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Hybrid Minimal Repair for a Serial System

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Abstract**—**This study proposes a hybrid minimal repair policy which combines *periodic maintenance policy* with *age-based maintenance policy* for a serial production system. Parameters of such policy are defined as $\boldsymbol{Z}$ and $\boldsymbol{U}$ which indicate as hybrid minimal repair time and planned preventive maintenance time respectively ($\mathbf{0} < Z < U$). Under this hybrid policy, the system is repaired minimally if it fails during $(\mathbf{0}, \boldsymbol{Z}]$. A perfect repair is conducted on the first failure after $\boldsymbol{Z}$ at any machines. At the same time, we take opportunity to advance the preventive maintenance of other machines simultaneously. If the system is still operating properly up to $\boldsymbol{U}$, then the preventive maintenance is carried out as its predetermined schedule. For a given $\boldsymbol{U}$, we obtain the optimal value $\boldsymbol{Z}$ which minimizes the expected cost per time unit. Numerical example is presented to illustrate the properties of the optimal solution.

Keywords**—**Hybrid minimal repair, opportunistic maintenance, preventive maintenance, series system

I. Introduction

PREVENTIVE maintenance (PM) has been intensively studied in the last five decades. Many studies have been developed to obtain an optimal maintenance strategy [1], [2].

Mostly, an optimal maintenance policy was developed from a simple policy, such as: age-based policy or periodic replacement policy [3]. Under age-based policy, an item is replaced at age T or at failure, whichever occurs first, where T is constant. While in periodic replacement policy, an item is periodically replaced at fixed interval without considering the failure history. The failure between consecutive replacements is minimally repaired [4], [5], [6], [7], [8], [9], so this policy is often called as periodic replacement with minimal repair.

Along with the advancement of maintenance studies, some papers have combined such maintenance policies at once. This modification was proposed to reduce the maintenance cost. In 1975, [4] introduced the policy of maintenance (t_0, T) which replace the item on the first failure after t_0 or when its age reaches T $(0 \leq t_0 \leq T)$, whichever occurs first. While failure during $[0, t_0]$ corrected with minimal repair. This (t_0, T) policy has combined periodic maintenance policy with age-based maintenance policy. Closely to (t_0, T) policy, [5] proposed $T - N$ policy. Under this policy, an item is replaced at N^{th} failure or preventively at T, whichever occurs first. Failure between replacements is repaired minimally. Extending this policy, [6] introduced (t, T) policy. Under this policy, during $(0, t]$, the item is replaced by a new one if a failure of type II occurs with probability $p(y)$ or minimally repaired if a failure of type I occurs with probability $q(y)$ where $q(y) = 1 - p(y)$. While corrective replacement is conducted on the first failure after t or preventively at T, whichever occurs first. Reference [7] has also optimized (t, T) policy using average cost criteria. This (t, T) policy has investigated further [8] by involving opportunistic maintenance as a consideration to execute opportunistic replacement for personal computer (PC) which has warranty period $(0, S]$. Under such policy, failure during $(0, S]$ repaired minimally, while replacement on interval $(S, T]$ can occur because of opportunity with probability p or failure. Otherwise, PC would be replaced preventively at T. The design variable of this policy is T which minimizes long run average cost. Extending such policy, [9] has considered opportunity to do replacement on interval $[S, T)$ with probability p or not do with probability $1 - p$. The design variable in this policy are S and T.

Some papers [10], [11], [12] have also modified the periodic maintenance policy by proposing (T_o, T) policy. Under this policy, failure which occur during $(0, T_0]$ repaired minimally and do corrective replacement if item fails during (T_0, T), otherwise PM is conducted preventively at T. Replacement during (T_0, T) is done by using three scenarios: replace with new, replace for certain item or left until PM without replacement.

Some papers deal with hybrid maintenance policies which combine periodic and age-based maintenance policy as well. However the term *hybrid minimal repair* just introduced by [13] by considering repair time. This study was investigated further to determine the optimum time to replace the item after its warranty has expired [14].

Studies on hybrid minimal repair policy has been carried out for systems with a single component, while in many practical situation, the system consists of several components with a complex structure: series, parallel and k-out-of-n. In this paper, the hybrid maintenance policy is developed for a serial system. Considering that the system is structured in series, when the system fails before the predetermined PM, we take opportunity to conduct maintenance simultaneously for some machines by advancing the planned PM schedule of other machines. However, the opportunity is taken if the failure occurs after the system passes a certain age. Hence, we need to determine the optimum time to take the opportunity which minimizes expected maintenance cost [15], [16], and so expected downtime cost as well [17]. This policy is extremely

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needed for production systems which have a very expensive downtime cost. For instance, the production loss cost due to downtime at chemical plants ranged from $ 5000 - $ 100,000 per hour [18], and this number will continue to increase as the system becomes more complex.

This study is organized as follows. In section 2, there is system description and assumptions. Section 3 contains the model formulation, beginning with modeling system failure. Numerical examples are presented in section 4 to illustrate the model solution.

II. System Description And Assumption

The system studied is a production system consisting of n machines arranged in series. This study focuses on $n = 2$ (see fig.1).

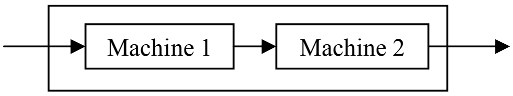

Fig. 1 Serial production system

Both machines are repairable. Any failure is detected and it may occur at any time. Failure of the machine is defined as the condition where the machine is not functioning as it should. Failure of machine 1 is independent to failure of machine 2. Both machines are not identical so that failure of both machines cannot occur at the same time.

In this study, in addition to the maintenance cost, we also consider cost of system downtime as an impact of failure and maintenance actions.

III. Model Formulation

Notations used to formulate the model are

U	Parameter of planned PM time, same for both machines
Z	Parameter of hybrid minimal repair time $(0 < Z < U)$
α_i	Scale parameter of i^{th} machine
β_i	Shape parameter of i^{th} machine
$f(t)$	Probability density function of system failure
$f_i(t)$	Probability density function of i^{th} machine failure
$F(t)$	Cumulative distribution function of system failure
$F_i(t)$	Cumulative distribution function of i^{th} machine failure
$\bar{F}(t)$	Survival function of system failure
$\bar{F}_i(t)$	Survival function of i^{th} machine failure
$g_\gamma(y)$	Probability density function of residual life
$G_\gamma(y)$	Cumulative distribution function of residual life
$\overline{G_\gamma}(y)$	Survival function of residual life
c_m	Average cost of minimal repair per repair
c_{r_i}	Average cost of perfect repair per repair for i^{th} machine
c_{p_i}	Average PM cost per maintenance action for i^{th} machine
c_{p_g}	Average PM cost per maintenance action for group machines
c_d^1	Average downtime cost related to minimal repair
c_d^2	Average downtime cost related to perfect repair
c_d^3	Average downtime cost related to planned PM
$E[B(Z,U)]$	Expected total cost per cycle
$E[K(Z,U)]$	Expected cycle length
$C(Z,U)$	Expected total cost per unit of time

Maintenance Policy

A hybrid minimal repair policy for a series system in this paper has two parameters: Z and U (see fig. 2), where Z is a parameter for hybrid minimal repair policy and U are the parameters for age-based maintenance policy. This policy is defined as follows:

(i) Any failure that occurs during $(0, Z]$ on any machine is repaired minimally.

(ii) The first failure that occurs in the interval $(Z, U]$ on any machine is repaired perfectly and at the same time we take opportunity to conduct PM for the other machine that is still functioning.

(iii) For the case where the two machines are still functioning properly until U, then the PM is carried out on schedule at U for both machines simultaneously.

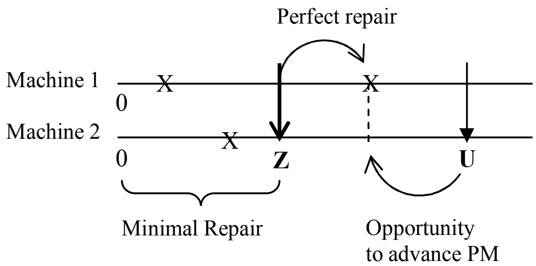

Fig. 2 Hybrid minimal repair policies for series systems

A. Modeling System Failure

Let T denotes a random variable of time to the first failure of system and T_i for i^{th} machine, then $T = min\{T_1, T_2\}$. Refer to [17], the probability that the system fails because of machine 1 is given by

$$P\{T_1 \le t < T_2\} = \int_0^t \overline{F_2}(x)\, f_1(x) dx \quad (1)$$

The probability that the system fails because of machine 2 is given

$$P\{T_2 \le t < T_1\} = \int_0^t \bar{F}_1(x)\, f_2(x) dx \quad (2)$$

Then the probability that the system fails can be obtained by

$$F_s(t) = P\{T_1 \le t < T_2\} + P\{T_2 \le t < T_1\} = \int_0^t \overline{F_2}(x)\, f_1(x)dx + \int_0^t \overline{F_1}(x)\, f_2(x)dx \quad (3)$$

While the survival function of T is given by

$$\overline{F_s}(t) = P\{T_1 > t, T_2 > t\} = \overline{F_1}(t)\overline{F_2}(t) \quad (4)$$

Maintenance action during $(0, Z]$ and $(Z, U]$ are different, where failure during $(0, Z]$ is corrected by minimal repair and in $(Z, U]$ with perfect repair.

Failure modeling for interval $(0, Z]$

As any failure which occurs on any machine in $(0, Z]$ is repaired minimally, so that failure process becomes non-homogeneous poison process with intensity function $\lambda(t)$ which is a non-decreasing function of t [20]. Thus, mean number of failures in the interval (0, Z], $H_s(Z)$ is given by

$$H_s(Z) = \int_0^Z h_s(t)\, dt \quad (5)$$

where $h_s(t)$ is

$$h_s(t) = \frac{f_s(t)}{\overline{F_s}(t)}, \quad (6)$$

and $f_s(t)$ is the density function of T and $\overline{F_s}(t)$ is the survival function of T which is given by (4).

Failure modeling for interval $(Z, U]$

Failure distribution in $(Z, U]$ is represented by the distribution of a residual life at Z, $\gamma(Z)$ [8].

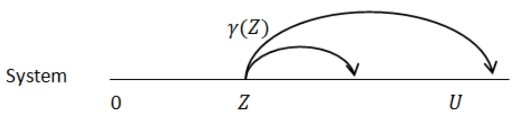

Fig. 3 Residual life after Z

Fig. 3 shows that $\gamma(Z)$ can be less than or greater than $(U - Z)$

The probability that the system fails in the interval (Z,U], $P\{\gamma(Z) \le (U - Z)\} = G_\gamma(U - Z)$. $G_\gamma(U - Z)$ is given by

$$G_\gamma(U - Z) = \frac{F_s(U) - F_s(Z)}{\overline{F_s}(Z)} \quad (7)$$

$$= \frac{P\{T_1 \le t < T_2,\ T_1 \le (U - Z)\}}{P\{T_1 > Z, T_2 > Z\}} + \frac{P\{T_2 \le t < T_1,\ T_2 \le (U - Z)\}}{P\{T_1 > Z, T_2 > Z\}}$$

$$= \frac{\int_Z^U \overline{F_2}(t) f_1(t)dt}{\overline{F_1}(Z)\overline{F_2}(Z)} + \frac{\int_Z^U \overline{F_1}(t) f_2(t)dt}{\overline{F_1}(Z)\overline{F_2}(Z)}$$

Let $\overline{G_\gamma}(U - Z)$ be the survival function of $\gamma(Z)$. $\overline{G_\gamma}(U - Z)$ is given by

$$\overline{G_\gamma}(U - Z) = \frac{1 - F_s(U)}{\overline{F_s}(Z)} = \frac{\overline{F_s}(U)}{\overline{F_s}(Z)} = \frac{\overline{F_1}(U)\overline{F_2}(U)}{\overline{F_1}(Z)\overline{F_2}(Z)} \quad (8)$$

B. Expected Total Cost

Let $C(Z, U)$ denotes the expected total cost per time unit and it is given by

$$C(Z, U) = \frac{E[B(Z, U)]}{E[K(Z, U)]}, \quad (9)$$

where $E[B(Z, U)]$ is the expected total cost per cycle and $E[K(Z, U)]$ is the expected cycle length.

Since $K(Z, U)$ is

$$K(Z, U) = \begin{cases} t & for\ \gamma(Z) \le (U - Z) \\ U & for\ \gamma(Z) > (U - Z) \end{cases} \quad (10)$$

then the expected cycle length $E[K(Z, U)]$ is given by

$$\begin{aligned} E[K(Z, U)] &= Z + E[\gamma(Z) \le (U - Z)] + E[\gamma(Z) > (U - Z)] \\ &= Z + \int_Z^U t[g_\gamma(t - Z)]dt + (U - Z)\overline{G_\gamma}(U - Z) \\ &= Z + \int_Z^U \overline{G_\gamma}\ (t - Z)dt \quad (11) \end{aligned}$$

Let c_m, c_r and c_p denote respectively the average cost of minimal repair, perfect repair and PM, where $c_m < c_p < c_r$. As system may fail because of machine 1 or 2, then c_r is written as c_{r_1} caused by machine 1 and c_{r_2} caused by machine 2. c_p is also written as c_{p_1} caused by machine 1 and c_{p_2} caused by machine 2. For the case where PM is carried out simultaneously for both machines at U, PM cost in group c_{p_g} becomes cheaper compared to individual maintenance cost, $c_{p_g} < c_{p_1} + c_{p_2}$ [17].

In this study, cost of downtime is also considered as an impact of failure and maintenance actions. The cost of downtime is distinguished for each maintenance activity. Let c_d^1, c_d^2, c_d^3 are defined as the average cost of downtime due to minimal repair, perfect repair and planned PM, respectively. The downtime cost due to minimal repair is commonly cheaper than the cost of other maintenance actions. Meanwhile, the downtime cost due to perfect repair which is unscheduled maintenance, will be more expensive than that of the downtime cost due to scheduled PM at U, so then $c_d^1 < c_d^3 < c_d^2$.

Since the total cost $B(Z,U)$ is given by

$$B(Z,U)=\begin{cases} c_m+c_d^1 & for\ 0<T\le Z \\ c_{r_1}+c_d^2+c_{p_2} & for\ Z<T_1<U, T_1\le t<T_2 \\ c_{r_2}+c_d^2+c_{p_1} & for\ Z<T_2<U, T_2\le t<T_1 \\ c_{p_g}+c_d^3 & for\ T>U \end{cases} \quad (12)$$

then, the expected total cost per cycle $E[B(Z,U)]$ can be expressed as

$$E[B(Z,U)]=[(c_m+c_d^1)\,H_s(Z)] + [(c_{r_1}+c_d^2+c_{p_2})\,P\{Z<T_1<U, T_1\le t<T_2\} + (c_{r_2}+c_d^2+c_{p_1})\,P\{Z<T_2<U, T_2\le t<T_1\}] + \left[\left(c_{p_g}+c_d^3\right)\,P\{T>U\}\right]$$

$$E[B(Z,U)] = [(c_m+c_d^1)\,H_s(Z)] + \left[\left(c_{r_1}+c_{p_2}+c_d^2\right)\frac{\int_Z^U \overline{F_2}(t)f_1(t)dt}{\overline{F}_1(Z)\overline{F}_2(Z)} + \left(c_{r_2}+c_{p_1}+c_d^2\right)\frac{\int_Z^U \overline{F}_1(t)f_2(t)dt}{\overline{F}_1(Z)\overline{F}_2(Z)}\right] + \left[\left(c_{p_g}+c_d^3\right)\frac{\overline{F}_1(U)\overline{F}_2(U)}{\overline{F}_1(Z)\overline{F}_2(Z)}\right] \quad (13)$$

Where $H_s(Z)$ is obtained from (5).

Using (13) and (11) we can obtain the expected total cost per unit of time, $C(Z,U)$ given by

$$C(Z,U)=\frac{1}{\left\{Z+\int_Z^U \overline{G_\gamma}\,(t-Z)dt\right\}}\left[[(c_m+c_d^1)\,H_s(Z)] + \left[\left(c_{r_1}+c_{p_2}+c_d^2\right)\frac{\int_Z^U \overline{F}_2(t)f_1(t)dt}{\overline{F}_1(Z)\overline{F}_2(Z)} + \left(c_{r_2}+c_{p_1}+c_d^2\right)\frac{\int_Z^U \overline{F}_1(t)f_2(t)dt}{\overline{F}_1(Z)\overline{F}_2(Z)}\right] + \left[\left(c_{p_g}+c_d^3\right)\frac{\overline{F}_1(U)\overline{F}_2(U)}{\overline{F}_1(Z)\overline{F}_2(Z)}\right]\right] \quad (14)$$

By minimizing $C(Z,U)$, we have optimal solution Z^*. Considering that the model is complex to be solved analytically, a numerical computation is used to get Z^*.

IV. Numerical Example

In this numerical example, the machines assumed to have failure distribution following the weibull distribution as in (15) where α_i and β_i is the scale and the shape parameter for the i^{th} machine.

$$F_i(t)=1-e^{-(\frac{t}{\alpha_i})^{\beta_i}} \quad (15)$$

TABLE I
The Effect Of Parameters Changing Value With Respect To Z^* And $C\,(Z^*,U)$ For
$\alpha_1=5;\alpha_2=2;\beta_1=2;\beta_2=2;c_m=1;c_{r_1}=4;$
$c_{r_2}=3;\ c_{p_1}=2;c_{p_2}=2;c_d^1=0.5;c_d^2=2;c_d^3=1;U=10$

		Z*	C(Z*,U)			Z*	C(Z*,U)			Z*	C(Z*,U)			Z*	C(Z*,U)
cm	2.0	2.024	3.759	cr1	5.0	3.241	3.192	cr2	5.0	3.746	3.636	cp1	2.5	3.340	3.270
	1.5	2.516	3.501		4.0	3.194	3.155		4.0	3.480	3.382		2.0	3.194	3.155
	1.0	3.194	3.155		3.0	3.146	3.117		3.0	3.194	3.155		1.5	3.042	3.035
	0.5	4.264	2.676		2.0	3.098	3.078		2.0	2.883	2.911		1.0	2.883	2.911
		Z*	C(Z*,U)			Z*	C(Z*,U)			Z*	C(Z*,U)			Z*	C(Z*,U)
cp2	2.5	3.218	3.173	cd1	2.0	1.641	3.949	cd2	3.5	3.679	3.541	cd3	2.0	3.194	3.155
	2.0	3.194	3.155		1.5	2.024	3.759		3.0	3.524	3.417		1.5	3.194	3.155
	1.5	3.170	3.136		1.0	2.516	3.501		2.5	3.363	3.288		1.0	3.194	3.155
	1.0	3.146	3.117		0.5	3.194	3.155		2.0	3.194	3.155		0.5	3.194	3.155

Table I shows the effect of changing parameter values $c_m, c_{r_1}, c_{r_2}, c_{p_1}, c_{p_2}, c_d^1, c_d^2, c_d^3$ in obtaining optimal solutions.

Fig. 4 shows the effect of minimal repair cost (c_m) with respect to Z and $C(Z,U)$. It is seen that the more expensive c_m, the smaller the Z, and the greater the $C(Z,U)$.

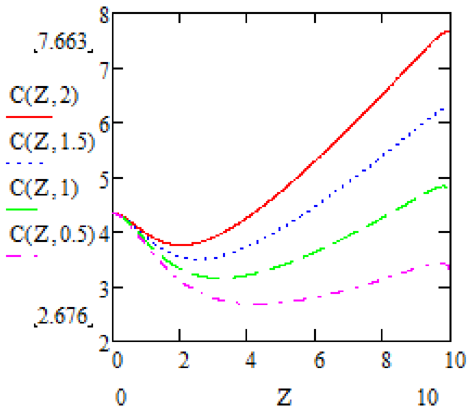

Fig. 4 The effect of c_m with respect to Z and $C(Z,U)$

In Fig. 5 and Fig. 6, we can see the effect of c_{r_1} and c_{r_1} with respect to Z and $C(Z,U)$.

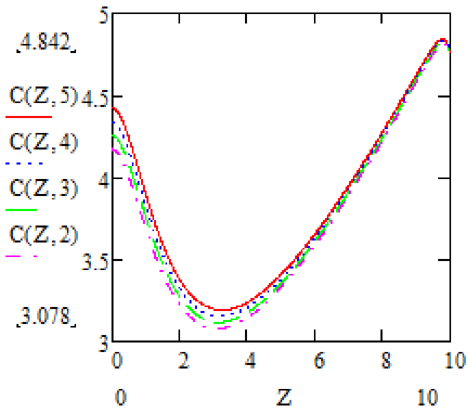

Fig. 5 The effect of c_{r_1} with respect to Z and $C(Z,U)$

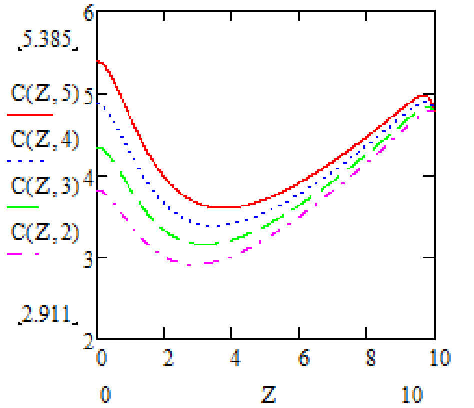

Fig. 6 The effect of c_{r_2} with respect to Z and $C(Z,U)$

Fig. 7 and Fig. 8 show the effect of c_{p_1} and c_{p_2} with respect to Z and $C(Z,U)$.

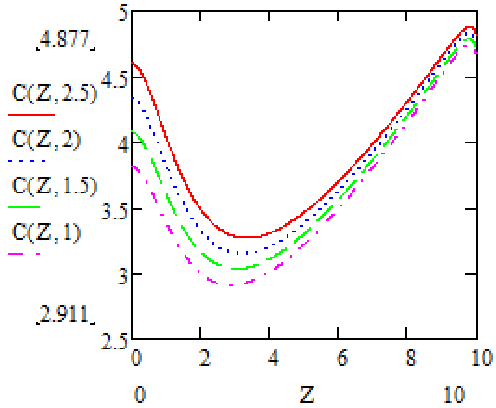

Fig. 7 The effect of c_{p_1} with respect to Z and $C(Z,U)$

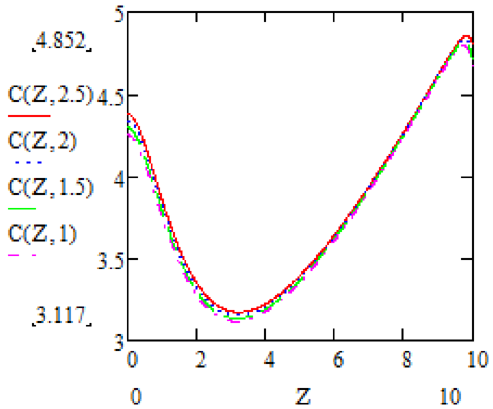

Fig. 8 The effect of c_{p_2} with respect to Z and $C(Z,U)$

Fig. 9, Fig.10, and Fig. 1 shows the effect of downtime cost c_d^1, c_d^2, c_d^3 with respect to Z and $C(Z,U)$. In Fig. 9, it is seen that c_d^1 gives much effect on Z and $C(Z,U)$. The more expensive c_d^1, the smaller the the Z, and the greater the $C(Z,U)$.

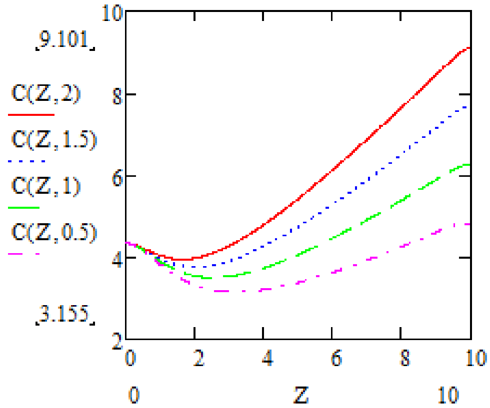

Fig. 9 The effect of c_d^1 with respect to Z and $C(Z,U)$

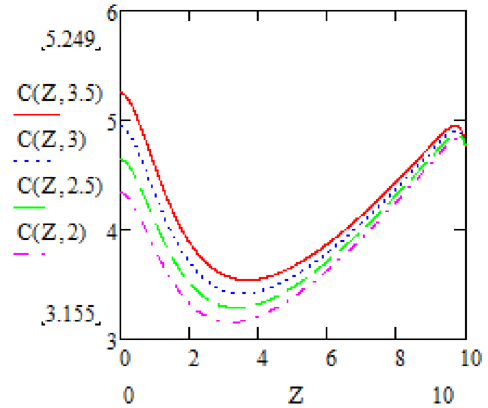

Fig. 10 The effect of c_d^2 with respect to Z and $C(Z,U)$

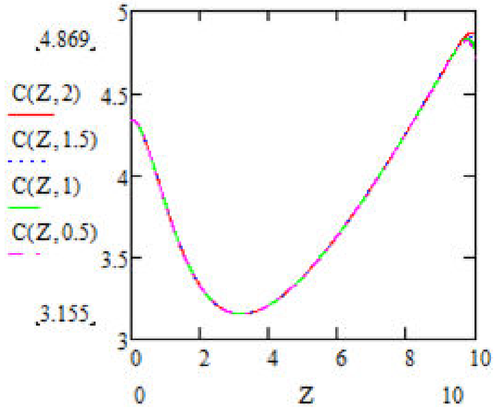

Fig. 11 The effect of c_d^3 with respect to Z and $C(Z,U)$

Based on the graphs, it is clearly seen that the function $C(Z,U)$ is a minimum function. All optimal solution Z^* meet the requirement $0 < Z^* < U$.The proposed model is strongly influenced by c_m and c_d^1 associated with costs of minimal repair and its downtime. The more expensive minimal repair costs and downtime, the smaller the Z^* and the greater $C(Z^*)$. This means that the optimal solution depends on the magnitude of related costs at the beginning of the cycle. The more expensive the cost to be incurred at the beginning of the cycle, the earlier opportunity to advance preventive maintenance can be taken, and vice versa.Changes in c_d^3 does not affect the optimal solution Z^* and $C(Z^*)$. This is understandable since the value of the parameter U has been given previously. In this paper, the first failure in $(Z,U]$ on any machines or PM at U when there is no failure for both machines in $(Z,U]$ is perfectly repaired. One interesting topic to research is the policy where repair action is imperfect.

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