

# Hydrodynamic Processes in Bubbly Liquid Flow in Tubes and Nozzles

Raisa Kh. Bolotnova, Marat N. Galimzianov, Andrey S. Topolnikov, Valeria A. Buzina, Uliana O. Agisheva

**Abstract**—The hydrodynamic processes in bubbly liquid flowing in tubes and nozzles are studied theoretically and numerically. The principal regularities of non-stationary processes of boiling liquid outflow are established under conditions of experiments when the depressurization of a tube with high pressure inside occurs. The steady-state solution of bubbly liquid flow in the nozzle of round cross section with high pressure and temperature conditions inside bubbles is studied accounting for phase transition and chemical reactions.

**Keywords**—bubbly liquid, cavitation, chemical reactions, phase transition.

## I. INTRODUCTION

WHEN the liquid flows through the channels and tubes a number of interesting effects appear, which are originated from the processes of phase exchange, heat and mass transfer and the geometry of nozzles. The presence of bubbles in the liquid influences the velocity and amplitude of acoustic and shock waves, and ultrahigh pressures and temperatures can be achieved inside the bubbles [1], [2].

The theoretical and experimental research of the explosive boiling of the underheated liquid, which occurs as a result of tube depressurization, has a great value in power engineering as it connected with problems of safety of energetic machines. The equilibrium regimes of outflow of underheated liquid were studied theoretically in [3] and [4]. The influence of the non-equilibrium state on the critical outflow of the liquid with bubbles was studied in [5]–[7] with assumption of weak compressibility of liquid phase. In [8] the theory of “mechanical nucleation” was presented, according to which the liquid starts to boil on the surface of small amount of impurities, and the generated bubbles break into small pieces due to instability of bubble interface during bubble growth. At the present paper the numerical simulation of non-stationary processes of the boiling liquid outflow has been carried out under conditions of experiments [9] by using the wide-range equation of state of steam and water [10], when the depressurization of tube with high pressure inside occurs.

R. Kh. Bolotnova, Dr. Sc., Principal scientist of Mavlutov Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: bolotnova@anrb.ru).

M. N. Galimzianov, PhD, Senior scientist of Mavlutov Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: monk@anrb.ru).

A. S. Topolnikov, PhD, Senior scientist of Mavlutov Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (corresponding author, tel./fax: +7-347-2355255, e-mail: tandrew@anrb.ru).

V. A. Buzina, postgraduate student, Junior scientist of Mavlutov Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: valeria@imech.anrb.ru).

U. O. Agisheva, postgraduate student, Junior scientist of Mavlutov Institute of Mechanics Ufa Branch of the Russian Academy of Sciences, 450054 Russia, Ufa, Prospect Oktyabrya 71 (e-mail: uliana@imech.anrb.ru).

One of the methods of obtaining of high pressure and temperature in the matter is the stimulation of radial oscillations of bubbles and bubble clusters by single or periodical impulses of pressure [1], [11]. It was observed experimentally, that under conditions of cavitation the flow of benzene through the nozzle produces nano-particles with diamond inclusions [12]. The theoretical investigation of the main features of such flow under the experimental conditions enables to explain the nature of the phenomenon and can be put into the base for the development of continuous technological processes, which carried out at high pressure and temperature conditions. At the present paper the problem of steady-state flow of bubbly liquid is solved in the nozzle with round cross section taking into account for phase transition and chemical reactions. The theoretical analysis is done to explore the possibility of realization of high temperature and pressure inside the gas bubbles in the narrow part of the nozzle.

## II. THE OUTFLOW OF BOILING LIQUID FROM THE TUBE WITH HIGH PRESSURE

### A. Governing Equations

Consider the closed tube with diameter  $D = 0.075$  m and length  $L = 4.1$  m containing water with pressure  $p_0 = 7$  MPa and temperature, which is less than the saturation temperature  $T_0 = 515$  K ( $T_0 \leq T_s(p_0)$ ). According to the modeled experiment [9] the outflow of boiling liquid starts when the valve at one of the endings of the tube is opened. The system of differential equations of conservation of mass, momentum and energy of two-phase mixture in Lagrangian coordinates can be written in the following form, if we assume the one-dimensional, single velocity and temperature model with equal pressure in both phases [7]:

$$\frac{\alpha_1}{\rho} \frac{\partial \rho_1^0}{\partial t} + \frac{\rho_1^0}{\rho} \frac{\partial \alpha_1}{\partial t} = -\frac{\rho_1}{\rho_0} \frac{\partial u}{\partial r} - \frac{J_{12}}{\rho} = b_1, \quad (1)$$

$$\frac{1-\alpha_1}{\rho} \frac{\partial \rho_2^0}{\partial t} - \frac{\rho_2^0}{\rho} \frac{\partial \alpha_1}{\partial t} = -\frac{\rho_2}{\rho_0} \frac{\partial u}{\partial r} + \frac{J_{12}}{\rho} = b_2, \quad (2)$$

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial r}, \quad u = \frac{\partial x}{\partial t}, \quad (3)$$

$$\begin{aligned} & \frac{\rho_1}{\rho} \left( \frac{\partial e_1}{\partial \rho_1^0} \right)_T \frac{\partial \rho_1^0}{\partial t} + \frac{\rho_2}{\rho} \left( \frac{\partial e_2}{\partial \rho_2^0} \right)_T \frac{\partial \rho_2^0}{\partial t} + \\ & + \left[ \frac{\rho_1}{\rho} \left( \frac{\partial e_1}{\partial T} \right)_{\rho_1^0} + \frac{\rho_2}{\rho} \left( \frac{\partial e_2}{\partial T} \right)_{\rho_2^0} \right] \frac{\partial T}{\partial t} = b_3, \end{aligned} \quad (4)$$

where

$$b_3 = -\frac{\rho_1}{\rho_0} \frac{\partial u}{\partial r} - (e_2 - e_1) \frac{J_{12}}{\rho}.$$

The condition of equal pressures in both phases leads to the following formula:

$$\left(\frac{\partial p_1}{\partial \rho_1^0}\right)_T \frac{\partial \rho_1^0}{\partial t} - \left(\frac{\partial p_2}{\partial \rho_2^0}\right)_T \frac{\partial \rho_2^0}{\partial t} + \left[\left(\frac{\partial p_1}{\partial T}\right)_{\rho_1^0} - \left(\frac{\partial p_2}{\partial T}\right)_{\rho_2^0}\right] \frac{\partial T}{\partial t} = 0 \quad (5)$$

Here and further the bottom indexes  $i = 1, 2$  refer to the liquid and gas (vapor) phases of water,  $p_i(\rho_i^0, T)$  and  $e_i(\rho_i^0, T)$  are the pressure and internal energy of the phases, which are the functions of density  $\rho_i^0$  and temperature  $T$ ;  $\alpha_i$ ,  $\rho_i = \rho_i^0 \cdot \alpha_i$  are the void fraction and reduced density of the  $i$ -th phase,  $\rho = \rho_1^0 \cdot \alpha_1 + \rho_2^0 \cdot \alpha_2$ ,  $\rho_0$  are the average current and initial values of density,  $r$  and  $x$  are the Lagrangian and Eulerian coordinates,  $u$  is the mass velocity,  $J_{12}$  is the rate of phase transition. The radius of bubbles at the time moment  $t$  is defined by void fraction of vapor phase  $\alpha_2$  and number of bubbles  $n$

$$a(r, t) = \sqrt[3]{\frac{3\alpha_2(r, t)}{4\pi n(r, t)}} \quad (6)$$

The thermodynamic properties of vapor-liquid mixture are described by the equation of state of steam and water [10] in Mie-Gruneisen form as a sum of potential (cold) and heat components of pressure and internal energy:

$$p = p^{(p)} + p^{(T)}, \quad e = e^{(p)} + e^{(T)} + e^{(ch)}, \quad (7)$$

where  $e^{(ch)}$  is the constant of interpolation of internal energies of vapor and liquid phases.

The cold component of pressure and internal energy has the form of Born-Mayer potential [10]. The liquid and vapor phases are described by equation of state with single pressure and different heat capacities and Gruneisen coefficients:

$$p^{(T)}(V_i^0, T) = \frac{\Gamma_i(V_i^0)c_{Vi}}{V_i^0} T, \quad e_i^{(T)} = c_{Vi} T. \quad (8)$$

The equation of kinetics of phase transition at the water-steam interface  $J_{12}$  depends on bubble number  $n$  and bubble radius  $a$ , saturation temperature  $T_s(p)$ , heat of vaporization  $l_s(T)$  and coefficient of heat conductivity  $\lambda_1$

$$J_{12}(r, t) = 2\pi a(r, t)n(r, t)\text{Nu} \frac{\lambda_1(T - T_s(p))}{l_s(T)}, \quad (9)$$

where Nu is the Nusselt number [13].

### B. Modeling of Overheated Liquid State

According to the experimental data [9] at the initial stage of liquid outflow after depressurization of the tube till the time moment  $t \approx 5$  ms a rarefaction wave is propagating along the tube. When it reaches the closed ending of the tube the uniform pressure is stated  $p \approx 2.8$  MPa. The phase transition water-steam (9) at overheated state takes place when the liquid temperature exceeds the temperature of saturation:

$$T > T_s(p) + \Delta T_s. \quad (10)$$

The existence of the outflow regime, when the liquid for a long time ( $t \approx 0.3$  s) is overheated at pressure  $p \approx 2.8$  MPa, which is less than the saturation pressure ( $p_s(T_0) = 3.4$  MPa), can be explained by two main reasons. The first reason is the action of capillary effects at the phase interface [14]. Indeed, the observed overheat of liquid in the experiments,  $\Delta T_s = 10$  K at  $T_0 = 515$  K, according to estimation, made in [5], corresponds to the conditions of equilibrium between water and steam pressures in the bubbles accounting for capillary forces  $\Delta p_s \approx 2\sigma/a \approx 0.5$  MPa, where  $a \approx 0.2 \times 10^{-6}$  m,  $\sigma \approx 4 \times 10^{-2}$  N/m [7]. It was supposed that at the stage of non-equilibrium outflow of liquid the size of bubbles is limited by this value, and the void fraction increases by generation new bubbles on the impurity interfaces. The second reason is the generation of flow with constant velocity and uniform pressure, which is less than the saturation pressure according to (3). At these conditions the overheated liquid shifts from metastable state according to (9) into the equilibrium two-phase state, and the excess of internal energy, which arises due to overheating, goes to the generation of vapor.

When the void fraction  $\alpha_2$  exceeds its critical value  $\alpha_{2*}$  the generation of new bubbles stops, and further boiling happens by increasing the radii of existing bubbles. The transition to the equilibrium regime of two-phase flow is done according to approach, which is described in [3], [4], and equations (1)–(9) at  $T > T_s(p)$ .

### C. The Method of Solution

The initial and boundary conditions for the considered problem were chosen to confirm the experimental data [9]:

$$u(r, 0) = 0, \quad p(r, 0) = p_0 = 7 \text{ MPa}, \quad T(r, 0) = T_0 = 515 \text{ K},$$

$$L = 4.1 \text{ m}, \quad \rho_1^0(r) = \rho_1^0(p_0, T_0), \quad \rho_2^0(r) = \rho_2^0(p_0, T_0),$$

$$u(0, t) = 0, \quad p(L, t) = p_{cs}.$$

The number of bubbles  $n$  is obtained as function of void fraction  $\alpha_2$  and bubble radius:

$$\alpha_2(r, 0) = 10^{-11}, \quad n(r, 0) = 1.6 \times 10^9 \text{ m}^{-3},$$

$$\alpha_2(r, t) \leq 0.25: n(r, t) = \frac{3\alpha_2(r, t)}{4\pi a^3}, a = 0.2 \times 10^{-6} \text{ m},$$

$$\alpha_2(r, t) > 0.25: n(r, t) = 3.5 \times 10^{10} \text{ m}^{-3}, a = \sqrt[3]{\frac{3\alpha_2(r, t)}{4\pi n(r, t)}}.$$

The system of governing equations (1)–(5) along with expressions for pressure and internal energy (6)–(8), kinetic (9), (10), initial and boundary conditions was solved numerically by method of particles. As opposed to Eulerian approach, which was used in [3], [5], [8] for investigation of flow inside the tube, at the present paper Lagrangian approach was employed. This approach allows to obtain the total picture of non-stationary wave effects inside and outside the tube, because it correctly sets the boundary conditions, which conserve the balance of momentum and energy in the whole region of flow.

#### D. Discussion the Results of Numerical Modeling

In Fig. 1 the calculated profiles of pressure are presented in the whole region of flow for the fixed time moments (in microseconds) inside ( $x \leq 4.1$  m) and outside the tube ( $x > 4.1$  m). As a result of initial depressurization a rarefaction wave is spreading into the tube, and the outflow of boiling liquid occurs outside the tube with void fraction up to 80 % (Fig. 2). The pressure at the ending of the tube (denoted by letter  $K$  in Figs. 1–3) approximately equals 1.8 MPa (Fig. 1).

In Fig. 1 the comparison of calculations of the present paper, experimental data [9] and calculations based upon non-equilibrium model [5] is presented. In the rapid rarefaction wave the adiabatic type of flow has the dominant influence on the liquid temperature drop compared with mass and heat transfer. The interaction between the rapid rarefaction wave and the closed ending of the tube leads to the drop of pressure below 2.8 MPa, and the overheated liquid starts to boil. This leads to increase of void fraction and pressure. The drop of pressure after reflection of the rarefaction wave from the closed ending of the tube does not exceed 0.1 MPa.

After passing the rapid rarefaction wave at  $t \geq 5$  ms the uniform pressure is settled in the tube, which is less than the saturation pressure ( $p_s(T_0) = 3.4$  MPa), and followed by constant flow velocity for overheated liquid [9].

In Fig. 1–3 the described stage of the process is plotted by spatial distributions of parameters starting from the time moment  $t = 50$  ms. In Fig. 1 the profile of the pressure wave of slow boiling is formed for the time moment  $t = 50$  ms near the closed ending of the tube. The movement and interaction between waves of boiling are illustrated in Fig. 2, which shows the distribution of volume concentration of liquid phase  $\alpha_1$  at different time moments.

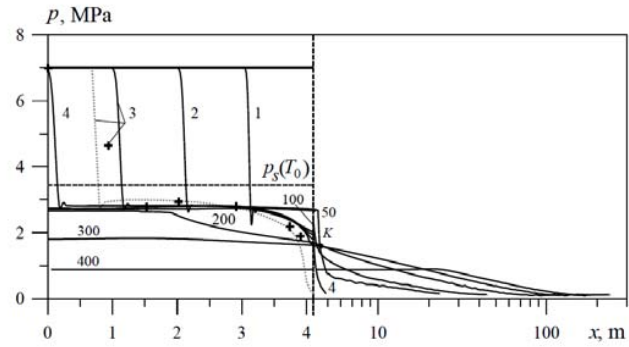


Fig. 1 Solid lines – pressure distributions in the liquid at time moments in ms. Points – experimental data [9], dotted line – calculation according to non-equilibrium model [5] at  $t = 3$  ms, dashed line – saturation pressure

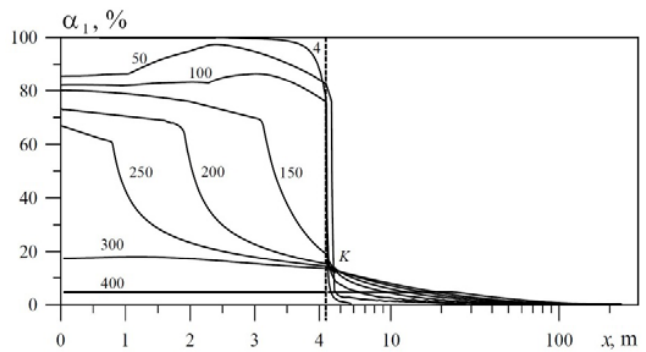


Fig. 2 Volume concentration of liquid  $\alpha_1$  at time moments in ms

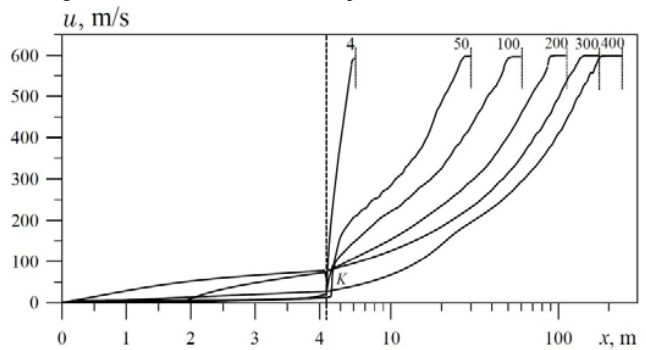


Fig. 3 Mass velocity  $u$  at time moments in ms

The process of slow boiling occurs under conditions of phase transition and is described by vapor concentration growth due to generation of new bubbles. According to calculations the number of bubbles increases from  $n = 1.6 \times 10^9 \text{ m}^{-3}$  at  $t = 0$  up to  $n = 3.5 \times 10^{10} \text{ m}^{-3}$ , when we limit the bubble radius  $a \leq 0.2 \times 10^{-6} \text{ m}$ . In flow regions, where the process of non-equilibrium slow boiling takes place, the vapor-liquid mixture gradually shifts to equilibrium two-phase state. The numerical simulation of the problem shows that the intensification of boiling due to the bubble radius increase  $a(r, t) > 0.2 \times 10^{-6} \text{ m}$  at constant number of bubbles  $n(r, t) = 3.5 \times 10^{10} \text{ m}^{-3}$  takes place in equilibrium regime, when void fraction  $\alpha_2(r, t) > 0.25$ , and overheating of the liquid due to

capillary forces does not exist ( $\Delta T_s \rightarrow 0$  as soon as  $2\sigma/a \rightarrow 0$  when  $a \rightarrow \infty$ ).

Look at the part of the tube with length  $\sim 0.5$  m, which is sided with open ending of the tube in front of point  $K$  (see Figs. 1–3). Till time moment  $t \approx 100$  ms the supersonic regime of liquid outflow is supported at this part of the tube (Fig. 3), when the flow velocity  $u$  exceeds the velocity of rarefaction wave, which equals the speed of sound  $C \approx 82$  m/s in two-phase mixture.

To estimate the adiabatic speed of sound in the gas-liquid mixture Wood equation [15] was used

$$C^2 = \frac{\rho_0}{\rho^2 \left( \frac{\alpha_{20} \rho_{20}}{\rho_2^2 C_2^2} + \frac{\alpha_{10} \rho_{10}}{\rho_1^2 C_1^2} \right)} = \frac{1}{\rho \left( \frac{\alpha_2}{\rho_2 C_2^2} + \frac{\alpha_1}{\rho_1 C_1^2} \right)}$$

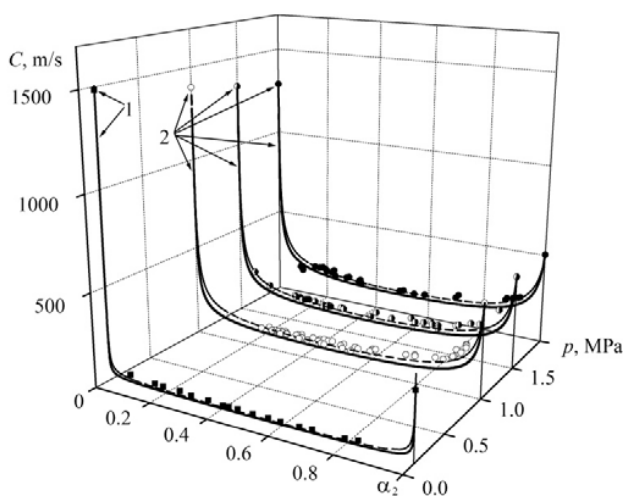


Fig. 4 Experimental and calculated isobars of adiabatic speed of sound as functions of  $\alpha_2$  [16]. 1 – air-water mixture; 2 – steam-water mixture. Solid line – equilibrium model, dashed line – model with account for capillary forces

In Fig. 4 the calculated isobars of adiabatic speed of sound are shown as functions of void fraction  $\alpha_2$  for gas-liquid (air-water) and vapor-liquid mixtures along with experimental data [16]. Calculations of speed of sound has been carried out according to Wood equation and equation of state from [10] for two variants: at equilibrium state at saturation line (solid line in Fig. 4), and at metastable state of gas and vapor due to the action of capillary forces (dashed line in Fig. 4). The taking into account for the capillary effects leads to increase in sound speed of mixture, which promotes to achieve much more excellent agreement between experimental and calculated data.

The supersonic regime of liquid outflow is supported with large pressure gradient (Fig. 1), intensive boiling (the inversion of bubble structure into the mixed vapor-drop flow occurs at  $\alpha_2 \sim 0.7 \div 0.8$ ) and rapid acceleration of steam-water mixture.

As a result the rarefaction wave is carried away from the inner part of tube. In Fig. 3 for the fixed time moments the position of contact boundary between mixture and air is shown, which velocity reaches 600 m/s.

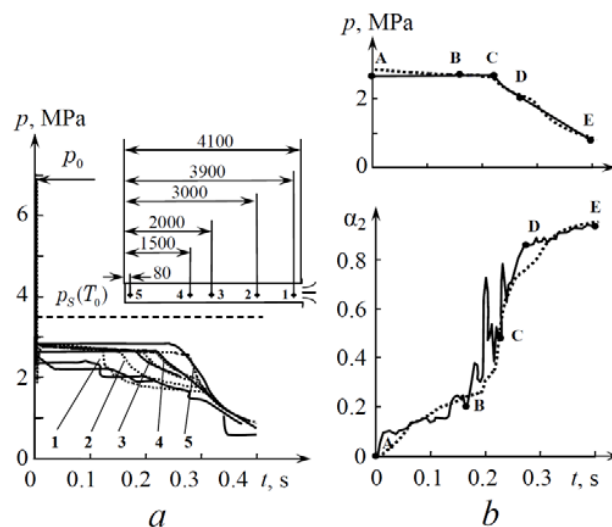


Fig. 5 Experimental [9] (solid line) and calculated (dots) dependencies of pressure from time in the tube for different cross sections (a). Comparison of experimental and calculated pressures and void fractions  $\alpha_2$  in time for cross section 4 (b)

The acceleration of the flow also takes place at the outer part of the tube, accompanied with increase of  $\alpha_2$  (see Fig. 2 for comparison). The further growth of  $\alpha_2$  shifts the process to the final stage: the flow velocity in front of point  $K$  becomes weaker and the rarefaction wave is directed inside the tube (see Figs. 1–3). At  $t > 300$  ms a low speed outflow regime inside the tube with maximum velocity near its open ending occurs (from 60 m/s at  $t = 300$  ms down to 20 m/s at  $t = 400$  ms). The calculated results for inner part of the tube show good correspondence with experimental data [9] (Fig. 5).

The pressure at the outer part of the tube decreases from 1.7 MPa near the point  $K$  down to  $p_{cs} = 0.2$  MPa at the contact boundary between air and vapor-liquid mixture. From the time moment  $t \approx 300$  ms, when the process of generation of vapor-drop structure involves the whole flow, the uniform pressure is stated inside the tube, which reaches 0.5 MPa at  $t \approx 400$  ms. Some excess of flow velocity at the outer part of the tube at the final stage of the process can be explained by the reason that the problem was solved without account for hydraulic friction.

### III. REALIZATION OF HIGH PRESSURE AND TEMPERATURE IN GAS PHASE OF BUBBLY LIQUID WHILE FLOWING THROUGH THE NOZZLE

#### A. The Problem Formulation

Consider the steady-state flow of liquid with vapor-gas bubbles in the nozzle of round cross section. For description of the dynamics of bubbly liquid the model of two-phase medium with equal mass velocities was used.

It was supposed, that at every medium point the spherically bubbles has the same radius (much less than the nozzle diameter) and the processes of its splitting and coagulation are absent.

The gas phase of the bubble in the common case is considered as the mixture of non-condensable gas and vapor. Non-condensable gas can be treated as inert gas or as the stoichiometric mixture of chemically active gases.

The governing system of equations for quasi one-dimensional steady-state regime of bubbly liquid flow through the nozzle consist of the following equations:

mass conservation of liquid (we neglect here mass transfer)

$$\frac{d}{dz}(\rho_1(1-\alpha_2)uS) = 0, \quad (11)$$

conservation of bubble number

$$\frac{d}{dz}(nuS) = 0, \quad (12)$$

conservation of mixture momentum

$$\rho_1(1-\alpha_2)u \frac{du}{dz} = -\frac{dp_1}{dz} - \frac{\tau\Sigma}{S}, \quad (13)$$

Rayleigh-Lamb equation to describe radial oscillations of bubbles adjusted for mass exchange:

$$au \frac{dw}{dt} + \frac{3}{2}w^2 = \frac{p_2 - p_1 - 2\sigma/a - 4\mu_1 w/a}{\rho_1}, \quad (14)$$

$$u \frac{da}{dz} = w + \frac{j}{\rho_1}, \quad \alpha_1 + \alpha_2 = 1, \quad \alpha_2 = \frac{4}{3}\pi na^3,$$

where  $w$  is the radial velocity of the bubble,  $S$  and  $\Sigma$  are the square and perimeter of the nozzle,  $\tau$  is the surface tension [2],  $\mu_1$  is the coefficient of dynamic viscosity of liquid,  $j$  is the rate of condensation/evaporation at the bubble interface.

The set of equations is supplemented by energy equation in the form:

$$u \frac{de_2}{dz} = -4\pi a^2 p_2 w + 4\pi a^2 q_2 + 4\pi a^2 j c_2 T_a + \frac{4}{3}\pi a^3 (1-k)\Lambda \cdot \Delta H \quad (15)$$

Here  $e_2 = 4/3\pi a^3 \rho_2 c_{v2} T_2$  is the internal energy of the bubble,  $c_{v2}$  is the heat capacity coefficient at constant volume,  $q_2$  is the intensity of heat exchange,  $T_a$  is the bubble interface temperature,  $k$  is the mass concentration of vapor in the bubble,  $\Lambda$  is the rate of chemical reactions,  $\Delta H$  is the difference of enthalpies of initial gas components and the products of chemical reactions.

The following equations are also employed:

equation for density of vapor-gas mixture in the bubble

$$\frac{d\rho_2}{dz} = \frac{3(j - \rho_2 w)}{au}, \quad (16)$$

equation for mass concentration of vapor inside the bubble

$$\frac{dk}{dz} = \frac{3(1-k)}{\rho_2 au} j, \quad (17)$$

equation of state for perfect gas

$$p_2 = \rho_2 R_2 T_2 = (\rho_g R_g + \rho_v R_v) T_2, \quad (18)$$

where  $\rho_v = k\rho$ ,  $\rho_g = (1-k)\rho$  are the densities of vapor and non-condensable gas correspondingly,  $R_v$ ,  $R_g$  and  $R_2$  are the gas constants of vapor, non-condensable gas and vapor-gas mixture.

#### B. Interface Conditions and Kinetics of Chemical Reactions

The parameter  $q_2$  in (15) defines the intensity of heat transfer in the bubble. To find it we shall use the heat balance equation at the bubble interface:

$$q_1 - q_2 = jl, \quad (19)$$

where  $l$  is the heat of vaporization.

The heat fluxes in gas and liquid can be expressed via formulas:

$$q_1 = \text{Nu}_1 \lambda_1 \frac{T_1 - T_a}{2a}, \quad q_2 = \text{Nu}_2 \lambda_2 \frac{T_a - T_2}{2a}.$$

Using these equations, we can find from (19) the interface temperature

$$T_a = \frac{\lambda_1 \text{Nu}_1 T_1 + \lambda_2 \text{Nu}_2 T_2 - 2ajl}{\lambda_1 \text{Nu}_1 + \lambda_2 \text{Nu}_2}, \quad (20)$$

where  $\text{Nu}$  is the Nusselt number [17].

The intensity of phase transition is obtained from

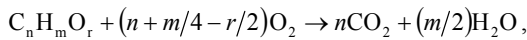
$$j = -\frac{\rho_{2a}}{1-k} D \cdot \text{Sh} \frac{k - k_a}{2a}, \quad (21)$$

where  $\rho_{2a}$  and  $k_a$  are the density of vapor-gas mixture and mass concentration of vapor at the bubble interface,  $D$  is the diffusion coefficient,  $\text{Sh}$  is the Sherwood number [18].

The kinetics of chemical reactions satisfies the common Arrhenius law:

$$\Lambda = A_f \exp(-C_f/T_2) \theta_1^x \theta_2^y. \quad (22)$$

Here  $A_f$ ,  $C_f$ ,  $x$ ,  $y$  are the kinetic parameters,  $\theta_1$  and  $\theta_2$  are the mole concentrations of initial components of the following chemical reaction:



which are expressed by formulas

$$\theta_1 = \frac{\varphi(1-k)\rho_2}{M_{CnHmOr} + (n + m/4 - r/2)M_{O_2}},$$

$$\theta_2 = \frac{(n + m/4 - r/2)\varphi(1-k)\rho_2}{M_{CnHmOr} + (n + m/4 - r/2)M_{O_2}},$$

where  $\varphi$  is the concentration of initial components of non-condensable gas:

$$\frac{d\varphi}{dz} = -\frac{\Lambda}{(1-k)\rho_2 u}. \quad (23)$$

For numerical integrating of the set of equations (11)–(23) the 4-th order Runge-Kutta method was used for the fixed geometrical characteristics of the nozzle.

### C. Results of Numerical Simulation

While outflow of bubbly liquid through the nozzle, because of pressure drop in the narrow part of the nozzle, the nonlinear oscillations of bubbles are developed. The intensity of these oscillations are defined by the characteristics of gas-liquid mixture and geometry of the nozzle.

The numerical simulation has been carried out for the nozzle with length 0.15 m. The diameters of inlet and outlet of the nozzle are 0.0357 m. The diameter of minimum cross section equals 0.0252 m, which is situated 0.023 m away from the nozzle inlet.

Two set of calculations have been done. In the first case the influence of mass transfer on the amplitude of radial oscillations of vapor-gas bubbles was investigated to estimate temperature and pressure inside the bubbles during its collapses. The chemical reactions in the non-condensable gas were not accounted for. The second series of calculations deal with analysis of influence of chemical reactions on the dynamics of gas bubbles in the absence of mass transfer.

Figs. 6 and 7 present the steady-state distribution of parameters of liquid in the nozzle filled with gas and vapor-gas bubbles. The velocity at the nozzle inlet equals  $u_0 = 11.4$  m/s. The following values of thermophysical parameters are used for gas (air):  $\lambda_2 = 2.6 \cdot 10^{-2}$  kg·m/(s<sup>3</sup>·K),  $c_{v2} = 1006$  J/(kg·K),  $\rho_2 = 1.29$  kg/m<sup>3</sup> and liquid (water):  $\lambda_1 = 0.567 \cdot 10^{-2}$  kg·m/(s<sup>3</sup>·K),  $c_{v1} = 4200$  J/(kg·K),  $\rho_2 = 10^3$  kg/m<sup>3</sup>,  $\sigma = 7.3 \cdot 10^{-2}$  N/m,  $\mu_1 = 10^{-3}$  Pa·s. At the inlet of the nozzle ( $z = 0$ ) we set the constant values of liquid pressure  $p_{20} = 0.2$  MPa, bubble radius  $a_0 = 10^{-4}$  m and void fraction  $\alpha_{20} = 10^{-4}$ .

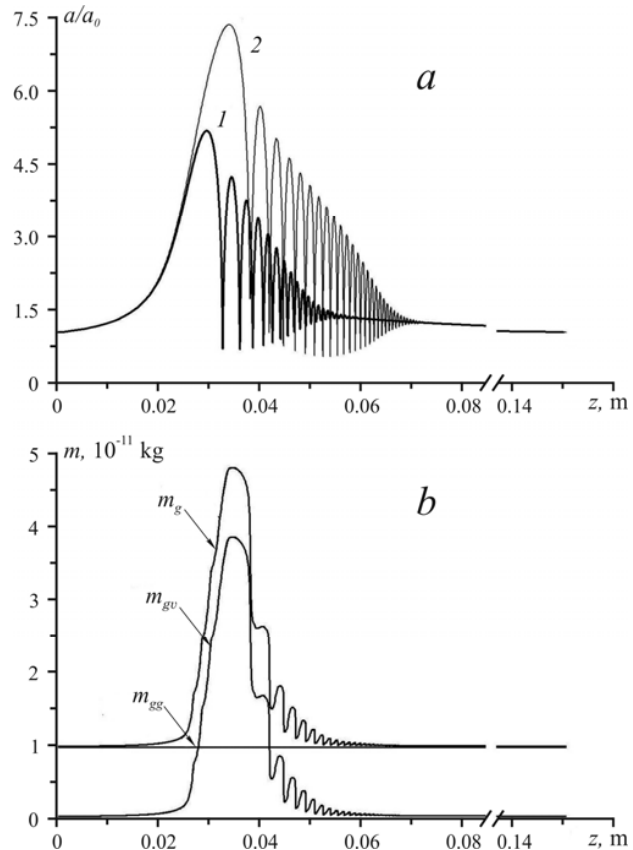


Fig. 6 Steady-state distribution of dimensionless bubble radius (a) and mass of bubble components (b) along the nozzle. 1 – vapor-gas bubble, 2 – gas bubble

The curves 1 in Fig. 6 correspond to the case, when the phase transition is absent, curves 2 – when the phase transition is taking into account. As it was mentioned in [7], two main effects can be observed due to the mass transfer. The first effect is connected with increase of initial growth of bubbles in the region of minimum cross section of the nozzle is caused by intensive evaporation of liquid at the bubble interface. The mass of vapor-gas mixture can increase several times compared with initial mass of the bubble (Fig. 6). The second effect deals with asymmetric evolution of bubble radius in time: the duration of its expansion is usually longer than the period of bubble contraction. This often leads to the situation, when the vapor inside the bubble does not have enough time to condense into the liquid, which essentially weakens the bubble collapse. Therefore in vapor-gas bubbles the peak temperatures and pressures are not usually achieved during first collapse, as it typically observed for gas bubbles, but during subsequent collapses, when the bubble mass becomes less (Fig. 7).

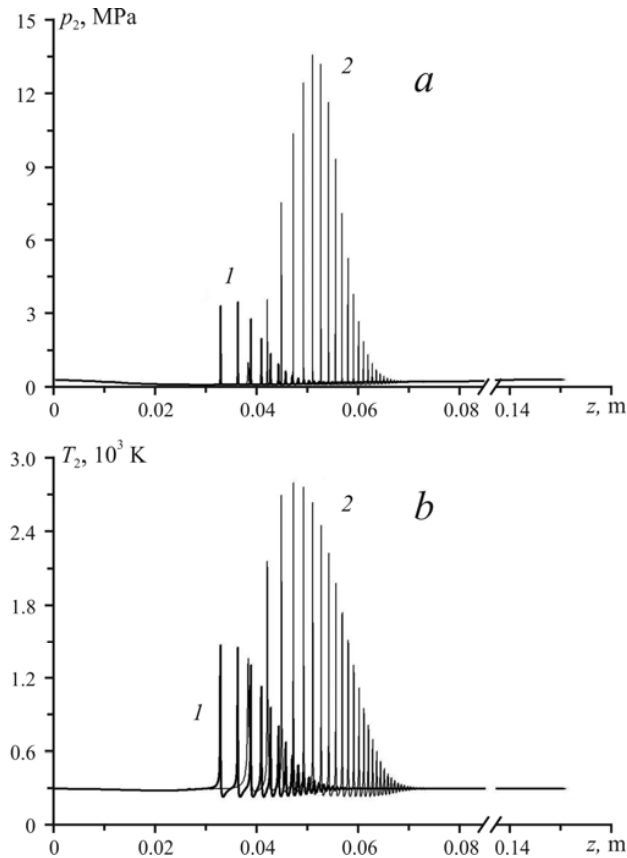
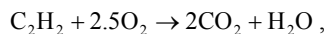


Fig. 7 Steady-state distribution of pressure (a) and temperature (b) in the bubble along the nozzle. 1 – vapor-gas bubble, 2 – gas bubble

In Fig. 8 the steady-state distributions of dimensionless bubble radius, pressure and temperature inside the gas bubble are displayed. The results are presented for the gas bubble with chemically passive components (curve 1, no chemical reactions) and taking into account the chemical reactions (curve 2). In calculations the gas is considered as a stoichiometric mixture of acetylene and oxygen, which react with generation of carbonic acid and water



and the water is used as a liquid. To model the dynamics of the gas bubbles the following parameters are used:  $\lambda_2 = 2.49 \cdot 10^{-2} \text{ kg}\cdot\text{m}/(\text{s}^3\cdot\text{K})$ ,  $R_{21} = 277.1 \text{ J}/(\text{kg}\cdot\text{K})$ ,  $x = 0.5$ ,  $y = 1.25$ ,  $R_{22} = 237.5 \text{ J}/(\text{kg}\cdot\text{K})$ ,  $A_f = 6.5 \cdot 10^{12} (\text{cm}^3/\text{mol})^{x+y-1}$ ,  $C_f = 1.52 \cdot 10^4 \text{ J/kg}$ ,  $\Delta H = 2.26 \cdot 10^6 \text{ J/kg}$ . The boundary conditions at the inlet of the nozzle are the same, except for the inlet velocity  $u_0 = 9.5 \text{ m/s}$ .

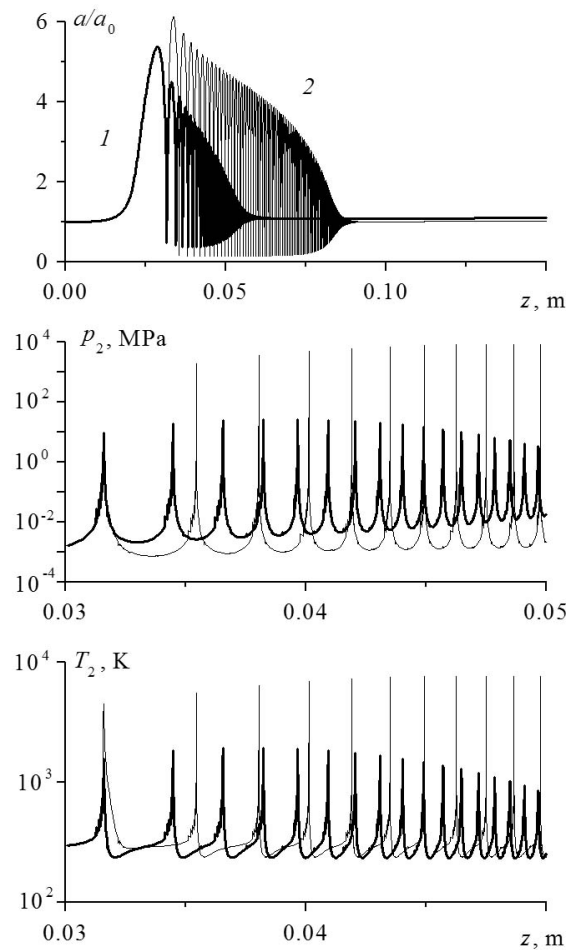


Fig. 8 Steady-state distribution of dimensionless bubble radius ( $a/a_0$ ), pressure ( $p_2$ ) and temperature ( $T_2$ ) inside the bubble with account of chemical reactions (curve 2) and without chemical reactions (curve 1)

According to the obtained results the main differences in radial dynamics of the gas bubble between two curves start from the moment of the first collapse of the bubble, when the heating of its inner occurs. As a result the complete burning of acetylene is observed (parameter  $\varphi$  drops rapidly from unity down to zero), and the radial dynamics of the bubble, which contains the products of the chemical reaction, differs from the dynamics of chemically passive bubble. The amplitude and the duration of radial oscillations growth and the maximum pressure and temperature become several times greater compared with the bubble with no chemical reactions. Such the behavior of the bubble can be explained by increase of its kinetic energy due to exothermic reaction of acetone burning.

#### IV. CONCLUSION

In the present paper the problem of outflow of boiling liquid after depressurization of tube with high pressure inside and the problem of steady-state flow of bubbly liquid in the nozzle are solved theoretically by the help of numerical simulation.

The obtained results of solution of the first problem show, that after the passing of the rapid rarefaction wave inside the tube the water becomes metastable and overheated, and the non-equilibrium outflow is supported, when the bubble radius is limited. Further the boiling occurs at the unlimited growth of bubble radius at the equilibrium regime of two-phase flow.

The analysis of results at the outlet of the tube shows the formation of the region with supersonic velocity of the liquid outflow, when the rarefaction wave cannot reach the inner part of the tube. The good agreement of calculated results with experimental data of pressure and void fraction time evolution is obtained for a cross section of inner part of the tube.

The numerical analysis of the flow of bubbly liquid in the nozzle confirms the possibility of existence of ultrahigh pressures and temperatures in the gas phase as a result of arising of the intensive nonlinear radial oscillations of the bubbles near the narrow part of the nozzle. The obtained results allowed to believe that the system bubbly liquid–nozzle can be treated as the appropriate technology to realize the energy cumulating with ultrahigh pressures and temperatures.

#### ACKNOWLEDGEMENT

The work is done by financial support of Russian Foundation for basic Research (Grants No. 11-01-97004-r\_povolzhie\_a and 11-01-00171-a), the Council of the President of the Russian Federation for leading scientific schools (Grant No. NSh-834.2012.1) and the Program of the Foundation of Fundamental Research of Division of Energetics, Mechanical Engineering, Mechanics, and Control Systems of the Russian Academy of Sciences (OE-13).

#### REFERENCES

- [1] R. I. Nigmatulin, I. Sh. Akhatov, A. S. Topolnikov, R. Kh. Bolotnova, N. K. Vakhitova, R. T. Lahey Jr., and R. P. Taleyarkhan, "Theory of supercompression of vapor bubbles and nanoscale thermonuclear fusion," *J. Physics of Fluids*, vol. 17, 107106, 2005.
- [2] R. I. Nigmatulin, R. Kh. Bolotnova, N. K. Vakhitova, A. S. Topolnikov, S. I. Konvalova, and N. A. Makhota, "Amplification of compression waves in clean and bubbly liquids", *Proc. of World Academy of Science, Engineering and Technology*, vol. 58, pp.188-193, 2009.
- [3] A. I. Ivandayev, A. A. Gubaidulin, "Investigation of a nonstationary outflow of flashing liquid in a thermodynamic equilibrium approximation," *Teplofizika Vysokikh Temperatur*, vol. 16, no. 3, pp. 556–562, 1978.
- [4] V. Sh. Shagapov, G. Ya. Galeeva, "The emptying of channels and reservoirs accompanied by boiling," *High Temperature*, vol. 48, no. 3, pp. 388–396, 2010.
- [5] B. I. Nigmatulin, K. I. Soplenkov, "Investigation of a nonstationary flow of flashing liquid out of channels in a thermodynamic nonequilibrium approximation," *Teplofizika Vysokikh Temperatur*, vol. 16, no. 2, pp. 370–383, 1978.
- [6] K. H. Ardron, "A two-fluid model for critical vapor-liquid flow," *Int. J. Multiphase Flow*, vol. 4, pp. 323–337, 1978.
- [7] R. I. Nigmatulin, *Dynamics of Multiphase Media*. Moscow: Nauka, vol. 1-2, 1987.
- [8] O. E. Ivashnev, "Self-sustaining shock waves in nonsteady boiling liquid," Synopsis of the dissertation to Dr. Phys.-Math. of Sciences. Moscow, 2009.
- [9] A. R. Edwards, T. P. O' Brien, "Studies of phenomena connected with the depressurization of water reactors," *J. of The British Nuclear Energy Society*, vol. 9, no. 1–4, pp. 125-135, 1970.
- [10] R. I. Nigmatulin, R. Kh. Bolotnova, "Wide range equation of state for water and steam. Simplified form," *High Temperature*, vol. 49, no. 2, pp. 303–306, 2011.
- [11] R. I. Nigmatulin, V. Sh. Shagapov, N. K. Vakhitova, and R. T. Lahey, "A method of ultrahigh compression of gas bubbles in a liquid by the nonperiodic vibrational action of pressure of moderate magnitude," *Doklady Physics*, vol. 40, no. 3, pp. 122–126, 1995.
- [12] E. M. Galimov, A. M. Kudin, V. N. Skorobogatskiy et al, "Experimental confirmation of the diamond synthesis during cavitation," *Doklady Physics*, vol. 395, no. 2, pp. 187–191, 2004.
- [13] L. E. Scriven, "On the dynamics of phase growth," *Chem. Eng. Sci.*, vol. 1, pp. 1–13, 1959.
- [14] N. K. Vakhitova, V. Sh. Shagapov, "Propagation of small disturbances in vapor-and-liquid bubbling media," *J. of Applied Mechanics and Technical Physics*, vol. 25, no. 5, pp. 682–691, 1984.
- [15] G. B. Wallis, *One-Dimensional Two-Phase Flow*. New York: McGraw-Hill Book Co, 1969.
- [16] N. I. Semenov, S. I. Kosterin, "Results of studying the speed and sound in moving gas-liquid systems," *Teploenergetika*, no. 6, pp. 46–51, 1964.
- [17] S. A. Lepikhin, M. N. Galimzyanov, and I. K. Gimaltdinov, "Initiation of detonation waves in channels of variable cross section filled with liquid with combustible gas bubbles", *High Temperature*, vol. 48, no. 2, pp. 215-221, 2010.
- [18] V. Sh. Shagapov, S. A. Lepikhin, and M. N. Galimzyanov, "Realization of high pressures and temperatures in the gas phase of a bubble liquid flowing through a nozzle", *J. of Engineering Physics and Thermophysics*, vol. 80, no. 6, pp. 134–137, 2007.