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Image Modeling Using Gibbs-Markov Random Field and Support Vector Machines Algorithm

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Abstract**—**This paper introduces a novel approach to estimate the clique potentials of Gibbs Markov random field (GMRF) models using the Support Vector Machines (SVM) algorithm and the Mean Field (MF) theory. The proposed approach is based on modeling the potential function associated with each clique shape of the GMRF model as a Gaussian-shaped kernel. In turn, the energy function of the GMRF will be in the form of a weighted sum of Gaussian kernels. This formulation of the GMRF model urges the use of the SVM with the Mean Field theory applied for its learning for estimating the energy function. The approach has been tested on synthetic texture images and is shown to provide satisfactory results in retrieving the synthesizing parameters.

Keywords**—**Image Modeling, MRF, Parameters Estimation, SVM Learning.

I. INTRODUCTION

THE subject of image modeling involves the construction of models or procedures for the specification of images. These models serve a dual role in that they can describe images that are observed and also they can generate synthetic images from the model parameters. This paper concerns with a specific type of image models, the class of texture models. There are important areas of image processing in which texture plays an important role: for example, classification, image segmentation, and image encoding. Julesz [1] considers the problem of generation of familiar textures from the theoretical and practical viewpoints. In addition, understanding texture is an essential part of understanding human vision [2]. These considerations have led to an increased activity in the area of texture analysis and synthesis.

Markov random field (MRF) models have been successfully used to represent contextual information in many 'site' labeling problems. A site labelling problem involves classification of each site (pixel, edge element, and region) into a certain number of classes based on an observed value (or vector) at each site. Contextual information plays an important role here because the true label of a site is assumed to be compatible with the labels of the neighboring sites. Markov random fields are appropriate models of context because they can be used to specify this spatial dependency or spatial distribution. The class of MRF with exponential priors can be described, equivalently, by Gibbs models. Hence, for this vast class, the parameters of the MRF can be specified in terms of the clique potentials in the Gibbs distribution [3]. A Gibbs-Markov model is specified by the model order and the set of clique potentials. The flexibility of the Gibbs-Markov random fields generated considerable research interest in the past three decades. However, the problem of parameter estimation has remained to a large extent unsolved. Therefore, a systematic method for specification of these parameters is of significant interest. Several schemes have been proposed in the computer vision literature to estimate the parameters of an MRF model [4-7]. For MRFs defined on pixel sites (e.g. texture modeling), these schemes have been applied with considerable success. For MRFs defined in edge sites [5] (line variable used to denote discontinuity between adjacent pixels), however, the available parameter estimation techniques are difficult to apply because of the lack of true edge labels. Also the Least squares (LS) method is not accurate [6].

The Support Vector Machines (SVM) has been introduced and used intensively for solving pattern recognition problems either as direct classifiers [8], or as a density estimation algorithm [9]. In this paper we introduced a new approach based on using the SVM to estimate the energy function of the GMRF model. The potential function for each clique is assumed as a Gaussian-shaped kernel. This assumption leads to the formulation for the energy function as a weighted sum of Gaussian kernels. The SVM is used then to estimate the parameters of this energy function. In the proposed approach the Mean Field theory is used in the learning of the SVM to estimate the weight of each Gaussian kernel associated with a clique shape.

Experimental evaluation on modeling synthetic texture images show that the proposed algorithm managed to retrieve the parameters by which the image was synthesized. Also, several synthesizing experiments are provided to illustrate the performance of the proposed algorithm.

II. MARKOV RANDOM FIELD

The study of Markov random fields has had a long history, beginning with Ising thesis on ferromagnetism [10]. Although it did not prove to be a realistic model for magnetic domains, it is approximately correct for phase-separated alloys, idealized gases, and some crystals. The model has traditionally been applied to the case of either Gaussian or binary variables on lattice. Besag [4] allows a natural extension to the case of

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variables that have integer ranges, either bounded or unbounded. These extensions, coupled with estimation procedures, permit the application of the Markov random field to texture modeling. Before going through the modeling procedure, a couple of important definitions are provided.

Definition 1: A clique c is a subset of S for which every pair of sites is a neighbor. Single pixels are also considered cliques. The set of all cliques on a grid is denoted by C .

Definition 2: A random field X is an MRF with respect to the neighborhood system $\eta = \{\eta_s, s \in S\}$ if and only if:

a. $p(X = x) > 0$ for all $x \in \Omega$, where Ω is the set of all possible configurations on the given grid;

b. $p(X_s = x_s \mid X_{s|r} = x_{s|r}) = p(X_s = x_s \mid X_{\partial s} = x_{\partial s})$, where $s|r$ refers to all N^2 sites excluding site r, and ∂s refers to the neighborhood of site s .

Definition 3: X is a Gibbs random field (GRF) with respect to the neighborhood system $\eta = \{\eta_s, s \in S\}$ if and only if:

$$p(x) = \frac{1}{z} e^{-E(x)} \tag{1}$$

where z is a normalization constant called the partition function and $E(x)$ is the energy function of the form:

$$E(x) = \sum_{c \in C} V_c(x) \tag{2}$$

where V_c is called the clique potential. Generally, V_c is a function of the cliques around the site under consideration.

Only cliques of size 2 are involved in a pairwise interaction model. The energy function for a pairwise interaction model can be written in the form [4]:

$$V(x) = \sum_{i=1}^{N^2} G(x_t) + \sum_{i=1}^{N^2} \sum_{r=1}^{m} H(x_t, x_{t:+r}) \tag{3}$$

where G is the potential function for single-pixel cliques and H is the potential function for all cliques of size 2. The parameter m depends on the size of the neighborhood around each site. For example, m is 2, 4, 6, 10, and 12 for neighborhoods of orders 1, 2, 3, 4, 5, respectively. Numbering and order coding of the neighborhood up to order five is shown in Fig. (1). Also Fig. (1-1) shows the location of site $x_{t:+r}$ in the neighborhood system.

In this paper we proposed the following model for G and H potential functions.

$$G(x_t) = \frac{w_0}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_{w_0} - I(x_t)}{\sigma}\right)^2} \tag{4}$$

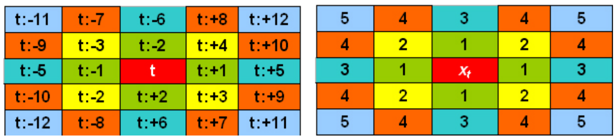
$$H(x_t, x_{t:+r}) = \frac{w_r}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\mu_{w_r} - I(x_t, x_{t:+r})}{\sigma}\right)^2} \tag{5}$$

where $I(a,b)$ is the indicator function where $I(a,b) = 1$ if $a = b$, otherwise equal 0 . $I(a)$ is always equal 1 .

The estimated mean values (μ_{w_r}) of the clique shapes of second order MRF (shown in sequence at the bottom of Fig.(3)) are shown in TABLE I.

TABLE I
THE REFERENCE MEANS FOR 2ND ORDER MRF CLIQUES.

Parameter	μ_{w_0}	μ_{w_1}	μ_{w_2}	μ_{w_3}	μ_{w_4}
Value	1/21	3/21	5/21	7/21	9/21
Parameter	μ_{w_5}	μ_{w_6}	μ_{w_7}	μ_{w_8}	μ_{w_9}
Value	11/21	13/21	15/21	17/21	21/21



(a) (b)

Fig. 1 Numbering and order coding of neighborhood structure

III. Support Vector Machines Regression

In this paper the Support Vector Machines (SVM) technique is used as a regression algorithm for estimating the parameters of the GMRF model. The details of the SVM regression can be found in our previous work [9], but here only the main outlines of the algorithm are presented. In the following, the SVM as a regression tool is considered as the maximum a posteriori prediction with a Gaussian prior under the Bayesian framework.

Thus, the output from the SVM regression for the sample $\mathcal{D}$ is represented as a Gaussian process with a zero mean in the following form:

$$p(\mathbf{g}(\mathcal{D})) = \frac{1}{\sqrt{2\pi \det(\mathcal{K}_n)}} \exp\left\{-\frac{1}{2}\mathbf{g}(\mathcal{D})\mathcal{K}_n^{-1}\mathbf{g}(\mathcal{D})^T\right\} \tag{6}$$

where $\mathcal{K}_n = [\mathcal{K}(\mathbf{x}_i, \mathbf{x}_j)]$ is the covariance matrix at the points of $\mathcal{D}$ and $\mathbf{g}(\mathcal{D})$ is the SVM output vector.

The performance of the SVM regression algorithm is characterized by the Vapnik's ε-loss function which has the form:

$$\mathcal{L}(t, g(\mathbf{x})) = \begin{cases} 0 & \text{if } |t - g(\mathbf{x})| \le \varepsilon \\ |t - g(\mathbf{x})| - \varepsilon & \text{otherwise} \end{cases} \tag{7}$$

Depending on this loss function, the likelihood of the target output vector $\mathcal{T}$ given the actual SVM output will be in the form:

$$p(\mathcal{T} \mid \mathbf{g}(\mathcal{D})) = \left(\frac{C}{2(\varepsilon C + 1)}\right)^n \exp\left\{-C\sum_{i=1}^{n} \mathcal{L}(t_i, g(\mathbf{x}_i))\right\} \tag{8}$$

where $\mathcal{T} = [t_1, t_2, \ldots, t_n]$.

Using Equations (6) and (8) and from Bayes' theorem:

$$p(\mathbf{g}(\mathcal{D})\,|\,\mathcal{D}) = \frac{p(\mathcal{D}\,|\,\mathbf{g}(\mathcal{D}))\,p(\mathbf{g}(\mathcal{D}))}{p(\mathcal{D})}$$

$$= \frac{\mathcal{M}\exp\left\{-C\sum_{i=1}^{n}\mathcal{L}(t_i, g(\mathbf{x}_i)) - \frac{1}{2}\mathbf{g}(\mathcal{D})\,\mathcal{K}_n^{-1}\,\mathbf{g}(\mathcal{D})^T\right\}}{\sqrt{2\,\pi\det(\mathcal{K}_n)}\,p(\mathcal{D})} \quad (9)$$

where:

$$M = \left(\frac{C}{2(\varepsilon C+1)}\right)^n .$$

Using the posterior prediction distribution $p(\mathbf{g}(\mathcal{D})\,|\,\mathcal{D})$, which is defined in Eq. (9), the predicted (expected) SVM output on a new test point $\mathbf{x}$ is given by:

$$g(\mathbf{x}) = \int g(\mathbf{x})\; p(g(\mathbf{x})\,|\,\mathcal{D})\; d\,g(\mathbf{x})$$
$$= \int g(\mathbf{x})\; p(g(\mathbf{x}), \mathbf{g}(\mathcal{D})\,|\,\mathcal{D})\; d\,g(\mathbf{x})\, d\,\mathbf{g}(\mathcal{D}) \quad (10)$$

Substituting in Eq. (10) from the previous equations and after some mathematical reductions which are omitted for space limitations, the output $g(\mathbf{x})$ of the SVM regression algorithm has the form:

$$g(\mathbf{x}) = \sum_{i=1}^{n} w_i\,\mathcal{K}(\mathbf{x}_i, \mathbf{x}) \quad (11)$$

where n is the number of samples in the training set $\mathcal{D}$, $\mathcal{K}(\mathbf{x}_i, \mathbf{x})$ is the kernel function used by the SVM regression algorithm and w_i 's are the weight coefficients.

There are different methods for estimating the weight coefficients in Eq. (11). One of the new algorithms is suggested in our recent work [9] which uses the Mean Field theory to approximate an efficient and fast learning algorithm for the SVM to estimate these weight coefficients. This learning algorithm suggests that the distribution of the SVM output $p\,(\,g(\mathbf{x}_i)\,|\,\overline{\mathcal{D}})$ corresponding to a training instant given the rest of the training instants is approximated using the Mean Field theory by a more simple distribution. In our implementation, a Gaussian distribution function is used for approximating $p\,(\,g(\mathbf{x}_i)\,|\,\overline{\mathcal{D}})$. The details for the learning procedure can be found in [9].

IV. GMRF Parameters Estimation using SVM

Comparing the form of the potential function of the GMRF model in Eq. (3) with that of the SVM regression output in Eq. (11) shows that the SVM can be used for estimating the GMRF parameters provided that the SVM regression algorithm uses a Gaussian Radial Basis-shaped kernel. In order to estimate the weights in the SVM regression representation which correspond to the strengths of the cliques in the GMRF representation, the joint histogram for all clique shapes in the given image are calculated. The MF-based SVM regression algorithm is used to approximate (fit a regression to) the joint histogram by estimating the weights in Eq. (11), which correspond to the MRF parameters as stated before.

V. Experimental Results

Figure (2) shows a texture image generated by Metropolis algorithm [11]. Figure (3) shows the joint histogram for the ten cliques shape of the second order neighborhood system.

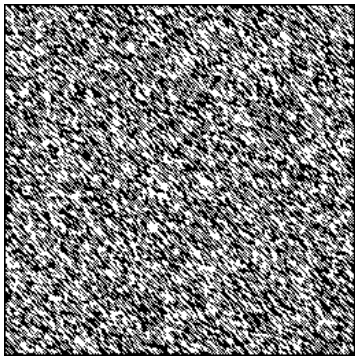


Fig. 2 A texture image generated by Metropolis algorithm

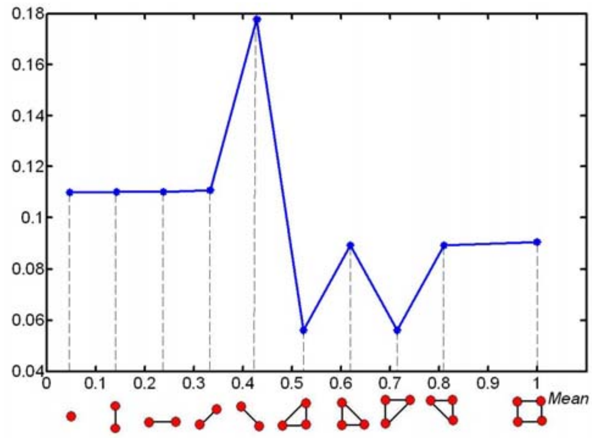


Fig. 3 The joint histogram for the ten clique shapes of the 2nd order MRF model

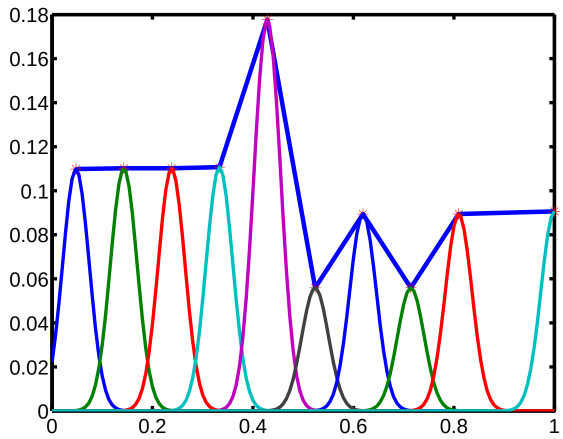


Fig. 4 The estimated mixture of Gaussians distributions using MF-based SVM

Figure (4) shows the estimated Mixture of Gaussian distribution using the SVM which shows that the SVM manages to estimate optimal values for the clique strengths. TABLE II, shows the estimated parameters for each distribution. Figure (5) shows the regenerated image using the estimated parameters shown in TABLE II. More results obtained by the proposed approach are shown in Fig. (6).

TABLE II
ESTIMATED PARAMETERS FOR THE MIXTURE OF GAUSSIAN DISTRIBUTIONS

Component	Mean	Weight	Variance
1	1/21	0.1098	0.1592
2	3/21	0.1102	0.1592
3	5/21	0.1102	0.1592
4	7/21	0.1107	0.1592
5	9/21	0.1777	0.1592
6	11/21	0.0559	0.1592
7	13/21	0.0894	0.1592
8	15/21	0.0559	0.1592
9	17/21	0.0894	0.1592
10	21/21	0.0906	0.1592

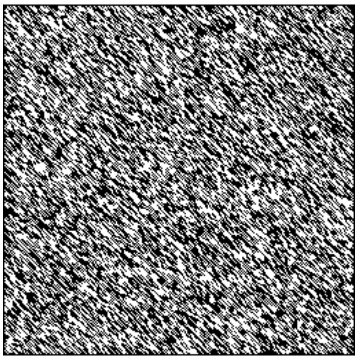


Fig. 5 Regenerated texture image using the estimated parameters in Table 2

VI. CONCLUSION

In this paper we introduced a novel approach to estimate the clique potentials in Gibbs-Markov image models. The proposed approach is based on modeling the potential function associated with each clique shape as a Gaussian-shaped kernel. In turn, the energy function of the GMRF will be in the form of a weighted sum of Gaussian kernels. Using the SVM with the Mean Field theory applied for its learning, we estimate the energy function. The approach has been tested on synthetic texture images and the results show that it provides satisfactory results.

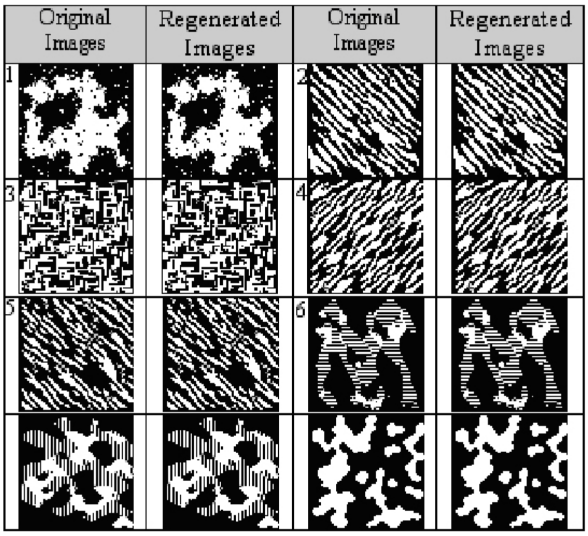


Fig. 6 Original and regenerated images using the proposed approach

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