'}^n(t_\ell) \qquad (1)$$

where the cross sections involving capture and fission, which because of the burnup gradient [5] vary spatially within the node, are given by:

$$\Sigma_{xg}^n(t_\ell) \equiv \frac{1}{V_n \bar{\phi}_g^n(t_\ell)} \int_{V_n} \Sigma_{xg}(x,y,z,t_\ell)\phi_g(x,y,z,t_\ell)dV. \qquad (2)$$

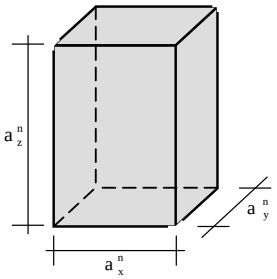

Fig. 1 Node n dimensions

A. Nodal Coupling Equations

The nodal coupling equations are obtained from Fick´s law, integrating these equations in the area transverse to direction u, and subsequent division of the integrated equations by this area, i.e.

$$\bar{J}^n_{gus}(t_\ell)=\bar{J}^{+n}_{gus}(t_\ell)-\bar{J}^{-n}_{gus}(t_\ell)=-\bar{D}^n_g(t_\ell)\frac{d}{du}\bar{\psi}^n_{gu}(u,t_\ell)\Big|_{u=u^n_s}, \quad (3)$$

for $u=x,y,z$ and $s=r,l$. Where:

$$\bar{J}^{\pm n}_{gus}(t_\ell)\equiv\frac{1}{a^n_v a^n_w}\int_0^{a^n_v}\int_0^{a^n_w}J^{\pm}_{gu}(u^n_s,v,w,t_\ell)dvdw, \quad (4)$$

$$\bar{\psi}^n_{gu}(u,t_\ell)\equiv\frac{1}{a^n_v a^n_w}\int_0^{a^n_v}\int_0^{a^n_w}\phi_g(u,v,w,t_\ell)dvdw \quad (5)$$

and

$$\bar{D}^n_g(t_\ell)\equiv\frac{1}{3\bar{\Sigma}^n_{trg}(t_\ell)}, \quad (6)$$

with

$$\bar{\Sigma}^n_{xg}(t_\ell)\equiv\frac{1}{V_n}\int_{V_n}\Sigma_{xg}(x,y,z,t_\ell)dV, \quad (7)$$

for Σ_{tr} and $\Sigma_{g'g}$.

$\bar{\psi}^n_{gu}(u,t_\ell)$ is obtained by integrating the diffusion equation in the transverse area to direction u. The transverse integrated diffusion equation, mentioned above is written as:

$$-\bar{D}^n_g(t_\ell)\frac{d^2}{du^2}\bar{\psi}^n_{gu}(u,t_\ell)+\Sigma^n_{tg}(t_\ell)\bar{\psi}^n_{gu}(t_\ell)=$$

$$\sum_{g'=1}^{2}\{\frac{\chi_g}{k_{eff}}\nu\Sigma^n_{fg'}(t_\ell)+\bar{\Sigma}^n_{gg'}(t_\ell)\}\bar{\phi}^n_{g'}(t_\ell)-L^n_{gu}(u,t_\ell)-$$

$$d^n_{gu}(u,t_\ell) \quad (8)$$

Where the transverse leakage is defined as

$$L^n_{gu}(u,t_\ell)\equiv-\frac{\bar{D}^n_g(t_\ell)}{a^n_v a^n_w}\int_0^{a^n_v}\int_0^{a^n_w}\sum_{\xi=v,w}\frac{\partial^2}{\partial\xi^2}\phi_g(u,v,w,t_\ell)dvdw \quad (9)$$

and the cross section difference term is defined as

$$d^n_{gu}(u,t_\ell)\equiv\frac{1}{k_{eff}}\chi_g\sum_{g'=1}^{2}\{\nu\Sigma^n_{fg'u}(u,t_\ell)-\nu\Sigma^n_{fg'}(t_\ell)\}\bar{\psi}^n_{g'u}(u)$$

$$-\{\Sigma^n_{agu}(u,t_\ell)-\Sigma^n_{ag}(t_\ell)\}\bar{\psi}^n_{gu}(u)$$

.

There have been some variants of NEM [2], since its appearance to solve eq. (8). One of them is the analytical NEM, e.g., the nodal integration method (NIM) [6]. This method solves the transverse integrated equations analytically, approximating only the transverse leakage and the cross section difference term. Other variant is the semi-analytical NEM [7], which makes use of polynomial expansions for the transverse leakage, fission source, scattering and the cross section difference term, thus uncoupling the energy groups. According to the original NEM, $\bar{\psi}^n_{gu}(u,t_\ell)$ are calculated using a fourth order polynomial expansion as follows:

$$\bar{\psi}^n_{gu}(u,t_\ell)=\bar{\phi}^n_g(t_\ell)+\sum_{k=1}^{4}c^n_{kgu}(t_\ell)h_k(u/a^n_u), \quad (10)$$

where $h_k(u/a^n_u)$ are basis functions of NEM [2].

III. Treatment Of Transverse Leakage

All the NEM based nodal codes, including CNFR, use second order polynomial expansions to approximate $L^n_{gu}(u,t_\ell)$, for every node in the reactor (active core and reflector), in the form:

$$L^n_{gu}(u,t_\ell)=\bar{L}^n_{gu}(t_\ell)+\sum_{k=1}^{2}\alpha^n_{kgu}(t_\ell)h_k(u/a^n_u), \quad (11)$$

where,

$$\bar{L}^{n}_{gu}(t_\ell) \equiv \frac{1}{a^{n}_{v}}[\bar{J}^{n}_{gvr}(t_\ell) - \bar{J}^{n}_{gvl}(t_\ell) + \frac{1}{a^{n}_{w}}[\bar{J}^{n}_{gwr}(t_\ell) - \bar{J}^{n}_{gwl}(t_\ell)\ , \tag{12}$$

$$\alpha^{n}_{1gu}(t_\ell) = \frac{1}{2}\{L^{n}_{gur}(t_\ell) - L^{n}_{gul}(t_\ell)\} \tag{13}$$

and

$$\alpha^{n}_{2gu}(t_\ell) = \bar{L}^{n}_{gu}(t_\ell) - \frac{1}{2}\{L^{n}_{gur}(t_\ell) + L^{n}_{gul}(t_\ell)\} \tag{14}$$

with $L^{n}_{gus}(t_\ell) \equiv L^{n}_{gu}(u^{n}_{s}, t_\ell)$ for $u = x, y, z$ and $s = r, l$.

In this work, we have used for all nodal surfaces in the core and reflector, except by those reflector node surfaces at the external surface of the reactor, the following expression for $L^{m}_{gur}(t_\ell)$:

$$L^{m}_{gur}(t_\ell) = \frac{a^{n}_{u}\ \bar{L}^{m}_{gu}(t_\ell) + a^{m}_{u}\ \bar{L}^{n}_{gu}(t_\ell)}{a^{n}_{u} + a^{m}_{u}}, \tag{15}$$

where m is the node adjacent to node n and

$$L^{n}_{gul}(t_\ell) = L^{m}_{gur}(t_\ell)\ . \tag{16}$$

But in those reflector node surfaces at the external surface of the core, the following expression was used

$$L^{n}_{gus}(t_\ell) = \{\bar{\psi}^{n}_{gu}(u^{n}_{s}, t_\ell)/\bar{\phi}^{n}_{g}(t_\ell)\}\ \bar{L}^{n}_{gu}(t_\ell)\,. \tag{17}$$

One important difference among CNFR and other NEM based codes is the number of inner iterations per outer iterations, as reported in [3]. In CNFR, for each axial layer into which the core has been divided, two radial nodal mesh sweeps are done, the first one sweeping the columns for each row, while the other sweeps the rows for each column. Note that only these two mesh sweeps are done for each outer iteration. In this way, computing time required to complete one outer iteration is greatly reduced. For the first sweep, one searches the neighboring nodes to each reflector or fuel node being swept, in order to update incoming currents and/or impose the boundary or symmetry conditions. Although results were very good inside the core, there was a motivation to improve the results at reflector nodes. This resulted in the development of this modified treatment of the transverse leakage terms for the external surfaces of reflectors at the core periphery. We point out that in [3], eq. (15) was always used, while in this work eq. (17) was used at those mentioned reflector surfaces.

IV. Results

To test the proposed procedure, the IAEA 3D benchmark problem [8] was used. The calculated effective multiplication factor was 1.029006, with a relative deviation of 0.0003 %. The normalized assembly power and relative percent deviations are shown in figure 2 below.

0.730 0.18	1.285 0.17	1.429 0.45	1.196 0.10	0.610 0.02	0.952 -0.06	0.956 -0.25	0.770 -0.35
1.285 0.17	1.403 0.34	1.436 0.28	1.295 0.31	1.072 0.04	1.055 -0.05	0.971 -0.34	0.749 -0.57
1.429 0.45	1.436 0.28	1.370 0.08	1.314 0.24	1.184 0.26	1.088 -0.03	0.991 -0.59	0.708 0.13
1.196 0.10	1.295 0.31	1.314 0.24	1.181 0.20	0.972 -0.04	0.921 -0.25	0.863 -0.17	
0.610 0.02	1.072 0.04	1.184 0.26	0.972 -0.04	0.475 -0.09	0.694 **-0.77**	0.611 0.44	
0.952 -0.06	1.055 -0.05	1.088 -0.03	0.921 -0.25	0.694 **-0.77**	0.597 -0.04	Power Devia-tion(%)	
0.956 -0.25	0.971 -0.34	0.991 -0.59	0.863 -0.17	0.611 0.44			
0.770 -0.35	0.749 -0.57	0.708 0.13					

Fig. 1 Normalized assembly power

It can be seen, from these results, that the modified leakage treatment was able to calculate the assembly power distribution with a maximum relative deviation of less than 1%, as indicated (in bold) in figure 2. Other test cases were calculated with this modified external surface of reflector nodes, always with accuracies similar to the IAEA test case, including burnup dependent problems [9]. Results at reflector at periphery are better than with the ones obtained using only Eq. (15) throughout the whole core.

V. Conclusions

This paper presents a modified procedure for treatment of reflector surfaces at core periphery with respect to the transverse leakage terms. The proposed method was implemented into a computational system which, besides solving the diffusion equation, also solves the burnup equations governing the gradual changes in material compositions of the core due to fuel depletion. Results presented refer to the IAEA benchmark 3D problem and confirm the effectiveness of the method for practical purposes. For instance, the calculated effective multiplication factor was 1.029006, with a relative deviation of 0.0003 %. Results at reflector at periphery are better than with the ones obtained using only Eq. (14) throughout the whole core. Calculations with burnup dependent problems, not presented here, were also done and results presented good accuracy.

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