International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:11, No:5, 2017

High Sensitivity Crack Detection and Locating with Optimized Spatial Wavelet Analysis

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***Abstract*—**In this study, a spatial wavelet-based crack localization technique for a thick beam is presented. Wavelet scale in spatial wavelet transformation is optimized to enhance crack detection sensitivity. A windowing function is also employed to erase the edge effect of the wavelet transformation, which enables the method to detect and localize cracks near the beam/measurement boundaries. Theoretical model and vibration analysis considering the crack effect are first proposed and performed in MATLAB based on the Timoshenko beam model. Gabor wavelet family is applied to the beam vibration mode shapes derived from the theoretical beam model to magnify the crack effect so as to locate the crack. Relative wavelet coefficient is obtained for sensitivity analysis by comparing the coefficient values at different positions of the beam with the lowest value in the intact area of the beam. Afterward, the optimal wavelet scale corresponding to the highest relative wavelet coefficient at the crack position is obtained for each vibration mode, through numerical simulations. The same procedure is performed for cracks with different sizes and positions in order to find the optimal scale range for the Gabor wavelet family. Finally, Hanning window is applied to different vibration mode shapes in order to overcome the edge effect problem of wavelet transformation and its effect on the localization of crack close to the measurement boundaries. Comparison of the wavelet coefficients distribution of windowed and initial mode shapes demonstrates that window function eases the identification of the cracks close to the boundaries.

***Keywords*—**Edge effect, scale optimization**,** small crack locating, spatial wavelet.

I. Introduction

NOWADAYS crack detection and localization has been a common topic among researchers to enhance the stability, durability, and safety of engineering structures. Vibration-based damage identification methods are widely used in the mechanical and civil engineering researches. A damage changes the physical properties of the structure such as stiffness, mass, and damping ratio. The vibration-based damage identification is based on the fact that these alterations can affect modal properties of the structure such as mode shapes and natural frequencies. Investigation of the change in the natural frequency was the earliest vibration-based method to identify the damage [1].

Using mode shapes as a feature to detect the damage has some advantages over the natural frequency-based method. Mode shapes can reveal local information which makes them more sensitive to local defects [2]. Moreover, mode shapes are less affected by environmental factors, such as temperature variations, as compared to the natural frequency methods. Over last decades, wavelet transformation has been a popular tool among researchers to analyze the vibrational signals to ease the damage detection. Ashino and Yamamoto developed the theory of wavelets and their applications [3]. Surace and Ruotolo used the wavelet transform to analyze vibration response signals of a cracked beam [4]. Wang and Deng proposed a damage detection technique using spatial wavelet analysis [5]. A fluctuation on the deflection profile at the crack position would be induced by crack in structures. Even a small and invisible perturbation would be discerned through wavelet coefficients values. This would practically mean that the detection of the crack location becomes possible. Under both static and dynamic loading conditions, the numerically simulated deflection or displacement responses were analyzed with wavelet transform, and the presence of the crack was detected by a sudden change in the spatial variation of the transformed response. For most of the analysis, they employed the simple Haar wavelets. An application of spatial wavelet theory to crack identification in structures was proposed by Liew and Wang [6]. They calculated the wavelet coefficients along the length of the beam based on the numerical solution for the deflection of the beam. In order to find the position of crack from the wavelet data, an excitation that oscillates rapidly along the length of the beam was used to excite the beam. The crack location was then indicated by a peak in the variations of some of the wavelets along the length of the beam. Wu and Wang had experimental studies on damage detection of a beam structures with wavelet transform [7]. They used a high-resolution laser profile sensor to measure the deflection profile of a cracked aluminum cantilever beam subjected to a static displacement at its free end. De-noise techniques have been introduced to make the detection more efficient. The smoothed static profile of the cracked beam has been analyzed with Gabor wavelet to identify the crack. Rucka worked on wavelet-based damage detection technique on a cantilever beam with a single notch [8]. He presented experimental and numerical analysis of damage detection based on higher order modes. After reviewing previous studies in this area, it can be concluded that to have higher crack detection sensitivity, an optimization study for scale factor of wavelet transformation is needed. Furthermore, identification process for cracks, which are close to the boundary, is still a problem for researchers because of edge effect in wavelet analysis.

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In this study, an optimization process is performed on wavelet scale factor to reach higher crack detection sensitivity. Afterward, Hanning window is employed to erase the edge effect in order to ease the detection of cracks close to the boundaries.

II. Theoretical Model

A. Beam Equations

In this section, a cantilever beam with a through thickness crack in the middle is modeled. The schematic view of this cracked beam is shown in Fig. 1. The dimensions of the beam are mentioned in the TABLE II. The crack is represented by a torsion spring with torsion coefficient K_t (Fig. 1 (b)). Since a thick beam is considered in this study, the governing equations are based-on Timoshenko theory. The governing equations are as follows [9]:

$$-kAG\frac{\partial^2\omega}{\partial x^2}+kAG\frac{\partial\emptyset}{\partial x}+\ \rho A\frac{\partial^2\omega}{\partial t^2}=0 \tag{1}$$

$$-EI\frac{\partial^2\emptyset}{\partial x^2}\ -kAG\frac{\partial\omega}{\partial x}+kAG\,\emptyset+\rho I\frac{\partial^2\emptyset}{\partial t^2}=0 \tag{2}$$

where $\emptyset$ is the slope of the deflection curve, ω is the lateral displacement, ρ is the density of the beam material, A is the cross-section area, E is the elastic modulus, G is the shear modulus, I is the second moment of area, k is called the Timoshenko shear coefficient that depends on the beam geometry (mostly k=5/6 for a rectangular section). TABLE I shows the material properties of the beam under study.

TABLE I
Material Properties

Symbol	Description	Quantity
ρ	Density of the beam material	7870 kg/m^3
E	Elastic modulus	210 GPa
G	Shear modulus	79 GPa

To solve (1) and (2), variables are separated. $\omega(x,t)$ and $\emptyset(x,t)$ are assumed to be:

$$\omega(x,t)=W(x).\,e^{i\omega_n.t} \tag{3}$$

$$\emptyset(x,t)=P(x).\,e^{i\omega_n.t} \tag{4}$$

where ω_n is the natural frequency of the structure.

In this model, there are two boundary conditions at the fixed-end, and two at the free-end. In addition, the torsion spring makes four more boundary conditions at the crack position. According to the spring assumption, the cantilever beam has two separate parts which are connected by a spring. All boundary conditions are as:

$$at\ x=0\quad W1=0\,,\ \ P1=0 \tag{5}$$

$$at\ x=L\quad \frac{dP2}{dx}=0,\ \frac{dW2}{dx}-P2=0 \tag{6}$$

$$at\ x=e\quad W1=W2,\quad \frac{dP1}{dx}=\frac{dP2}{dx} \tag{7}$$

$$at\ x=e\quad P1-\frac{dW1}{dx}=P2-\frac{dW2}{dx} \tag{8}$$

The slope fluctuation at crack position is given by (9) [10]:

$$P1+\frac{EI}{KtL}.\frac{dP1}{dx}=P2 \tag{9}$$

where K_t is the stiffness of rotational spring, which is calculated as follows [11]:

$$Kt=\frac{bh^2E}{72\pi\left(\frac{d}{h}\right)^2q\left(\frac{d}{h}\right)} \tag{10}$$

where:

$$q\left(\frac{d}{h}\right)=0.6384-1.035\left(\frac{d}{h}\right)+3.7201\left(\frac{d}{h}\right)^2-5.1773\left(\frac{d}{h}\right)^3+7.553\left(\frac{d}{h}\right)^4-7.332\left(\frac{d}{h}\right)^5+2.4909\left(\frac{d}{h}\right)^6 \tag{11}$$

By applying eight boundary conditions, characteristic matrix is obtained. Afterward, the first three mode shapes of the beam are derived from a numerical simulation conducted in MATLAB [12]. Each of the mode shapes consists of 1001 data points along the beam length.

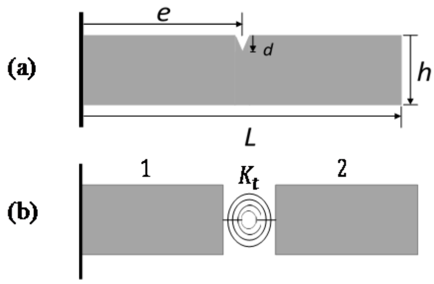


Fig. 1 (a) Schematic of a cantilever beam with a through thickness crack. (b) Torsion spring representing the crack

TABLE II
Beam Dimensions

Symbol	Description	Quantity
L	Length	1 m
h	Height	0.1 m
b	Width	0.05 m
d	Crack depth	0.0125 m

B. Wavelet Analysis

Wavelet is a transformation that decomposes a function $f(t)$ into a superposition of the elementary function $\psi_{a,b}(t)$ derived from an analyzing wavelet $\psi(t)$ known as the mother wavelet. Wavelets are generated from the mother wavelet through scaling and translation, as defined below:

$$\psi_{a,b}(t)=\frac{1}{|a|^{\frac{1}{2}}}\psi\left(\frac{t-b}{a}\right) \tag{12}$$

where a and b are real-valued parameters, a is the scale parameter, and b is the translation parameter. The wavelet coefficient for time scale wavelet transform is defined as:

$$C_{a,b} = \int_{-\infty}^{\infty} f(t)\overline{\psi_{a,b}(t)}\,dt \tag{13}$$

where $\overline{\psi_{a,b}(t)}$ is the conjugate function of $\psi_{a,b}(t)$.

In this project, the Gabor wavelet family is used to analyze the spatial information. This wavelet is generated from Gabor function, as defined below [13]:

$$\psi(t) = \frac{1}{\sqrt[4]{\pi}}\sqrt{\frac{\omega_0}{\gamma}}\exp\left[-\frac{\left(\frac{\omega_0}{\gamma}\right)^2}{2}t^2 + i\omega_0 t\right] \tag{14}$$

$$\gamma = \pi\sqrt{\left(\frac{2}{ln2}\right)} \tag{15}$$

Wavelets are mostly used to analyze the time-domain signals but with changing the t to a spatial coordinate in (14), spatial distributed signal can be analyzed by wavelet transform, as well.

$$C_{a,b} = \int_0^1 W(x)\overline{\psi_{a,b}(x)}\,dx \tag{16}$$

where x is the spatial coordinate, $W(x)$ is the mode shape of the cracked beam, a is the scale parameter and b is the translation parameter (indicating the position).

III. Results and Discussions

Three mode shapes which are derived in the previous section are not able to show any sign of crack in the beam. In structural health monitoring, wavelet transform is used to magnify the singularity at the crack position of the mode shape due to the crack effect. Gabor wavelet is applied to the first three mode shapes. The results show that, for a crack in the middle of the beam, all three modes reveal a large perturbation caused by the crack. Fig. 2 shows the relative wavelet coefficient values of the first three mode shapes for various scale factors. This figure is based-on a particular crack in the middle of the beam with the depth equal to 12.5 percent of the beam height. Relative wavelet coefficient compares coefficient values along the beam length with the minimum value at the intact locations of the beam for the sensitivity analysis (wavelet coefficient is used as a short expression in the rest of the paper). As it can be seen, very large wavelet coefficient values are obtained at all locations of the beam regardless of the crack position for scales greater than 9. Hence, it can be concluded, the scales greater than 9 are not suitable for this study. Furthermore, for scale values less than 4, edge effects clearly appear around the boundaries. According to Fig. 2, scales 6.5, 5.6, and 5.1 are the optimal scales for first, second, and third modes, respectively. For detection of a crack in the middle, two factors play important roles: first, the wavelet coefficient value at the damaged area, and second, the maximum number of wavelet coefficients, which are approximately zero at the intact area and domination of non-zero values at the crack area.

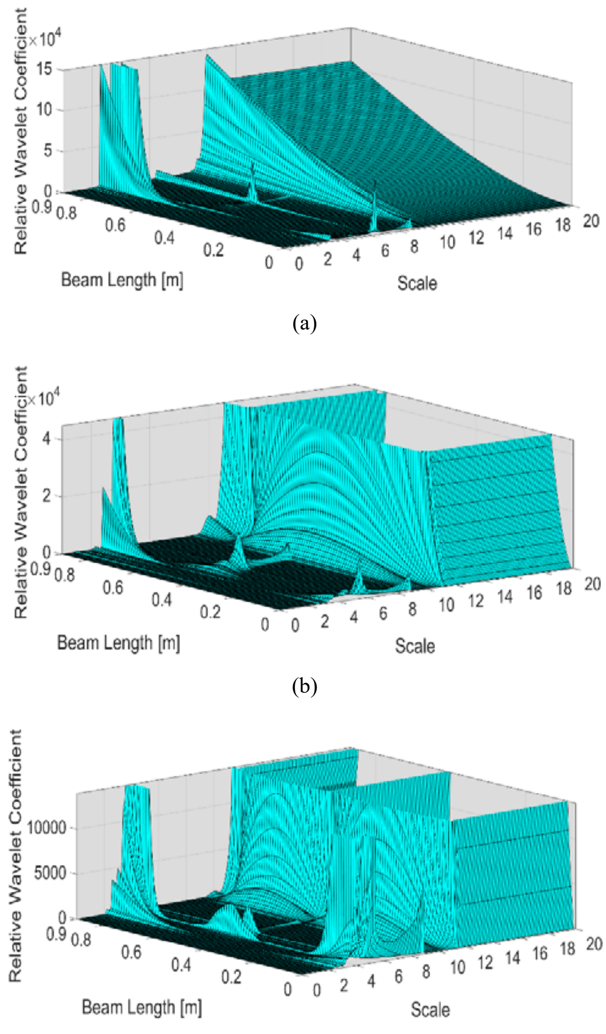
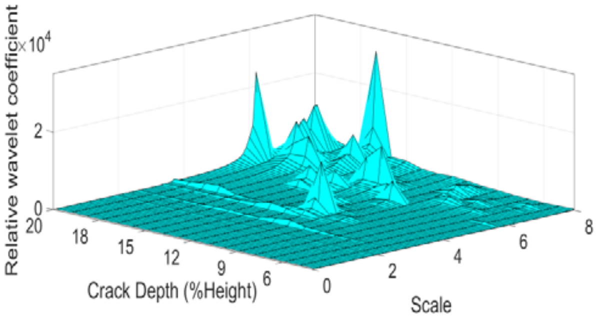


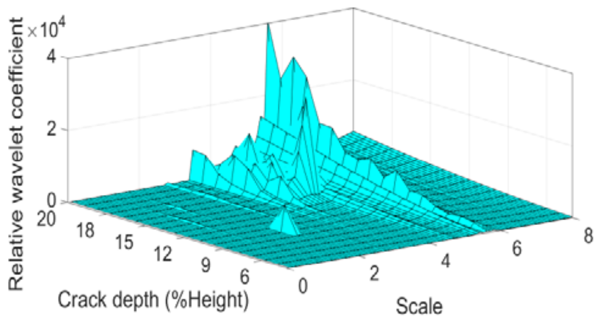
Fig. 2 Relative wavelet coefficient of the first three mode shapes with different scale factors at different positions of a beam with a crack at the middle (Crack depth = 0.125*Beam height). (a) first mode shape. (b) Second mode shape. (c) Third mode shape

Fig. 2 shows that wavelet coefficient values for the first and second modes are greater than the third mode. Wavelet coefficients are $4*10^4$, $1.5*10^4$, $3.5*10^3$ for first, second, and third mode, respectively (at the crack position with optimal scales). Also, the second mode has the maximum number of values, which are close to zero at the intact area when the optimal scale is used. Therefore, it can be concluded that the second mode is the most sensitive mode for identification of a crack located in the middle of the beam.

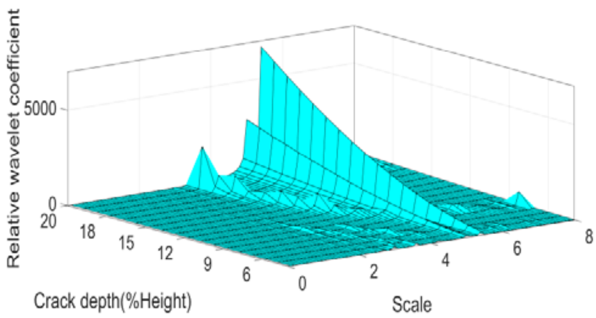
The previous discussion has clarified that the optimal scale varies for different mode shapes. Moreover, nature of the crack may also affect the results of this study. The depth and the position are two important crack properties. Optimal scales for depths from 3 to 20 percent of the beam height are shown in Fig. 3. This figure is comprised of three 3D plots and illustrates the effect of crack depth on the chosen optimal scale for the first three mode shapes while the crack is in the middle of the beam. For the first mode, 6 to 8 can be an optimal range of the scale factor for depths larger than 9 percent of beam height, although the results do not include any certain rule to choose an optimal scale for various crack depths. On the other hand, the derived plot based on the second mode demonstrates a certain range of the optimal scale for various crack depths, which is between 5 to 6. For the third mode, same range as the second mode is obtained for the optimal scale. However, the wavelet coefficient values for the third mode are very small in comparison to the first and second modes. Thus, the third is less sensitive to crack than the other two modes in this study.



(a)



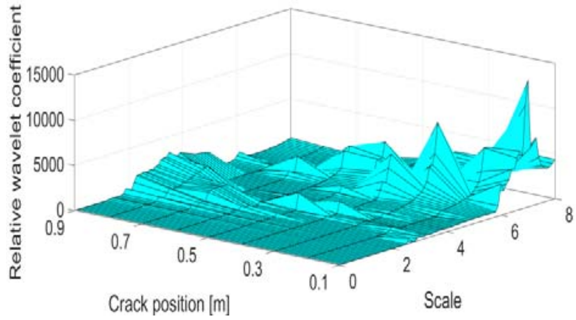
(b)



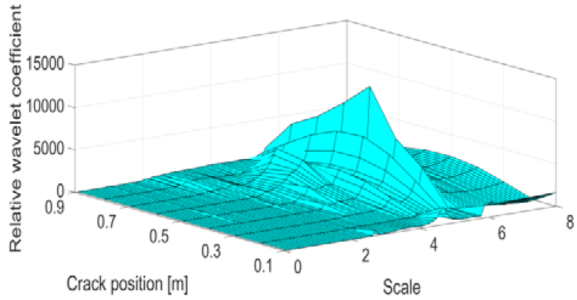
(c)

Fig. 3 Relative wavelet coefficient of the first three mode shapes vs. crack depth and scale with the crack at the middle of the beam. (a) First mode shape (b) Second mode shape (c) Third mode shape

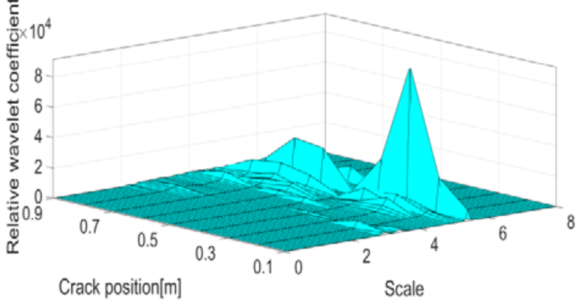
Fig. 3 shows a clear image of the crack depth effect on the chosen optimal scale. However, to have a complete understanding of influences of crack nature on the chosen optimal scale, another analysis is needed.



(a)



(b)



(c)

Fig. 4 Relative wavelet coefficients of first three mode shapes of a beam with a crack at nine different positions for various scale factors. (a) First mode shape (b) Second mode shape (c) Third mode shape

In this step, crack position varies from 0.1 to 0.9 m, and the wavelet coefficient value at the crack position for each scale factor is obtained. Fig. 4 illustrates the results of this analysis for the first three mode shapes. In this part of study, the crack depth is 12.5 percent of beam height. It can be seen that the first mode is more sensitive to detect and localize cracks, which are closer to the fixed-end than the free-end of the beam. Moreover, damage detection by scale factors smaller than 6 cannot be performed properly. However, by looking at plots which are obtained from the second and third modes, a specific optimal scale range of 5 and 6 can be found for different crack positions. Furthermore, the second mode is more sensitive to cracks close to the middle of the beam. On the other hand, the third mode is not sensitive to cracks close to the both ends and middle of the beam. It can be seen in Fig. 4 that the higher vibration modes carry more information on the damage in the structure than the lower vibration modes.

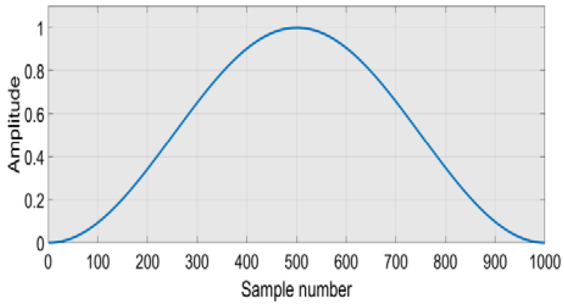
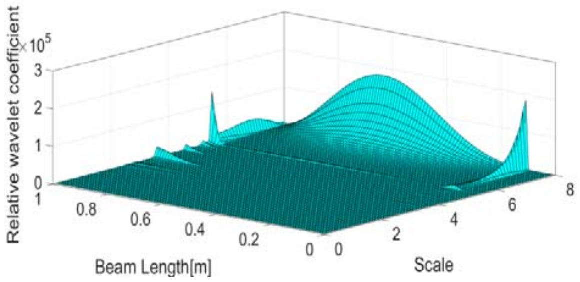
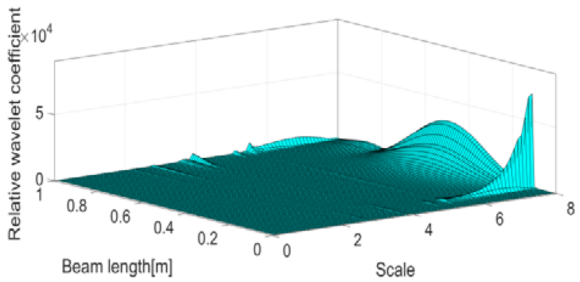


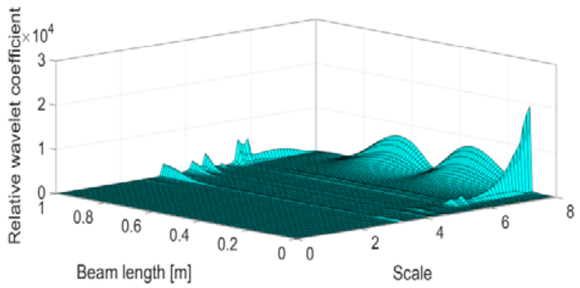
Fig. 5 Hanning window function



(a)



(b)



(c)

Fig. 6 Relative wavelet coefficient of the first three windowed mode shapes with different scale factors at different positions of a beam with a crack close to the fixed end (Crack depth = 0.125*Beam height). (a) First mode shape (b) Second mode shape (c) Third mode shape

As it has been mentioned before, edge effect is considered as a significant problem, because those large values could make it impossible to detect cracks, which are small or close to the boundaries. Fig. 2 illustrates that all three modes are not sensitive to cracks close to the fixed and free end because of the large coefficient values around the boundaries. At the mentioned locations, coefficient values are significantly large as it is illustrated in Fig. 2.

The window function can be an appropriate solution to erase the edge effect around the beam boundaries. As the last step of this study, a Hanning window function (Fig. 5) is applied to the first three mode shapes of the cantilever beam. A crack is defined close to the fixed end of the beam (0.1 m). The crack depth is 12.5 percent of the beam height. Afterward, the Gabor wavelet transform is applied to the windowed mode shapes and results are presented in Fig. 6. Same as the previous obtained results, large relative wavelet coefficient values at crack position prove that the first mode has the most sensitivity to detect and localize a crack close to the fixed end of the beam. This figure indicates an obvious perturbation at the crack position. However, except an intense peak at the damaged spot, a smooth perturbation appears in the middle of the beam. This smooth perturbation is due to the nature of the Hanning window function and can be seen in all three modes. As another finding from this figure, the edge effect is erased in areas close to the boundaries. Moreover, the optimal wavelet scale for all three modes is found to be around 8. Eventually, it can be concluded that by using proper window function, wavelet-based damage detection method is applicable even for cracks close to the both ends.

IV. Conclusion

In this work, an optimization process is performed on wavelet scale factor to reach higher crack locating sensitivity with spatial wavelet transformation. Optimal scale ranges with highest crack locating sensitivity are 6-8, and 5-6 for the first and second vibration modes, respectively. Furthermore, Hanning window is employed to erase the edge effect of the wavelet transformation in order to realize the detection of cracks close to the measurement boundaries. According to simulation results, it can be concluded that, by using the window function, spatial wavelet-based damage detection method is applicable even for cracks close to the both ends of the beam. As future works, experimental tests will be conducted to verify the results of numerical study. Moreover, the optimization procedure will be studied for different wavelet families.

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