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Improved Approximation to the Derivative of a Digital Signal Using Wavelet Transforms for Crosstalk Analysis

S. P. Kozaitis, and R. L. Kriner

***Abstract*—**The information revealed by derivatives can help to better characterize digital near-end crosstalk signatures with the ultimate goal of identifying the specific aggressor signal. Unfortunately, derivatives tend to be very sensitive to even low levels of noise. In this work we approximated the derivatives of both quiet and noisy digital signals using a wavelet-based technique. The results are presented for Gaussian digital edges, IBIS Model digital edges, and digital edges in oscilloscope data captured from an actual printed circuit board. Tradeoffs between accuracy and noise immunity are presented. The results show that the wavelet technique can produce first derivative approximations that are accurate to within 5% or better, even under noisy conditions. The wavelet technique can be used to calculate the derivative of a digital signal edge when conventional methods fail.

***Keywords*—**digital signals, electronics, IBIS model, printed circuit board, wavelets

I. INTRODUCTION

EVERY digital electronic system experiences crosstalk to some extent. Well designed systems can tolerate low to moderate levels of crosstalk without functional upset. Much research and effort has been focused on crosstalk defect prevention [1].

Despite the attention and effort, PCB crosstalk defect prevention is not completely effective. PCBs continue to shrink and must accommodate denser interconnect structures. Accordingly, crosstalk management involves troubleshooting and resolving crosstalk defects that, for a variety of reasons, occasionally surface in modern PCBs. Unfortunately, the tools and techniques commonly used to troubleshoot crosstalk problems and identify the root causes are often far less effective than desired. In practice, the troubleshooting approach often amounts to little more than a series of educated guesses combined with trial and error attempts to isolate and identify the aggressor signal. In modern systems with combinations of hardware, software, and field-programmable gate array (FPGA) firmware, this trial and error process can become a protracted and expensive exercise, consuming valuable engineering resources.

Real-Time Digital Oscilloscopes with long record lengths and sophisticated triggering capabilities can help in locating a voltage glitch caused by crosstalk. Once a glitch is found, the next step is to determine the cause of the glitch. This is often the most difficult and time consuming portion of the task. If important parameters such as the aggressor frequency, distance, directionality, or rise time could be discerned by analysis of the crosstalk glitch, then valuable clues as to its cause can be inferred. In the absence of such a utility, the search for clues as to the origin of the crosstalk must be a manual approach. The first step in the process often begins in the time domain by a visual inspection of the crosstalk glitch on an oscilloscope screen, followed by a careful examination of the PCB layout.

Unfortunately visual inspection in the time domain for the origin of crosstalk often falls short for a number of reasons. In cases such as far-end crosstalk, the glitch is the derivative of the aggressor signal. Therefore, visual inspection of the glitch does not directly provide useful information about the source. However, integration can be useful in such cases. In other cases, it is difficult to visually separate the crosstalk glitch from the victim signal. Reflections and other anomalies may introduce distortion that can make direct measurement of crosstalk glitch characteristics difficult, if not impossible. Derivatives with respect to time can be useful in cases where the crosstalk glitch is corrupted by the victim signal reflections or other anomalies.

In spite of the mathematical tools available for analysis, noise often inhibits their use. For example, differentiation is extremely sensitive to noise. To reduce the effect of noise, advanced filtering methods can be applied. One approach is to use Gaussian filters to smooth a signal prior to further processing [2]. Other edge detection approaches rely on the tuning of filter parameters, window sizes, and other characteristics for optimal performance [3]. With adaptive filtering, improved performance may often be obtained because a filter's characteristics change with the signal and noise. However, these approaches can quickly become complex to implement, and analysis to understand their impact on time domain characteristics can be daunting. In short, these approaches tend to alter the very characteristics of the

S. P. Kozaitis is with the Electrical and Computer Engineering Department Florida Institute of Technology, Melbourne, FL 32901 USA (321-674-7312; fax: 321-674-8291; e-mail: kozaitis@ fit.edu).

R. L. Kriner is with Harris Corporation, Palm Bay, FL 32905 USA (e-mail: rlkriner@harris.com).

crosstalk glitch that the derivative operation sought to reveal in the first place. Therefore a better approach is needed.

Frequency domain methods can be applied; however, the resulting spectrum tends to be a combination of the crosstalk glitch and the victim signal. Rarely does this approach offer meaningful insight into the source of the glitch. In some cases it may be possible to temporally isolate the crosstalk glitch using windowing functions. While this approach can significantly reduce the undesirable artifacts introduced by truncation in the time domain, it introduces artifacts in the frequency domain.

A better approach is to use a transform that would retain the desirable characteristics of both the time and frequency domain, but without the restrictions and limitations described above. Wavelet Transforms offer a potential solution capable of delivering on both points. Wavelet-based edge detection approaches have been proposed, as have wavelet-based derivatives which show promise in preserving edge characteristics while removing noise [4]-[5].

Previous papers have suggested approaches that analyze a crosstalk waveform in order to estimate the aggressor distance and frequency [6], and to classify crosstalk directionality [7]. Our approach asserts that there is a method by which a near-end crosstalk (NEXT) waveform can be analyzed to estimate the rise time of the aggressor. The method requires and relies upon an accurate first derivative operation. In some cases, simply knowing the aggressor rise time may be enough to make a positive identification. In other cases, it may be necessary to take advantage of the aforementioned methods suggested in previous papers to obtain a more complete picture of the aggressor signal.

We present a wavelet-based approach that approximates the first derivative of a digital signal edge. Working with the derivative of the edge can allow the rise time to be determined with greater accuracy and consistency than is possible when using more direct rise-time measurement approaches. This is particularly important in the presence of reflections and other anomalies commonly encountered in the PCB operating environment. Accordingly, the problem we focus on in this paper is calculating the reasonably accurate derivative for a digital signal edge containing noise and minor anomalies that are typical of a PCB operating environment. Our work also addresses concerns about decimation and signal length when considering higher order derivatives.

Applying the wavelet-based derivative to a NEXT waveform is a critical step in pursuing the ultimate goal of estimating the aggressor rise time. Then, subsequent analysis of the first derivative can estimate the rise time, which is a key piece of information about the crosstalk source. Similarly, the wavelet-based derivative can be applied directly to potential aggressor signals, and subsequently analyzed to estimate rise-time. Then, by comparing the estimated rise time of the NEXT glitch with the estimated rise times of the potential aggressors, it may be possible to rule out potential aggressors, or even make a positive identification of the actual aggressor.

II. Mathematical Differentiation using Wavelets

We are interested in the analysis of finite, non-stationary time domain signals over a short time interval, but not necessarily a fixed length time interval. Therefore, we considered wavelet transforms for analysis. The analysis and synthesis equations for the Discrete Wavelet Transform (DWT) are shown below,

$$a_j(k) = \int f(t)\varphi_{j,k}(t)dt\,, \tag{1}$$

$$b_j(k) = \int f(t)\psi_{j,k}(t)dt\,, \tag{2}$$

$$f(t) = \sum_k a_{j0}(k)\varphi_{j0,k}(t) + \sum_k \sum_{j=j0}^{\infty} b_j(k)\psi_{j,k}(t)\,, \tag{3}$$

where, $\varphi_{j,k}(t)$ and $\Psi_{j,k}(t)$ represent the scaling and wavelet functions, respectively. The scaling and wavelet functions are the basis functions of the DWT and the j and k indices represent their time scales and shifts, respectively. The $a_j(k)$ coefficients are commonly referred to as the approximation coefficients, and the $b_j(k)$ coefficients as the detail coefficients.

By inspection of (2), it is clear that the DWT expresses the time domain function $f(t)$ as a weighted sum of basis functions, where the weights are determined by the coefficients $a_j(k)$ and $b_j(k)$ from (1).

During implementation, the analysis coefficients are usually downsampled in order to eliminate redundancy. As a consequence, the DWT is not shift-invariant. In our application it is convenient to keep all the samples in the DWT. Therefore, we used the Stationary Wavelet Transform (SWT), which eliminates the down sampling operation of the outputs. As a result, the SWT has the desirable property of time-shift invariance, and offers the same benefits as the DWT with respect to time-frequency partitioning.

To approximate the first derivative of a signal using wavelet transforms, different-order SWT approximation coefficients can be subtracted from each other [4]. In essence, such an approach performs two separate SWTs of the input signal, each using a different order basis function. Then, the difference between the approximation coefficients is taken as an approximation to the first derivative of the input signal.

Daubechies orthogonal wavelets of order p have p vanishing moments and may be conveniently used to approximate derivatives as described in [4]. Let m and q be any positive integer such that $m \neq q$, and D_{2m} and D_{2q} represent Daubechies orthogonal wavelets of order m and q, respectively. Furthermore, let $C = \{a_j(k)\}$ be the set of approximation coefficients at scale j for all shifts k from (1). Then, the first derivative of sequence $y[n]$ can be approximated as

$$yw'[n] = C_{1,D_{2m}} - C_{1,D_{2q}} . \qquad (4)$$

In addition, higher-order derivative approximations are possible by iteration.

III. Results

A digital NEXT waveform is an amplitude scaled version of the digital aggressor signal from which it was caused. Therefore, if one can analyze a digital signal edge waveform to estimate rise time, then one can apply the same approach to analyze both the aggressor signal, and the resulting NEXT voltage glitch. (In practice, the analysis would first be applied to the NEXT glitch. It would then be applied to potential aggressors in an effort to identify the specific aggressor that caused the NEXT glitch.)

Our objective in this paper is to demonstrate the efficacy of the wavelet-based derivative method, which is a prerequisite step to the analysis that is used to estimate rise time. In this section we present results for a variety of digital signal edge waveforms starting with synthetic long and short rise times to verify the method. Noise is added in some cases to highlight the benefits of the proposed approach. In addition, we show results from both simulation and actual oscilloscope data.

A. 100 ps Gaussian Edge

We initially used a MATLAB-generated Gaussian edge test signal with a 20% to 80% rise time of 100 ps as shown in Fig. 1. We used a single decomposition of the wavelet transform with m = 8, and q = 18. The first derivative of the edge was taken using two different techniques and the results were compared. The derivative was approximated with a conventional method as $yc'[n] = y[n] - y[n-1]$, where n represents a unit sample time interval, and compared to the result obtained using (4).

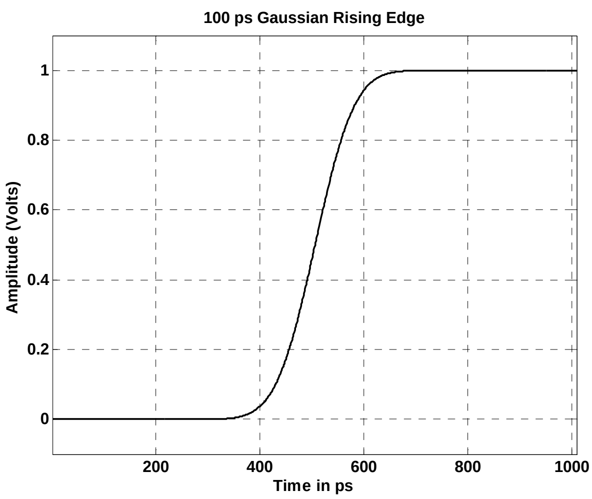

Fig. 1 100 ps Gaussian rising edge without noise.

After shifting and scaling the results of $yc'[n]$, the results appeared visually identical. The percent of full scale error was calculated as

$$\%FSE = \frac{yw'[n] - yc''[n]}{\max(yc''[n])} x100 , \qquad (5)$$

and is shown in Fig. 2. The value was approximately 0.24% at its peak. The lack of perfect symmetry in the error is due to the fractional time shift between the results in this discrete-time system. Nevertheless, it can be seen that the error is relatively small. This suggested that the wavelet-based derivative approach did indeed produce a close approximation to the first derivative of the 100 ps Gaussian edge.

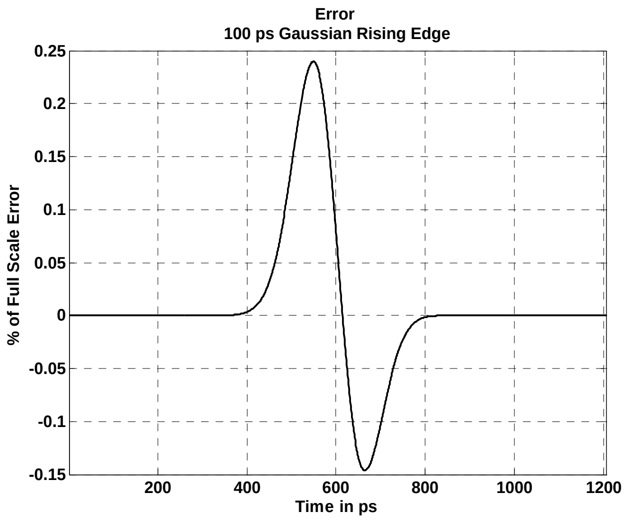

Fig. 2 Error between wavelet-based and conventional derivative for a 100 ps Gaussian rising edge (without noise) using one decomposition level.

In practice, waveforms often contain random noise. Therefore, 10 mV of random, band limited noise was added to the edge, and the derivatives were again calculated and compared. In order to accurately model the characteristics of the noise that would be present in an oscilloscope waveform, we filtered a uniformly-distributed random noise sequence using filter parameters based on the specifications of a commercially available, mature, and popular brand/model digital oscilloscope. Based on the manufacturer's specifications, the Tektronix TDS-7404 Real-Time Digital Oscilloscope has a maximum sample rate of 20 x 10^9 samples per second and an analog bandwidth of 4 GHz.

Using one level of decomposition of the wavelet transform, we calculated the approximation to the derivative of the noisy edge and compared it to the conventional result. The noise had a dramatic effect on both the conventional approximation and wavelet-approximated derivative results as can be seen in Fig. 3. Clearly the wavelet results were less affected by the noise. We attempted to improve results by increasing the wavelet decomposition level. Fig. 4 shows the result with a wavelet decomposition level of three. The results show that the wavelet-based derivative is smoother than when a one level decomposition was used. However, the error with a wavelet decomposition level of three increased to approximately 3.1% peak.

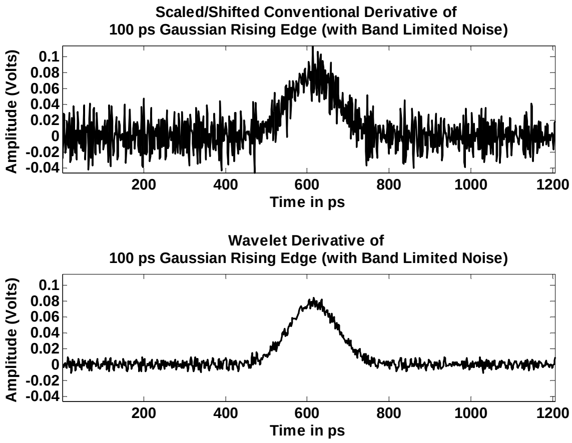

Fig. 3 Comparison of wavelet-based and conventional derivative of a 100 ps Gaussian rising edge with noise using one decomposition level.

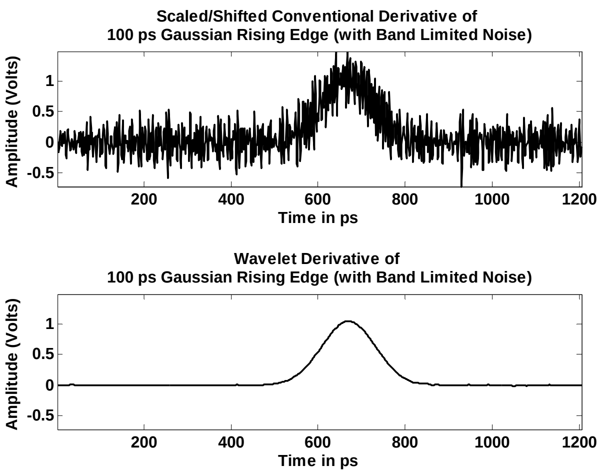

Fig. 4 Comparison of wavelet-based and conventional derivative of a 100 ps Gaussian rising edge with noise using three decomposition levels.

We also considered wavelets that had the minimum number of vanishing moments for our analysis as an extreme case. We used wavelets that had $q = 0$ and $m = 1$ to calculate the derivative. Keeping the decomposition level at three, we again calculated the difference between the wavelet approach and the conventional approach. We found the error was reduced to approximately 0.5% peak. The error when using the wavelet approach was clearly much smaller when a lower order wavelet was used. However when lower order wavelets were used on a noisy signal, the result was not as smooth as when higher order wavelets were used.

B. 2 ns Gaussian Edge

We considered a MATLAB-generated Gaussian edge signal like that in Fig. 1, but with a rise time of 2 ns. Noise was added as in the case of the 100 ps edge discussed earlier. In this case, the wavelet approach produced a recognizable result, whereas the conventional result was completely consumed by and hidden within the noise. Therefore, the wavelet approach was capable of extracting a recognizable derivative from a noisy Gaussian edge when it was not possible to do so with the conventional derivative function. The resulting level three decomposition error was quite small at $\approx$ 0.02% peak. In this case, the benefit of obtaining a recognizable derivative far outweighed the modest error associated with a higher decomposition level.

C. Clock Buffer IBIS Model Waveform

To demonstrate the utility of our approach using more realistic driver models, we analyzed an edge generated by the IBIS Model of the Xilinx Virtex-II Pro FPGA (v2pro.ibs, File Rev. 2.4). The actual waveform was captured using the Cadence SigXP Signal Integrity simulator. Fig. 5 shows the error between the wavelet result and the conventional result with a decomposition level of three.

The results obtained when 10 mV of random, band limited noise was added to the edge are shown in Fig. 6. By inspection, the wavelet approach produced a much smoother result than the conventional approach that was severely distorted and unusable.

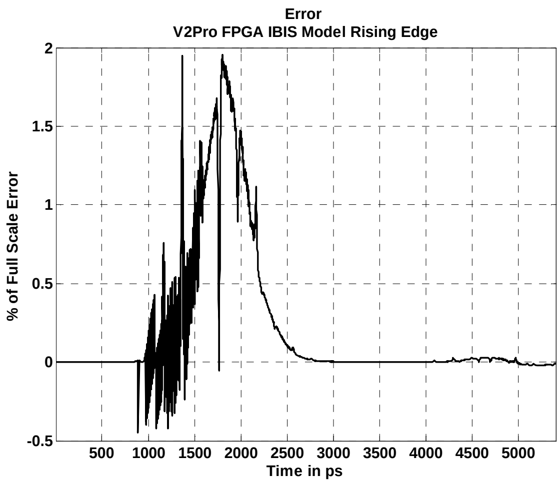

Fig. 5 Error between wavelet-based and conventional derivative for V2PRO_LVTTL_F_12 IBIS Model Edge using using three decomposition levels.

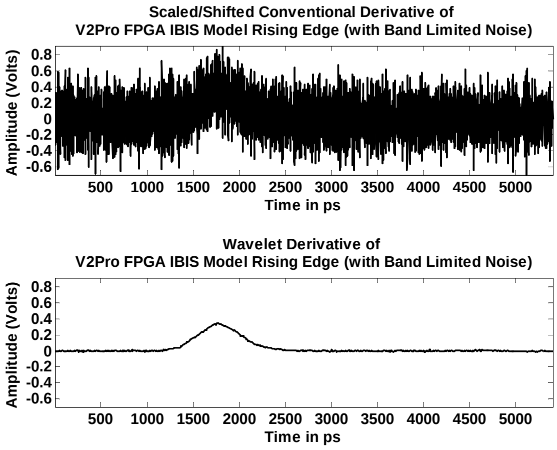

Fig. 6 Comparison of wavelet-based and conventional derivative of an IBIS Model edge with noise using three decomposition levels.

D. Actual Oscilloscope Data

The advantage of the wavelet-based derivative over conventional derivatives became even clearer when we applied both approaches to oscilloscope waveforms from an actual PCB. A Tektronix TDS-7404 Real-Time Digital Oscilloscope was used to capture the waveform presented in this section.

In this test case we analyzed a 53 MHz clock waveform. The sample interval was 100 ps, and the waveform points were stored at the oscilloscope sample rate. Fig. 7 shows the oscilloscope waveform for a rising and falling edge. Note the presence of small nonmonotonicities on the rising edge. These can be caused by a variety of factors including, but not limited to reflections, poor probing techniques, measurements taken too far from the receiver pin, parasitic inductance and capacitance, etc. Fig. 8 shows the first derivative of the clock edges using the conventional approach. It is indeed quite difficult to discern the edge derivatives from the noise. Fig. 9 shows the wavelet result with a decomposition level of one. The edge derivatives are clearly discernable from the noise.

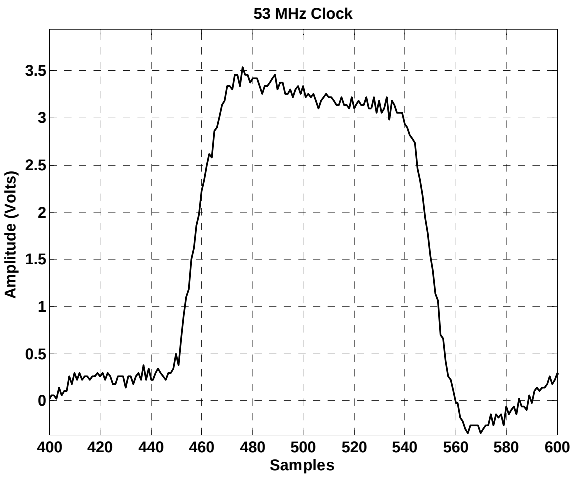

Fig. 7 53 MHz clock waveform (real-time sampled).

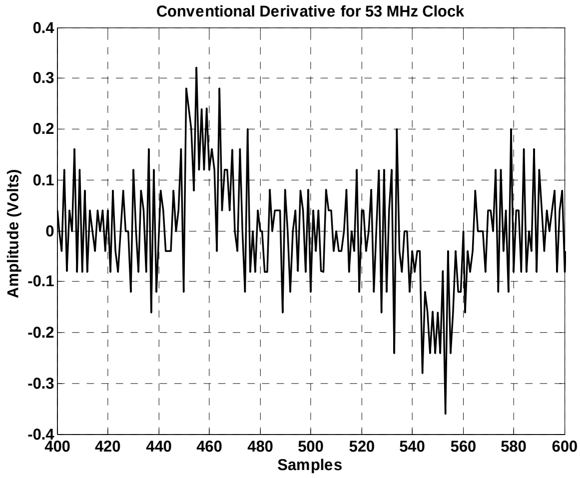

Fig. 8 Conventional derivative for the for 53 MHz clock.

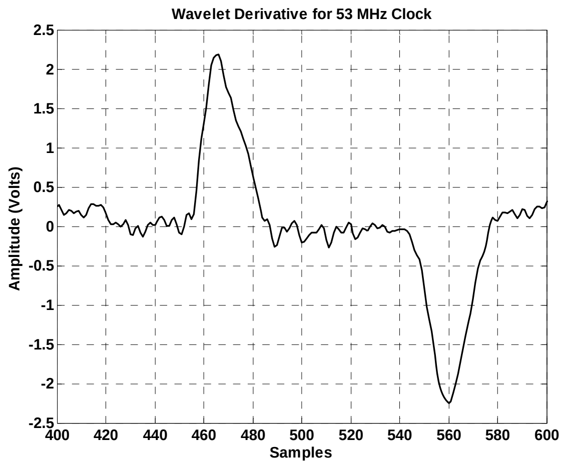

Fig. 9 Wavelet-based derivative for the for 53 MHz clock using wavelet decomposition level 1.

IV. Conclusion

We applied a wavelet-based derivative to Gaussian and IBIS Model digital edges, and digital edges in oscilloscope data from an actual PCB. Once the amplitude was normalized and the time shift removed, approximation errors were found to be suitably low for the purposes of crosstalk analyses. In the presence of noise the wavelet-based derivative produced an improved approximation to the derivative as compared to using a conventional approach.

This work showed that in some cases, the wavelet derivative was able to produce recognizable and useful results when the conventional derivative was overwhelmed by the effects of noise. We found that the wavelet-based derivative offers a tradeoff between accuracy and noise tolerance. Using a higher wavelet decomposition level provides a smoother result in the presence of noise. It also increases the error relative to the result that would be obtained by simply taking the derivative in the conventional way. We noted that error can be reduced by using shorter, lower order wavelets. However, we found that doing so tends to degrade performance in the presence of noise. Further investigation is recommended to determine the optimal tradeoff between wavelet decomposition level and length in a given application.

In the context of the digital signal waveforms considered in this paper, we found that the wavelet-based derivative combines an appropriate filtering stage with a derivative stage, producing suitably accurate derivatives for noisy signals in a single operation. It is reasonable to expect that other types of signal analysis could benefit from the availability of the wavelet-based derivative presented in this paper.

It is acknowledged that applications specific to PCB crosstalk analysis require additional considerations. Factors such as reflections, ringing, overshoot, undershoot, multi-aggressor crosstalk, rise time degradation, modal propagation differences, and crosstalk directionality all serve to complicate crosstalk analyses. This paper presented a method that applies to one piece aspect of the much larger problem.

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