

# Human Walking Vertical Force and Vertical Vibration of Pedestrian Bridge Induced by Its Higher Components

M. Yoneda

**Abstract**—The purpose of this study is to identify human walking vertical force by using FFT power spectrum density from the experimental acceleration data of the human body. An experiment on human walking is carried out on a stationary floor especially paying attention to higher components of dynamic vertical walking force. Based on measured acceleration data of the human lumbar part, not only in-phase component with frequency of  $2f_w$ ,  $3f_w$ , but also in-opposite-phase component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$  where  $f_w$  is the walking rate is observed. The vertical vibration of pedestrian bridge induced by higher components of human walking vertical force is also discussed in this paper. A full scale measurement for the existing pedestrian bridge with center span length of 33 m is carried out focusing on the resonance phenomenon due to higher components of human walking vertical force. Dynamic response characteristics excited by these vertical higher components of human walking are revealed from the dynamic design viewpoint of pedestrian bridge.

**Keywords**—Simplified method, Human walking vertical force, Higher component, Pedestrian bridge vibration.

## I. INTRODUCTION

NEEDLESS to say, human walking force is necessary when carrying out dynamic response analysis for the pedestrian bridges to check the discomfort for the users being on a pedestrian bridge. Generally speaking, the relationship between footstep and dynamic load factor or footstep and walking speed proposed by [1] has been selected in Japan for the dynamic response analysis of the pedestrian bridges although several researchers have investigated the vertical forces imparted by individual pedestrians on a stationary floor. In the overseas, not only the resonant component (=footstep component  $f_w$  of generally about 2 steps / s) but also higher component ( $2f_w$ ,  $3f_w$ ) are often considered when carrying out dynamic response analysis for the pedestrian bridges [2]. On the other hand, the synchronization problem in the lateral direction on a footbridge is pointed out by [3]. The phenomenon of synchronous lateral excitation caused by pedestrians walking on pedestrian bridges such as the London Millennium Bridge has increasingly attracted public attention [4]. The dynamic load factor (DLF) in lateral direction is short of measured data compared with that in vertical direction.

Special experimental apparatus such as force transducers

must be provided in order to measure a human walking force exactly. Accordingly, it will be acceptable from the practical viewpoint if a relatively simple and accurate method to evaluate human walking force for many pedestrians is used. It can be easily imagined that there is a method to measure the acceleration data using accelerometers set up to the lumbar part of a person. In fact, it is already pointed out by [5] that the human walking force measured by special experimental apparatus is nearly equal to an inertia force at the lumbar part of a person being tested (= [Weight of a person] × [acceleration measured at the lumbar part]). However, it may be noticed that the acceleration data of the human body involves so many responses in a lot of frequencies and that the main response in a principal frequency should be picked out by means of digital band-pass filter.

The purpose of this study is to propose the simplified method in order to identify human walking force by using FFT power spectrum density from the experimental acceleration data of the human body. An experiment on human walking is carried out on a stationary floor especially paying attention to higher components of dynamic vertical walking force. The vertical vibration of pedestrian bridge induced by higher components of human walking vertical force is also discussed in this paper. A full scale measurement for the existing pedestrian bridge with center span length of 33 m is carried out focusing on the resonance phenomenon due to higher components of human walking vertical force. Dynamic response characteristics due to these vertical higher components of human walking are revealed from the dynamic design viewpoint of pedestrian bridge.

## II. ACCURACY OF PRESENT METHOD FOR SINUSOIDAL WAVE

In this chapter, the accuracy of the proposed method by using FFT power spectrum density is discussed for the results of sinusoidal wave. Sinusoidal wave can be written in the form:

$$y = a \sin \omega t = a \sin 2\pi ft \quad (1)$$

where  $a$  is the amplitude,  $\omega$  is the circular frequency or angular velocity of the motion,  $f$  is the frequency of the motion. Power spectrum density  $P_f$  based on FFT (Fast Fourier Transform) in the frequency of  $f$  for the sinusoidal wave shown in (1) is given by

$$P_f = a^2 \times \frac{T}{2} \quad (2)$$

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where  $T$  is the duration of the wave without time corresponding to trailing zeros.

FFT analysis is carried out for the simulated acceleration wave with  $a = 1.0\text{m/s}^2$  and  $f = 0.9765$  Hz on condition that the sampling time  $\Delta t$  is 0.005 s and data number  $N$  is 2,048. Fig. 1 shows the power spectrum density obtained by FFT analysis. It can be seen from Fig. 1 that the peak value of the power spectrum density is  $5.12 \text{ (m/s}^2\text{)}^2 \cdot \text{s}$ . Substituting  $P_f = 5.12$  and  $T = 2,048 \times 0.005 = 10.24$  s into (2),  $a = 1.0\text{m/s}$  which is equal to the amplitude of the simulated original acceleration wave is obtained. However, attention needs to be paid to the fact that the power spectrum density based on FFT is given by every  $1/(N\Delta t)$ . For the example wave with  $a = 1.0\text{m/s}^2$  and  $f = 0.9765$  Hz ( $\Delta t = 0.005$  s), the power spectrum density in the frequency of 0.9765 Hz is correctly evaluated as the power spectrum density is given by every  $1/(N\Delta t) = 1/(2,048 \times 0.005) \cong 0.09765\text{Hz}$ . On the other hand, FFT analysis is also carried out for the simulated acceleration wave with  $a = 1.0\text{m/s}^2$  and  $f = 1.000$  Hz on condition that the sampling time  $\Delta t$  is 0.005 s and data number  $N$  is 2,048. Fig. 2 shows the power spectrum density obtained by FFT analysis. It can be seen from Fig. 2 that the peak value of the power spectrum density is reduced to be  $4.21 \text{ (m/s}^2\text{)}^2 \cdot \text{s}$  and scattered on both frequencies. This is the reason why the output frequency around 1 Hz by FFT analysis is 0.9765 Hz, 1.0742 Hz and 1.000 Hz is not to be the output frequency. Accordingly, in this research, it is assumed that the power spectrum density on the top 4 ( $P_1, P_2, P_3, P_4$ ) is summed up as shown in (3) with reference to Fig. 3.

$$P_f \cong P_1 + P_2 + P_3 + P_4 \tag{3}$$

Generally speaking, FFT analysis requires data number which is equal to the raised 2 to the nth power. Therefore, in case of  $N = 2,049$ , FFT analysis is carried out for data number of  $N = 4,096$  which is added to the trailing zero of 2,047. It follows that the differences may be varied because the dominant frequency is not equal to the output frequency owing to the change of resolution. Accordingly, the accuracy of the present method is checked by changing data number of the simulated

acceleration wave. These analytical results are shown in Table I. It can be seen from Table I that the difference in estimated value for the simulated sinusoidal wave is within the range of  $-5\%$ .

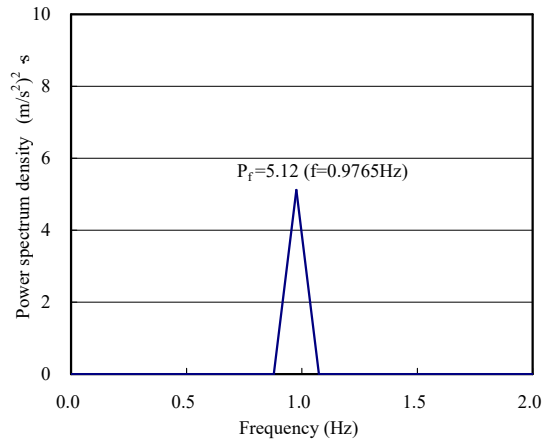


Fig. 1 Power spectrum density by FFT analysis ( $N=2,048, a=1.0\text{m/sec}^2, f=0.9765\text{Hz}$ )

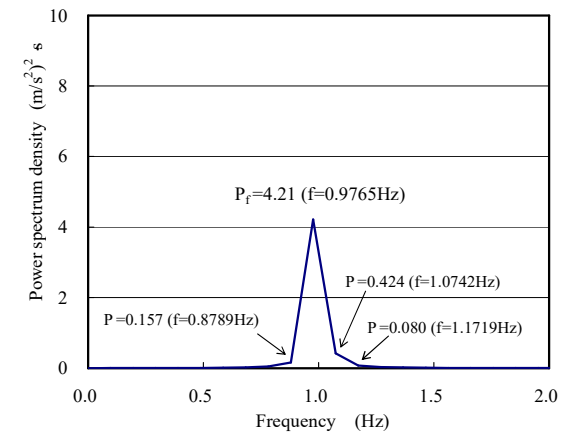


Fig. 2 Power spectrum density by FFT analysis ( $N=2,048, a=1.0\text{m/sec}^2, f=1.000\text{Hz}$ )

TABLE I  
DIFFERENCE FOR THE SIMULATED ACCELERATION WAVE

Frequency (Hz)	Amplitude (m/s <sup>2</sup> )	Data Number	$\Delta t$ (sec)	$P_1+P_2+P_3+P_4$ (m/sec <sup>2</sup> ) <sup>2</sup> · s	Calculated amplitude (m/s <sup>2</sup> )	Difference (%)
0.9765	1.0000	2,048	0.005	5.12	1.0000	0.00
1.0000	1.0000	2,048	0.005	4.87	0.9753	-2.47
0.9765	1.0000	2,049	0.005	4.64	0.9517	-4.83
1.0000	1.0000	2,049	0.005	4.87	0.9750	-2.50
0.9765	1.0000	3,072	0.005	7.09	0.9608	-3.92
1.0000	1.0000	3,072	0.005	7.26	0.9723	-2.77

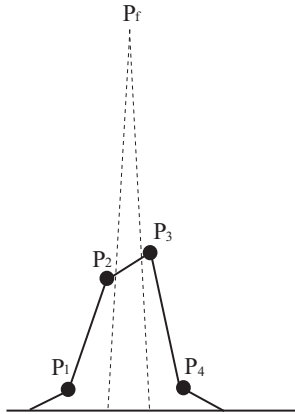


Fig. 3 Trapezoidal power spectrum density

### III. EXPERIMENTS ON HUMAN WALKING

#### A. Measuring Method

Experiments on human walking are carried out to investigate human walking characteristics according to the following procedure.

- Accelerometers in the vertical direction are set up to the lumbar part of a person being tested as shown in Fig. 4.
- The person being tested walks straight for 20 m on a stationary floor.
- The person being tested measures walking time with the stopwatch.

In principle, experiments on human walking were performed three times for every person being tested to obtain acceleration response which was filtered by 10 Hz analog low pass and sampled by  $\Delta t=0.005$  s. All persons being tested were senior healthy students belonging to my research laboratory with the height and weight of 168-175cm and 530-686N for male students, 145-163cm and 392-510N for female students.

#### B. Measured Acceleration

For instance, Fig. 5 shows the measured vertical acceleration at the right lumbar part of male student Ya (height=169cm, weight=686N) when walking to match the sound of 2Hz from an electronic metronome. It is appended that the first transient wave up to 1.5 seconds has been removed in Fig. 5. Fig. 6 shows the power spectrum density obtained by FFT analysis for the measured vertical acceleration shown in Fig. 5. In Fig. 6,  $f_w=2.0508$  Hz represents the footfall frequency in the vertical direction. It can be also seen from Fig. 6 that not only the component with frequency of  $2f_w$ ,  $3f_w$ , but also the component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$  where  $f_w$  is the walking rate is observed. So, FFT analysis was also conducted for the measured vertical acceleration at the right and left lumbar part of male student Na (height=173cm, weight=676N). Figs. 7 and 8 show the power spectrum density at the right and left lumbar part respectively obtained by FFT analysis. It is found from these figures that higher walking components (not only the component with frequency of  $2f_w$ ,  $3f_w$ , but also the component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$ ) are observed for male student Na. Therefore, FFT analysis has been carried out for the

averaging data observed by the right and left lumbar part data as shown in Fig. 9. It can be seen from Fig. 9 that the component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$  hardly exists and that it is the component with frequency of  $2f_w$ ,  $3f_w$  only be present. These results clearly demonstrate that the component with frequency of  $2f_w$ ,  $3f_w$  is in-phase and that the component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$  is in-opposite-phase in right and left lumbar part.



Fig. 4 Set up of accelerometer on a person being tested

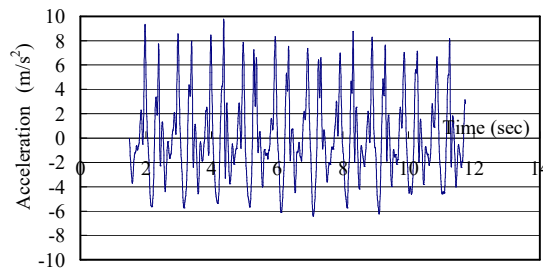


Fig. 5 Measured vertical acceleration at the right lumbar part of male student Ya (height=169cm, weight=686N)

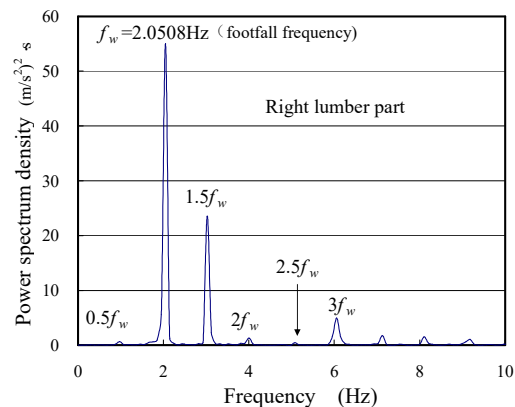


Fig. 6 Power spectrum density obtained by FFT analysis for the measured vertical acceleration shown in Fig. 5

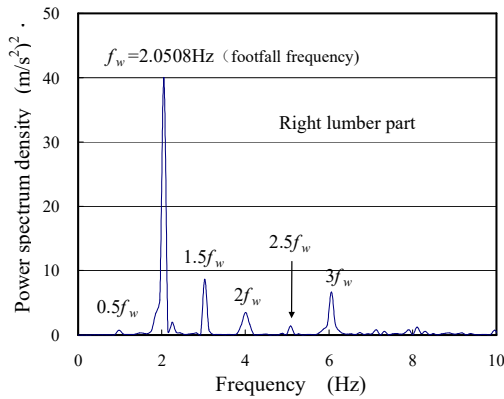


Fig. 7 Power spectrum density at the right lumber part of male student Na (height=173cm, weight=676N)

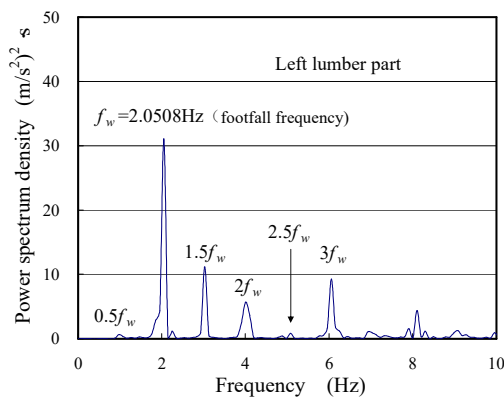


Fig. 8 Power spectrum density at the left lumber part of male student Na (height=173cm, weight=676N)

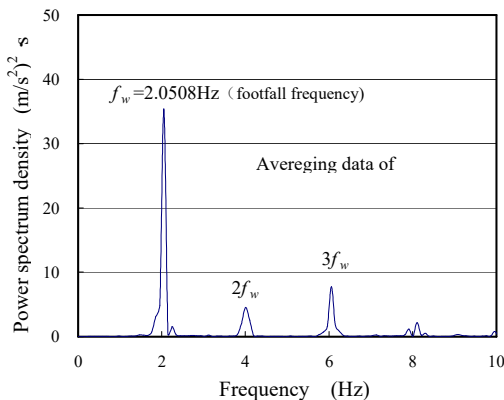


Fig. 9 Power spectrum density for the averaging data observed by the right and left lumbar part of male student Na (height=173cm, weight=676N)

#### IV. HUMAN WALKING FORCES (DYNAMIC LOAD FACTOR)

##### A. In-phase Component

FFT analysis is performed for the time history data ( $\Delta t=0.005s$ ) of all the subjects which is derived from measured acceleration after 1.5 second. Fig. 10 shows the relationship between the dominant frequency and the in-phase vertical

dynamic load factor (DLF). It is appended that the measured footfall component ( $f_w$ ) is also plotted in this figure. It can be seen from Fig. 10 that the DLF of footfall component ( $f_w$ ) estimated by the proposed method is fairly in good agreement with that measured based on force transducers by [1] although there is a slight difference from person to person in both values. Whereas, it can be also seen from Fig. 10 that 2 and 3 times components ( $2f_w$  and  $3f_w$ ) do not increase even the dominant frequency is increased and that both average DLFs are almost around 0.1 although measured values varied somewhat widely. These measured DLFs are fairly in good agreement with the reported values [6].

The walking vertical force can be estimated using the measured vertical acceleration on the lumbar part, from which a steady vertical acceleration  $a$  is calculated according to the method as shown in Chapter II.

The walking vertical force due to a pedestrian can be evaluated in:

$$\text{Walking force} = \frac{W}{g} \times a = W \times \frac{a}{g} = W \times DLF \quad (4)$$

where  $W$  is the weight of a person being tested,  $g$  is the gravity acceleration,  $DLF$  is the dynamic load factor ( $DLF = a/g$ ).

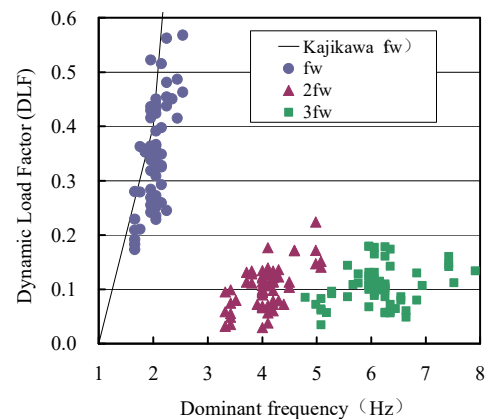


Fig. 10 Relationship between the dominant frequency and the in-phase vertical dynamic load factor (DLF)

##### B. In-opposite-phase Component

Fig. 11 shows the relationship between the dominant frequency and the in-opposite-phase vertical dynamic load factor (DLF). It can be seen from Fig. 11 that 0.5 and 1.5 times components ( $0.5f_w$  and  $1.5f_w$ ) show the tendency to increase according to the increase of dominant frequency. Although 1.5 times component ( $1.5f_w$ ) is greatly distributed over around 0.05-0.3, this is the reason why 1.5 times component depends on the walking characteristics such as in-opposite-phase movement of the lumbar part. On the other hand, 2.5 times component ( $2.5f_w$ ) does not show the tendency to increase according to the increase of dominant frequency and the average value of this component is about 0.05.

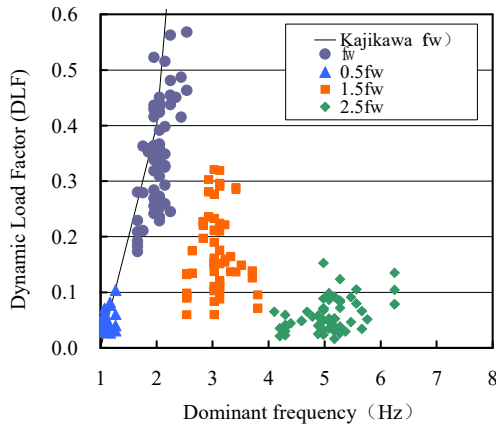


Fig. 11 Relationship between the dominant frequency and the in-opposite-phase vertical dynamic load factor (DLF)

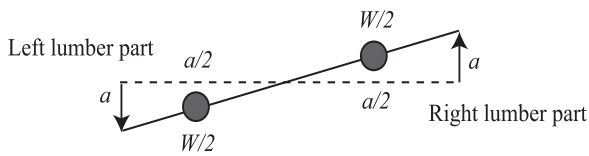


Fig. 12 In-opposite-phase component (rotary motion)



Fig. 13 Pedestrian bridge being tested

By the way, assuming that the in-opposite-phase component is the rotary motion as shown in Fig. 12, the walking vertical force of this component can be evaluated in:

$$Walking\ force = 2 \times \frac{W/2}{g} \times \frac{a}{2} = \frac{W}{2} \times \frac{a}{g} = \frac{W}{2} \times DLF \quad (5)$$

Therefore, it can be said that the walking force due to the in-opposite-phase component becomes half values compared with that of the in-phase component if DLF value of both is the same.

#### V. FULL SCALE MEASUREMENT FOR THE PEDESTRIAN BRIDGE

In Japan, the vibration of the pedestrian bridge induced by the in-phase component with frequency of  $2f_w$ ,  $3f_w$  where  $f_w$  is the footfall frequency have not been almost investigated.

Furthermore, the in-opposite-phase component with frequency of  $0.5f_w$ ,  $1.5f_w$  and  $2.5f_w$  is not known in other countries. Therefore, in this Chapter, the vibration excited by 2 times and 1.5 times component of the walking force is investigated for the existing pedestrian bridge.

#### A. Pedestrian Bridge Being Tested

Fig. 13 shows the pedestrian bridge being tested. This pedestrian bridge with the span length of about 33m and the effective width of 1.5m is located in Osaka prefecture. It is appended that this pedestrian bridge is simply supported bridge because it has a hinge connection at middle bridge pier as shown in Fig. 14.



Fig. 14 Hinge connection at middle bridge pier

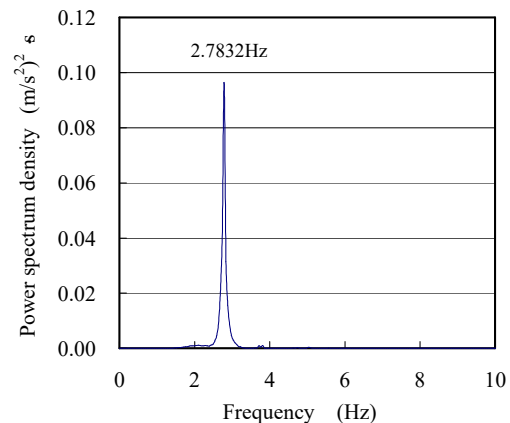


Fig. 15 Power spectrum density obtained by FFT analysis for pedestrian bridge being tested (10Hz analog low-pass filter)

One accelerometer is installed in the vertical direction of the bridge center point in order to measure the damped free vibration excited by bending and stretching movement of one person in condition that stationary 10 occupants are at the center point of the bridge. FFT analysis is carried out for the measured acceleration wave (sampling time  $\Delta t$  is  $\Delta t = 0.005s$ , 10Hz analog low-pass filter). Fig.15 shows the power spectrum density obtained by FFT analysis. It can be seen from Fig. 15

that fundamental frequency of this bridge is about 2.8 Hz in condition that stationary 10 occupants are at the center point of the bridge.

### B. Overview of Walking Experiment for the Bridge

Male students Ma (height 167cm, weight 530N) walked on the bridge from end to end (about 33m) as the same pace as the tone of the electronic metronome.

Because the tone of electronic metronome was set from 1.25 times/s (walking step=1.25 steps/s) to 2.2 times/sec (walking step=2.2 steps/s), it seems that walking experiment in these tone range does not cause resonance with the fundamental frequency of the bridge (about 2.8Hz) in case that only the footstep component is taken into account. However, 2 times walking component of walking steps ( $1.25 \times 2 = 2.5 \sim 1.65 \times 2 = 3.3$  steps/s) might cause resonance with the fundamental frequency of the bridge (about 2.8Hz). Moreover, 1.5 times walking component of walking steps ( $1.7 \times 1.5 = 2.55 \sim 2.2 \times 1.5 = 3.3$  steps/s) also might cause resonance with the fundamental frequency of the bridge (about 2.8Hz).

### C. Experimental Results and Discussion

Walking experiment for the pedestrian bridge mentioned above was performed three times. As almost the same results were obtained, second experimental results will be discussed in this Chapter. It is appended that 3 people of the standing position dealt with measurement work at the center point of the bridge in a stationary state.

Maximum acceleration is estimated from the band-pass filtered wave of 2.3Hz-3.5Hz. Fig. 16 shows the relationship between the footfall frequency and evaluated maximum acceleration at the center. It can be seen from Fig. 16 that two acceleration peaks are observed in both around 1.42Hz and 1.90Hz which differs significantly from the 2.7832Hz (fundamental frequency of the pedestrian bridge).

Fig. 17 shows the relationship between the 2 times footfall frequency ( $2 \times \text{footfall frequency} = 2f_w$ ) and evaluated maximum acceleration at the center in order to pay attention to the 2 times component. Fig. 18 shows that the power spectrum density for the acceleration measured at the left lumber part of subject Ma in case of 1.417 step/s. Therefore, it can be seen from these figures that the peak of the response at the frequency of around 2.83 Hz is the vibration excited by the 2 times component ( $2f_w$ ) of walking vertical force.

Fig. 19 shows the relationship between the 1.5 times footfall frequency ( $1.5 \times \text{footfall frequency} = 1.5f_w$ ) and evaluated maximum acceleration at the center in order to pay attention to the 1.5 times component. Fig. 20 shows that the power spectrum density for the acceleration measured at the left lumber part in case of 1.90 step/s. Therefore, it can be seen from these figures that the peak of the response at the frequency of around 2.85 Hz is the vibration excited by the 1.5 times component ( $1.5f_w$ ) of walking vertical force.

Based on the results of walking experiment for the pedestrian bridge, it can be said that it might be important to take into account the higher components in order to check the vibration

serviceability of slender and lighter pedestrian bridge.

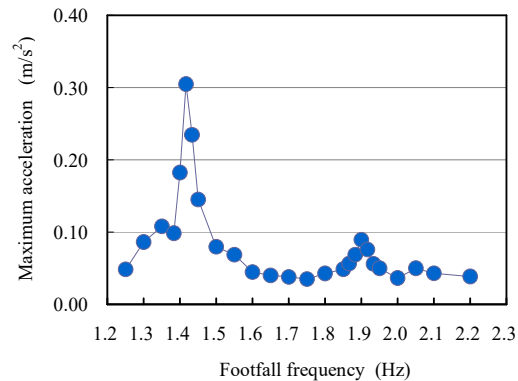


Fig. 16 Relationship between footfall frequency and evaluated maximum acceleration at the center

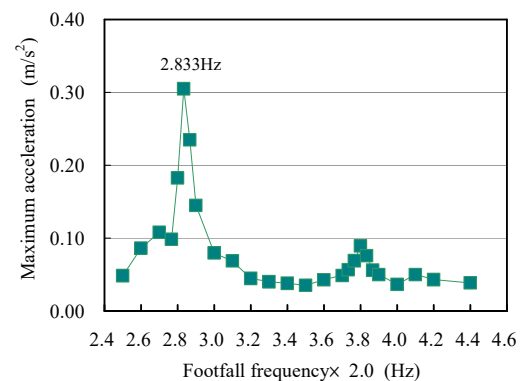


Fig. 17 Relationship between the 2 times footfall frequency and evaluated maximum acceleration at the center

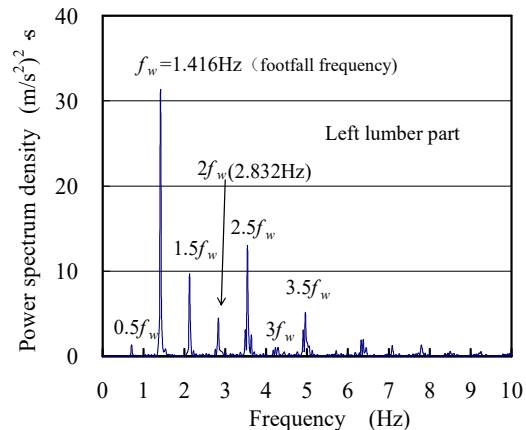


Fig. 18 Power spectrum density for the acceleration measured at the left lumber part of subject Ma in case of 1.417 step/s

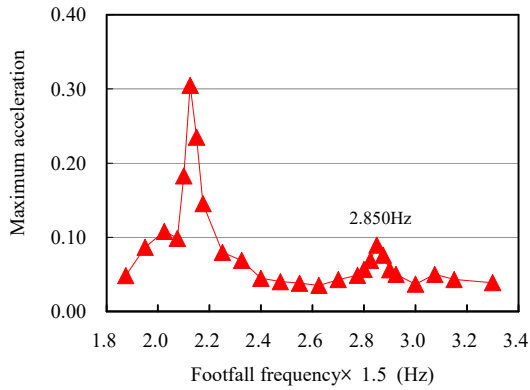


Fig. 19 Relationship between the 1.5 times footfall frequency and evaluated maximum acceleration at the center

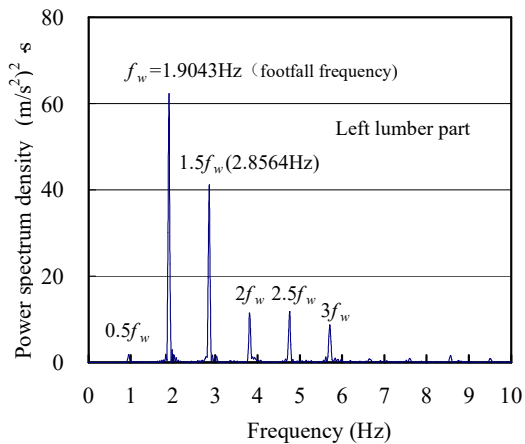


Fig. 20 Power spectrum density for the acceleration measured at the left lumber part in case of 1.90 step/s

## VI. CONCLUSIONS

This paper deals with the simplified method to identify human walking force by using FFT power spectrum density from the experimental acceleration data of the human body. The vertical vibration of pedestrian bridge induced by higher components of human walking vertical force is also discussed. The results are summarized below:

- (1) A simplified method to identify human walking force by using FFT power spectrum density from the experimental acceleration data of the human body is proposed.
- (2) The difference in estimated value for the simulated sinusoidal wave is within the range of  $-5\%$ .
- (3) It was found that the dynamic load factor (DLF) for walking step component in vertical direction estimated by the proposed method is in fairly good agreement with those measured by force transducers although there is a slight difference due to the body type.
- (4) In addition to walking step component with the footfall frequency of  $f_w$ , it is found that higher walking components (not only the component with frequency of  $2f_w$ ,  $3f_w$ , but also the component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$ ) are also observed.

- (5) It was verified that the component with frequency of  $2f_w$ ,  $3f_w$  is in-phase and that the component with frequency of  $0.5f_w$ ,  $1.5f_w$ ,  $2.5f_w$  is in-opposite-phase in right and left lumber part.
- (6) The pedestrian bridge vibration excited by 2 times and 1.5 times component ( $2f_w$  and  $1.5f_w$ ) is observed through a walking test for the existing bridge with a center span of about 33m.

Needless to say, a lot of further study might be necessary to identify human walking force by using FFT power spectrum from the experimental acceleration data of the human body. It is hoped that this study will provide useful information for bridge engineers in investigating dynamic behavior of pedestrian bridges.

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