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Implementation of Vertical Neutron Camera (VNC) for ITER Fusion Plasma Neutron Source Profile Reconstruction

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***Abstract*—**In present work the problem of the ITER fusion plasma neutron source parameter reconstruction using only the Vertical Neutron Camera data was solved. The possibility of neutron source parameter reconstruction was estimated by the numerical simulations and the analysis of adequateness of mathematic model was performed. The neutron source was specified in a parametric form. The numerical analysis of solution stability with respect to data distortion was done. The influence of the data errors on the reconstructed parameters is shown:

- is reconstructed with errors less than 4% at all examined values of δ (until 60%);
- is determined with errors less than 10% when δ do not overcome 5%;
- is reconstructed with relative error more than 10 %;
- integral intensity of the neutron source is determined with error 10% while δ error is less than 15%;

where -error of signal measurements, (R_0,Z_0), the plasma center position,- μparameter of neutron source profile.

***Keywords*—**ITER, neutronsource, neutron source profile reconstruction, Vertical Neutron Camera.

I. Introduction

The knowledge of neutron emissivity profile is essential for accurate measurement of total fusion power, position and dynamics of behavior of neutron source. The knowledge of neutron emissivity profile is necessary to study the D-T fusion plasma ignition conditions. The Vertical Neutron Camera of ITER should provide time- and space-resolved measurements of the neutron emissivity and neutron flux, fusion power density and alpha-particle density profile providing information on ITER operation performance. The VNC of ITER is composed of two fan-shaped collimating structures. The total number of collimators is 11. The number of lines of view in the bottom part of VNC is 5, and in the upper part – 6.

Task of the parameters of neutron source reconstruction by VNC measurements can be reduced to the inverse tomographic problem. Such problems arise in many fields of science, medicine and industry when it is important to determine the internal structure of an object without disturbing its integrity. Computed tomography is one of the most common techniques used to solve those problems. A lot of papers are dedicated to the developing of computed tomography methods and fundamental of them should be noted [1]-[7].

Usually tomographic problems are classified by the type of radiation source into transmission tomography and emission tomography. In the first case the object is scanned by external radiation (x-ray, ultrasound, radio waves, etc.). In the second case the intrinsic radiation of the object in some spectral range is used. There is also classification of tomographic schemes based on geometry of the emitters and/or detectors (parallel scheme are used in medical and technical X-ray tomography, cone scheme are used in technical tomography, spiral scheme, etc).

The problem considered in this document relates to the special scheme of flat conical emission tomography with a fixed position of limited number of the detectors. Similar problems are met before in the problem of the optical probing of plasma (see [4]).

Set of issues appears during the solving of the inverse problems but two of them are essential: the problem of uniqueness of the solution that describes the intrinsic structure of the object under study and the problem of instability of approximate solution obtained in the calculations to the errors of experimental data. The problem of uniqueness has been studied for different special cases of the tomographic schemes in [2]-[7]. The main feature of these schemes is the assumption that the investigator has an infinite amount of tomographic data ("projections") and these data correspond to the study of the object from the "different sides". For example radiation of the plasma beam is known in the optical emission scheme can be detected in the plane of its cross-section from each side [4]. Similarly projections are considered in the scheme of X-ray tomography of the parallel rays can be derived by the scanning of the object under any angle in any plane of its cross section. The proper "theorems of uniqueness" are proved with a certain degree of "smoothness" of the solution. In this case any assumptions about type of the solution are not made.

In our case the lack of information on the tomographic projections is obvious: they are limited in number and fixed in position. It makes us to find the problem solution in the special (parametric) form, i.e. we introduce some assumptions

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about the function f and sometimes its constituent parameters.

The peculiarity of inverse tomographic problems is the instability of solutions obtained using procedures of the precise handling of the tomographic operator (Radon operator, the operator in Abel equations, [1]-[7]) against perturbations of data measurement errors. Different techniques have been developed to prevent or reduce this instability: inverse filtering techniques, ridge regression etc.

In the case of the solution parameterization the problem of instability is often an issue of the ill conditioning solution of the corresponding system of equations with unknown parameters. Therefore for each considered parameterization numerical analysis of stability (conditioning) to perturbations of data of the inverse problem is necessary. Instability or ill-conditioning must be eliminated by including additional information about desired parameters in the procedure of the inverse problem solving (for example to set the range of possible values of the parameters and select the "good" initial approximation of the unknown parameters in the iterative search etc.). Procedures such as ridge regression can also be used [8]. It should also be noted that the conditioning of the problem of the object parameters finding is reducing as the number of desired parameters grows often in a catastrophic manner. It also requires a numerical analysis.

In this paper a numerical simulation of the possibility of neutron source parameters reconstruction is investigated. The analysis of adequacy of obtained approximate model parameters to experimental data and the numerical stability analysis of the inverse problem with respect to data perturbations are also carried out. In the numerical solution of the neutron source parameters reconstruction the following tasks are solved:

- Determination of the full intensity of the neutron source within 10% accuracy;
- Determination of the horizontal position of the neutron source center within 10% accuracy;
- Determination of the vertical position of the neutron source center within 10% accuracy;
- Determination of the radial asymmetry;
- Determination of the neutron source distribution shape.

II. Mathematical Problem of the Neutron Source Profile Recovery

Fig. 1 shows a scheme of a Vertical Neutron Camera (VNC), which is used in the calculations. Desired area where plasma locates and contour lines of a source are shown is marked as a large circle. The calculations have been performed for the standard density distributions of ITER neutron source (IDM №: ITER_D_2FV7QR). The number of lines of sight in the lower VNC is 5 and the upper VNC is 6. Digits 1 - 11 show the numbering of the detectors. The closed thin curves on the figure refer to contour lines of the magnetic field. Straight lines indicate the lines of sight determined by detectors' and collimators' locations. In a mathematical model these lines L_i are defined by the parametric equations.

$$R = R_{0i} + a_i t, Z = Z_{0i} + b_i t, t \in [t_{1i}, t_{2i}],\\ i = 1, \ldots M \tag{1}$$

Here (R, Z) is the coordinates of L_i line points, (a_i, b_i) is the direction vector of the i-th line, (R_{0i}, Z_{0i}) is the radiation registration point (the position of detector), t_{1i}, t_{2i} are the parameters which determine the intersection points of the i-th ray with the large circle. M is the number of sight lines. The numbers of lines of sight corresponds to the detector numbers in this paper and are shown as digits in Fig. 1.

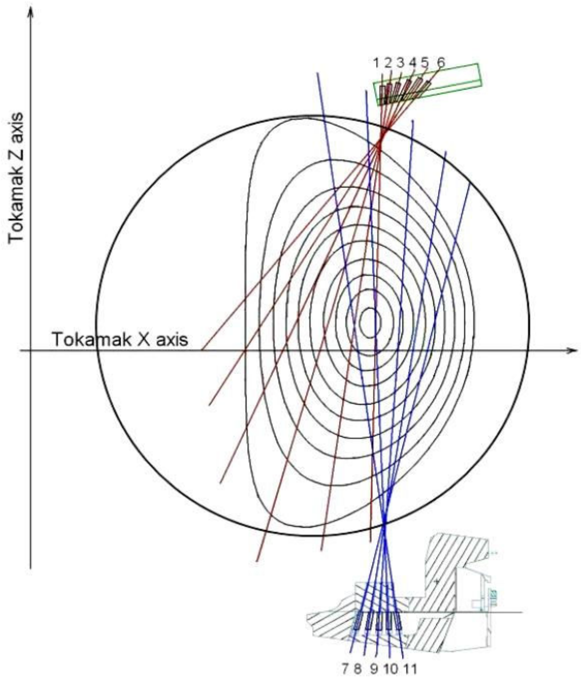

Fig. 1 VNC lines of sight

In general the density of the neutron radiation source located at the point (R_0, Z_0) and detected at the point (R, Z) is written as:

$$\varepsilon(R, Z, R_0, Z_0, \mu, \vec{\sigma}) = N\exp\{-\mu f(R - R_0, Z - Z_0, \vec{\sigma}\} \tag{2}$$

Here N is the factor of normalization that determines the total neutron yield, μ is the parameter of the neutron source distribution shape, f is a function that defines the shape of contour lines of the source and $\vec{\sigma}$ is a vector of parameters defining that shape. Thus the unknown source of neutrons is determined by the function f and parameters $, R_0, Z_0, \mu, \vec{\sigma}$.

In the experiment neutron fluxes at the detectors are modeled as integrals along lines L_i:

$$I_i = \int_{L_i} \varepsilon(R, Z, R_0, Z_0, \mu, \vec{\sigma}) dl, i = 1, \ldots M \tag{3}$$

Normalized data are used for calculations:

$$J_i = \frac{I_i}{\left[\sum_{j=1}^{M} I_j^2\right]^{\frac{1}{2}}} = \frac{\int_{L_j} \varepsilon\left(R_0, Z_0, \vec{\sigma}, R, Z\right) dl}{\left\{\sum_{j=1}^{M}\left[\int_{L_j} \varepsilon\left(R_0, Z_0, \vec{\sigma}, R, Z\right) dl\right]^2\right\}^{\frac{1}{2}}}, \quad (4)$$
$$i = 1, \ldots M$$

Their dependence on unknown parameters $f, R_0, Z_0, \mu, \vec{\sigma}$, can be written as a set of equations (5):

$$\int_{L_i} \exp\left\{-\mu f\left(R - R_0, Z - Z_0, \vec{\sigma}\right)\right\} dl \Big/ \left\{\sum_{j=1}^{M}\left[\int_{L_i} \exp\left\{-\mu f\left(R - R_0, Z - Z_0, \vec{\sigma}\right)\right\} dl\right]\right\}^{1/2} = J_i, \; i = 1, \ldots M. \quad (5)$$

The unknown parameters are the solution of (5).

The set of (5) does not depend on N. If the unknown parameters $f, R_0, Z_0, \mu, \vec{\sigma}$ were found the value of N will be calculated:

$$N = \left[\sum_{j=1}^{M} I_j^2\right]^{1/2} \Big/ \sqrt{\sum_{j=1}^{M}\left[\int_{L_i} \exp\left\{-\mu f\left(R - R_0, Z - Z_0, \vec{\sigma}\right)\right\} dl\right]^2} \quad (6)$$

It is obvious the inverse problem to be ill-posed without knowledge of the continuous function f. It means that this problem has not the only solution and the solution is instable with respect to perturbations of the experimental data. Moreover, if a particular type of the function f is postulated, set of equations (5) may not have any solution because the function f is not physically correct. For a given function f the problem of stability of the set of equations (5) solution is still valid. Otherwise it is replaced by the problem of ill-conditioning of this set of equations.

First, the parametric dependence of $f(R - R_0, Z - Z_0, \vec{\sigma})$ is postulated: the type of the function f as well as the type and a number of parameters $\vec{\sigma}$ are considered to be known. We introduce the normalized residual as a measure of experimental data divergence from the calculated values for the given parameters $\vec{\sigma}$:

$$F^2(R_0, Z_0, \mu, \vec{\sigma}) = \sum_{i=1}^{M}\left\{\int_{L_i} \exp\left\{-\mu f\left(R - R_0, Z - Z_0, \vec{\sigma}\right)\right\} dl \Big/ \sqrt{\sum_{j=1}^{M}\left[\int_{L_i} \exp\left\{-\mu f\left(R - R_0, Z - Z_0, \vec{\sigma}\right)\right\} dl\right]^2} - J_i\right\}^2 \cdot \left(\sum_{i=1}^{M} J_i^2\right)^{-2} \quad (7)$$

The problem of the residual minimization is solved. In other words acceptable parameters $R_0^*, Z_0^*, \mu^*, \vec{\sigma}^*$ that minimize the residual are found:

$$F(R_0^*, Z_0^*, \mu^*, \vec{\sigma}^*) = \min F(R_0, Z_0, \mu, \vec{\sigma}) \quad (8)$$

Then obtained optimal parameters $R_0^*, Z_0^*, \mu^*, \vec{\sigma}^*$ are verified for adequacy in this model: minimum residual $F(R_0^*, Z_0^*, \mu^*, \vec{\sigma}^*)$ is compared with the relative standard error δ of the value J_i. This relative standard error δis calculated using the values $\Delta(I_i)$ (absolute error of the measurement of neutron flux in the i-th detector) and is given by:

$$\delta = \sqrt{\frac{\sum_{i=1}^{M} \Delta^2(I_i)}{\sum_{i=1}^{M} I_i^2}} = \sqrt{\frac{\sum_{i=1}^{M} \Delta^2(\mathrm{J}_i)}{\sum_{i=1}^{M} J_i^2}} \quad (9)$$

Here $\Delta(J_i) = \frac{\Delta(I_i)}{\sqrt{\sum_{j=1}^{M} I_j^2}}$ and $\sum_{i=1}^{M} J_i^2 = 1$. Thus the relative standard error δ of measured value I_i is the same as the error of the J_i. Parameters $R_0^*, Z_0^*, \mu^*, \vec{\sigma}^*$ are considered to be appropriate in the model (1) if the inequality

$$F(R_0^*, Z_0^*, \mu^*, \vec{\sigma}^*) \le \delta \quad (10)$$

is correct.

If the type of the function f and its parameters are physically adequate (10) is satisfied. If the inequality (10) is not satisfied then the function f is believed to be chosen incorrectly and its type or its parameters need to be revised.

Newton type or quasi-Newton type optimization techniques are applied to find the minimum of (8).

In numerical experiments the noise of theith detector measurements is calculated according to the rule

$$\Delta(I_i) = \delta \cdot \zeta_i \left[\sum_{j=1}^{M} I_j^2\right]^{1/2}. \quad (11)$$

Here $\zeta_i = \frac{\xi_i}{\sum_{j=1}^{M} \xi_j^2}$, ξ_i is a random variable uniformly distributed on [-δ, δ]. Such representation of the noise is often used to solve ill-posed problem in practice. In a sense these error is the worst of possible because it is proportional to the sum $\left[\sum_{j=1}^{M} I_j^2\right]^{1/2}$ which is greater than each of the value I_i errors.

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III. The Results of Calculations on Reconstruction Parameters of the ITER Neutron Source

In this paper calculations mainly for sources such as limacon of Pascal are represented. Some calculations for elliptic source are also given for comparison.

Dependence like

$$\varepsilon(R,Z,R_0,Z_0,\mu,\sigma)= N\exp\left(-\mu\left[(R-R_0)^2+((Z-Z_0)/\sigma)^2\right]\right) \quad (12)$$

is considered as an elliptical source. Contour lines of this source are a centered family of ellipses with the same ratio of semiaxes defined by the parameter σ. It should be noted that this representation of source is equal to

$$\varepsilon(R,Z,R_0,Z_0,\mu,\sigma)=N\exp\left(-\nu\left[\frac{(R-R_0)^2}{a^2}+\frac{(Z-Z_0)^2}{b^2}\right]\right),\ \mu=\frac{\nu}{a^2},\ \sigma^2=\frac{b^2}{a^2}. \quad (13)$$

We present the calculation results for the solution of the inverse problem (5), (6) with test data obtained by the following formula

$$L_i=\int_{L_i} N\exp\left(-\mu\left[(R-R_0)^2+\frac{(Z-Z_0)^2}{\sigma^2}\right]\right)dl, \quad (14)$$
$$i=1,...M$$

Here M = 11 detectors, R_0 = 7, Z_0 = 0.5, σ = 1.5, μ=1 and N=10^{16}.

The basic calculations of the inverse problem (5), (6) have been performed for the source which contour lines are given by the curves of type limacon:

$$R=R_p+\Delta_{SH}(\psi)+a(\psi)(\cos t-\delta(\psi)\sin^2 t)$$
$$Z=Z_p+a(\psi)k(\psi)\sin t,\quad t\in[0,2\pi]. \quad (15)$$

Here $\Delta=\Delta_{SH}(\psi),\ a=a(\psi),\ k=k(\psi)$ are known functions which define the shape of each limacon from their family shown in Fig. 3. In this case the source function can be written as follows:

$$\varepsilon(R,Z,R_0,Z_0,\mu)=N\exp\{-\mu[(R-R_0+ +\gamma((Z-Z_0)^2)^2+\frac{(Z-Z_0)^2}{k^2}]\}=N\exp\{-\mu a^2(\psi)\}. \quad (16)$$

where $$\gamma=\frac{\delta(\psi)}{a(\psi)k^2(\psi)}.$$

Source is determined by the unknown parameters R_0, Z_0, μ.

Data 9MA SS at burn-ASTRA and 15MA Inductive at burn-ASTRA are used to set the limacon of Pascal.

The results of the inverse problem solution (5), (6) of parameters R_0, Z_0, μ and N reconstruction (using a solution of the external problem (8) with different values of input data errors are shown in Figs. 2-4 and Table I. The results of calculations are given for the source 15MA SS at burn-ASTRA. The normalized data J_i with no random perturbations (δ=0) added are marked as squares. The results of model calculations of analogs of J_i found with optimal parameters from the problem (8) are shown as circles. One can see a full match of data and their calculated analogs.

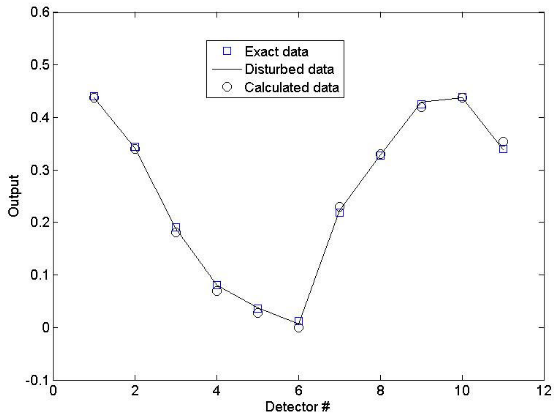

Fig. 2 Implementation of data corresponding with Table I; δ=1%

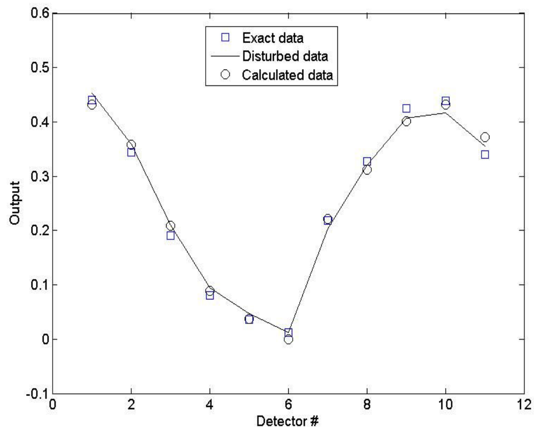

Fig. 3 Implementation of data corresponding with Table I; δ=5%

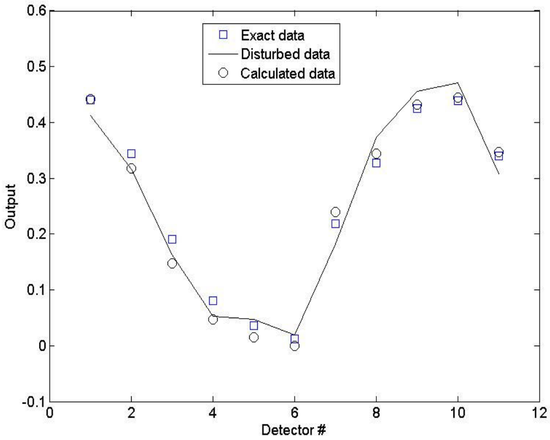

Fig. 4 Implementation of data corresponding with Table I; δ=10%

Similar curves for the perturbation of input data δ=5% are given in the Fig. 3. With the introduction of error input data is presented as the solid line. Fig. 4 shows similar curves with δ=10%. The results of calculation of the parameters and the residual corresponding with Figs. 2-4 are given in Table I (lines 4-6). The dependence of results versus initial approximation of the minimization in the problem (3) when data errors are averaged over 90 realizations is also shown there.

Thus all calculations show the adequacy of obtained approximate model parameters to experimental data. All parameters except Z_0 are reconstructed with acceptable accuracy roughly corresponding to the error δ. An exception is the parameter Z_0 that error of reconstruction is much greater. Recommended relative error in the data of the problem is at least 5%. The averaging of calculated parameters for the ensemble of many implementations of data errors reduces the error of these parameters reconstruction. As the number of parameters grows reconstruction error increases.

TABLE I
RECONSTRUCTED PARAMETERS FOR DIFFERENT VALUES OF ERRORS

	R	Z	μ	Residual	N
Exact solution	6.5	0.5	1.0	-	10^{16}
Initial approximation	5.5	0.3	0.9	-	-
δ=1%	6.4976	0.4693	1.0106	0.0081	$1.0080 \cdot 10^{16}$
δ=5%	6.4338	0.2117	0.9710	0.0423	$9.7264 \cdot 10^{15}$
δ=10%	6.4787	0.2839	1.0680	0.0717	$1.1051 \cdot 10^{16}$
Average value of the parameters with δ=5% (20 realizations)	6.4699	0.3545	0.9763	0.0491	$1.0203 \cdot 10^{16}$
Average value of the parameters with δ=10%	6.4763	0.3128	0.8947	0.0887	$1.0307 \cdot 10^{16}$

IV. The Influence of Error in Data on Accuracy of the Parameters in the Proposed Method of Solving the Inverse Problem

The given calculation results depend on the kind of data errors. The influence of data errors on accuracy of reconstruction of unknown parameters for the calculation of Section III is considered in this section. With that purpose for the elliptic source and for the source of limacon calculations with different values of error δ have been performed to determine the parameters with 20 implementations of noise (for each δ). The weak dependence of the found parameters on δ has been obtained for the elliptic source. Errors in determining the parameters R_0, μ are satisfactory. The parameter R_0 is restored with a small error for each δ and the parameter μ is restored within an accuracy of 10% if δ does not exceed 5%. At the same time the parameter Z_0 is restored with a great relative error. When δ = 1% - 2% all the parameters are restored within accuracy of 10%.

The calculated results of influence of data errors on the accuracy of reconstruction of unknown parameters for limacon (source 15MA SS) are shown in Figs. 5-8.

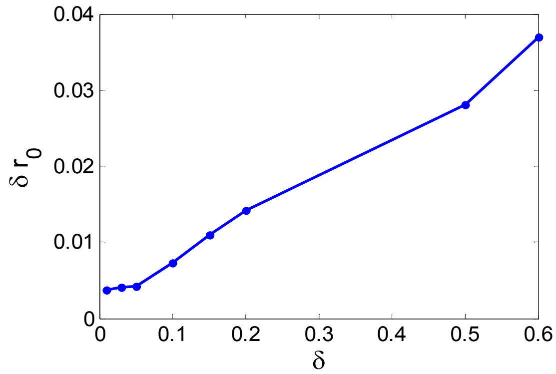

Fig. 5 The relative error in the determination of R_0

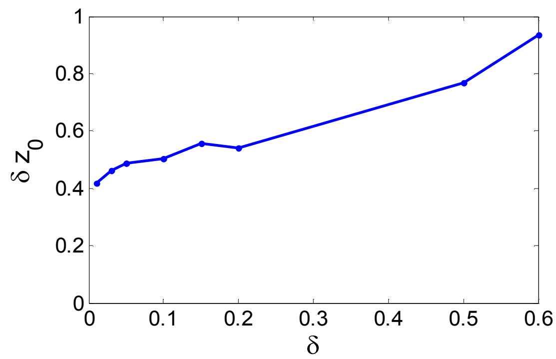

Fig. 6 The relative error in determining of Z_0

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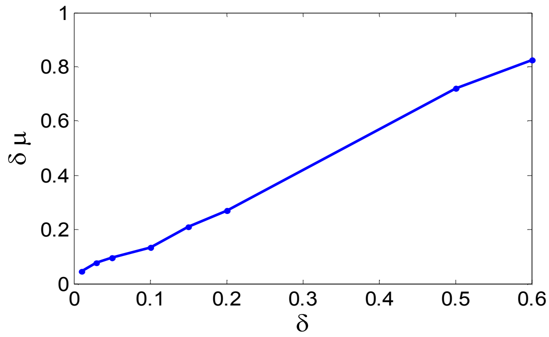

Fig. 7 The relative error in determining of μ

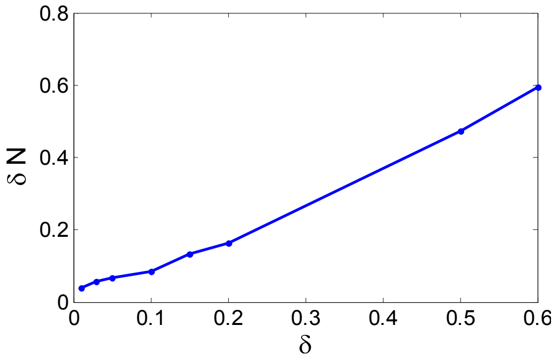

Fig. 8 The relative error in determining of *N*

It has been found that error of the parameters reconstruction are generally less in the case of asymmetric neutron source (limacon) than in the case of elliptic source. The results of the source parameters reconstruction for limacon (source 15MA) and cases where one detector lost are shown in Table II. Inverse problem data have an error of 5%. Parameters are compared with their exact values and initial approximation and the case when all detectors in operation. The calculations are averaged over 15 implementations of data errors.

One can see from the table the loss of a single detector in this model configuration has little effect on the results. The most critical is the loss of the detector number 7.

TABLE II
RECONSTRUCTED PARAMETERS IN CASE OF ONE DETECTOR LOST

	R_0	Z_0	μ	Residual	N
Exact solution	6.5	0.5	1.0	-	$\cdot 10^{16}$
Initial approximation	6.0	0.4	0.9	-	-
All detectors	6.4864	0.4451	0.9809	0.0409	$9.9013\cdot 10^{15}$
№ 1 lost	6.4835	0.4400	0.9958	0.0396	$9.9602\cdot 10^{15}$
№ 2 lost	6.4898	0.4455	0.9613	0.0407	$9.7698\cdot 10^{15}$
№ 3 lost	6.4917	0.4389	0.9915	0.0399	$9.9781\cdot 10^{15}$
№ 4 lost	6.5001	0.4661	0.9768	0.0392	$9.8552\cdot 10^{15}$
№ 5 lost	6.5011	0.4459	0.9745	0.0377	$9.8867\cdot 10^{15}$
№ 6 lost	6.4981	0.4809	0.9958	0.0398	$1.0027\cdot 10^{15}$
№ 7 lost	6.4954	0.3918	0.9716	0.0391	$9.8674\cdot 10^{15}$
№ 8 lost	6.4973	0.4754	1.0044	0.0414	$1.0025\cdot 10^{15}$
№ 9 lost	6.4921	0.4414	0.9709	0.0398	$9.8530\cdot 10^{15}$
№ 10 lost	6.4964	0.4869	0.9951	0.0385	$9.9963\cdot 10^{15}$
№ 11 lost	6.4906	0.4495	1.0165	0.0400	$1.0172\cdot 10^{15}$

V. CONCLUSION

1) Calculations have shown the adequacy of the obtained approximate model parameters to experimental data. The influence of data errors on the error of parameters reconstruction is shown:

- The parameter R_0 is reconstructed within accuracy of 4% for all considered δ(until 60%);
- The parameter μ is reconstructed within accuracy of 10% if δ does not exceed 5%;
- The parameter Z_0 is recovered with a relative error more than 10%;
- The total intensity of the neutron source is determined with an accuracy of 10% if δ does not exceed 15%;
- All parameters are recovered within an accuracy of 10% when δ = 1% - 2%.

2) The averaging of calculated parameters over the ensemble of many implementations of data errors reduces the error recovery of these parameters.
3) As the number of parameters grows reconstruction error increases.
4) Error of the parameters reconstruction is generally less in the case of asymmetric neutron source (limacon) than in the elliptic source.
5) The loss of a single detector has little effect on the results of the source parameters reconstruction for limacon case. The most critical is the loss of the detector number 7.

ITER needs various neutron diagnostic systems able to measure the neutron emissivity within 10% accuracy, with a temporal resolution of 1 ms and spatial resolution of a tenth of the minor plasma radius, i.e. 200mm. It has been shown in the numerical analysis that all parameters (except Z_0, μ) are reconstructed within required accuracy of 10% if the accuracy of measurements in each channel does not exceed 10%.

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