

Heat and Mass Transfer in MHD Flow of Nanofluids through a Porous Media Due to a Permeable Stretching Sheet with Viscous Dissipation and Chemical Reaction Effects

Yohannes Yirga, Daniel Tesfay

Abstract—The convective heat and mass transfer in nanofluid flow through a porous media due to a permeable stretching sheet with magnetic field, viscous dissipation, chemical reaction and Soret effects are numerically investigated. Two types of nanofluids, namely Cu-water and Ag-water were studied. The governing boundary layer equations are formulated and reduced to a set of ordinary differential equations using similarity transformations and then solved numerically using the Keller box method. Numerical results are obtained for the skin friction coefficient, Nusselt number and Sherwood number as well as for the velocity, temperature and concentration profiles for selected values of the governing parameters. Excellent validation of the present numerical results has been achieved with the earlier linearly stretching sheet problems in the literature.

Keywords—Heat and mass transfer, magnetohydrodynamics, nanofluid.

I. INTRODUCTION

THE study of the boundary layer flow of an electrically conducting fluid through a porous media has many applications in manufacturing and natural process which include cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, cooling of an infinite metallic plate in a cooling bath, textile and paper industries, glass-fiber production, manufacture of plastic and rubber sheets, the utilization of geothermal energy, the boundary layer control in the field of aerodynamics, food processing, plasma studies and in the flow of biological fluids. The concept of stretching sheet was first used by [1] to study the flow of an incompressible viscous fluid with a linearly varying surface velocity. Later, various authors extended this work to various aspects of flow and heat transfer over a stretching surface [2]-[5].

Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. Many experimental and theoretical studies on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. The study of magnetohydrodynamics has many

important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field [6]. Recently, the application of magnetohydrodynamics in the polymer industry and metallurgy has attracted the attention of many researchers. Several researches investigated the MHD flow [7]-[12].

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flows such as injection molding or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed aircraft raises the temperature of the skin. In a completely different application, the dissipation function is used to define the viscosity of dilute suspensions Einstein [13]: Viscous dissipation for a fluid with suspended particles is equated to the viscous dissipation in a pure Newtonian fluid, both being in the same flow (same macroscopic velocity gradient). Vajravelu and Hadjinicolaou [14] analyzed the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. The effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate are numerically studied by [15]. Gebhart [16] and Gebhart and Mollendorf [17] studied the effect of viscous dissipation in natural convection processes. They observed that the effect of viscous dissipation is significant in vigorous natural convection and mixed convection processes. They also showed the existence of a similarity solution for the external flow over an infinite vertical surface with an exponential variation of surface temperature. Javed and Sina [18] studied the viscous flow over nonlinearly stretching sheet with effects of viscous dissipation. They found that for large Prandtl numbers, the temperature profile decreases. Habibi et al. [19] studied the mixed convection MHD flow of nanofluid over a non-linear stretching sheet with effects of viscous dissipation and variable magnetic field.

The presence of porous media in a boundary layer flow can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface. Porous media are generally modelled using the classical Darcy formulation,

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which implies that the mean filter velocity is proportional to the summation of the pressure gradient and the gravitational force. The model is empirical and cannot be derived analytically via a momentum on a small element of porous medium. The Darcian model has been widely used non-Newtonian porous media heat transfer flows. For regimes where higher velocities may occur, the Darcy linear model is inadequate and refinements have to be employed. The Darcy–Forchheimer (DF) model is probably the most popular modification to Darcian flow utilized in simulating inertial effects. It has been used extensively in chemical engineering analysis and also in materials processing simulations. Recently, the flow through porous media has attracted the interest of different researchers due its diverse application in biorheology, chemical and petroleum industries, in geophysical fluid dynamics such as beach sand, wood, sandstone, limestone, the human lung and in small blood vessels [20], [21].

In the studying of boundary layer flows the phenomenon of suction or injection is very important. The boundary layer thickness is controlled by introducing suction at the surface of the moving body. The effect of the suction/enjection on the boundary layer flow has been studied by [22]-[26].

Heat and mass transfer problems with a chemical reaction have received a considerable amount of attention in recent years. In processes such as drying, evaporation, energy transfer in a cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes and industrial applications, such as in the curing of plastics, the cleaning and chemical processing of materials and the manufacture of pulp and insulated cables. Chamka [27] studied the MHD flow over a uniformly stretched vertical permeable surface subject to a chemical reaction. Afifi [28] analyzed the MHD free convective flow and mass transfer over a stretching sheet with a homogeneous chemical reaction of order n (where n was taken to be 0, 1, 2 or 3). The influence of a chemical reaction on heat and mass transfer due to natural convection from vertical surfaces in porous media subject to Soret and Dufour effects was studied by [29]. He showed that the thickness of the concentration boundary layer decreases as the Lewis number increases, a phenomenon also evident when a chemical reaction is absent. Kandasamy and Palanimani [30] carried out an analysis of the effects of chemical reactions on heat and mass transfer on a magnetohydrodynamic boundary layer flow over a wedge with ohmic heating and viscous dissipation in a porous medium.

Fluid heating and cooling are important in many industries such as power, manufacturing, transportation, and electronics. Effective cooling techniques are greatly needed for cooling any sort of high-energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited/poor heat transfer capabilities due to their low heat transfer properties. In contrast, metals have thermal conductivities up to three times higher than these fluids, so it is natural that it would be desired to combine the two

substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. A lot of experimental and theoretical researches have been made to improve the thermal conductivity of these fluids. In 1993, during an investigation of new coolants and cooling technologies at Argonne national laboratory in U.S, Choi invented a new type of fluid called Nanofluid [31]. Nanofluids are fluids that contain small volumetric quantities of nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid [32]. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to see effective heat transfer enhancements. Nanofluids are studied because of their heat transfer properties: they enhance the thermal conductivity and convective properties over the properties of the base fluid. Moreover, the presence of the nanoparticles enhance the electrical conductivity property of the nanofluids, hence are more susceptible to the influence of magnetic field than the conventional base fluids. Typical thermal conductivity enhancements are in the range of 15-40% over the base fluid and heat transfer coefficient enhancements have been found up to 40% [33]. Thermophysical properties of nanofluids such as thermal conductivity, diffusivity and viscosity have been studied by, among others, [34]-[36].

After the pioneer investigation of [31], thriving experimental and theoretical researches were undertaken to discover and understand the mechanisms of heat transfer in nanofluids. The knowledge of the physical mechanisms of heat transfer in nanofluids is of vital importance as it will enable the exploitation of their full heat transfer potential. Masuda et al. [37] observed the characteristic feature of nanofluid is thermal conductivity enhancement. This observation suggests the possibility of using nanofluids in advanced nuclear systems [38]. A comprehensive survey of convective transport in nanofluids was made by [39], which states that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on further heat transfer enhancement observed in convective situations. Khan and Pop [40] presented a similarity solution for the free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with a nanofluid. Makinde and Aziz [41] studied MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition.

Majority of the above studies are restricted to boundary layer flow and heat transfer in Newtonian fluids. However, due to the increasing importance of nanofluids, in recent years a great attention has been given to the study of convective transport of nanofluids. In addition to this the flow of nanofluids through a porous media with effects of chemical reaction and viscous dissipation has been given less attention. Therefore, the aim of the present paper is to study the magnetohydrodynamic flow and heat transfer of nanofluids through a porous Media due to a permeable stretching sheet

with viscous dissipation and chemical reaction effects. The combined effect of all the above mentioned parameters has not been reported so far in the literature.

The governing highly nonlinear partial differential equation of momentum, energy and concentration fields has been simplified by using suitable similarity transformations and then solved numerically with the help of a powerful, easy to use method called the Keller box method. This method has already been successfully applied to several non linear problems corresponding to parabolic partial differential equations. As discussed in [42], the exact discrete calculus associated with the Keller-Box Scheme is shown to be fundamentally different from all other mimic numerical methods. The box-scheme of Keller is basically a mixed finite volume method, which consists in taking the average of a conservation law and of the associated constitutive law at the level of the same mesh cell.

The paper is organized as follows: the mathematical formulation of the problem is presented in Section II. Section III outlines the numerical procedure whilst results and discussions are presented in Section IV. Section V concludes the paper.

II. MATHEMATICAL FORMULATION

Consider the two-dimensional steady laminar flow of an incompressible nanofluid over a stretching sheet. If we consider the Cartesian coordinate system with the origin fixed in such a way that, the x-axis is taken along the direction of the continuous stretching surface and the y-axis is measured normal to the surface of the sheet. Two equal but opposite forces are applied along the sheet so that the wall is stretched, keeping the position of the origin unaltered. The fluid is electrically conducting under the influence of an applied magnetic field $B_0(x)$ normal to the stretching surface. Since the magnetic Reynolds number is very small for most fluid used in industrial applications it is assumed that the induced magnetic field is negligible in comparison to the applied magnetic field. The fluid is a water based nanofluid containing two different types of nanoparticles; copper and silver nanoparticles. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the nanofluid are given in Table I (see [43]). With the above assumptions, the boundary layer equations governing the nanofluid flow, the heat and the concentration fields can be written in dimensional form as [44], [45].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(x)}{\rho_{nf}} u - \frac{\mu_{nf}}{\rho_{nf} K} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho c_p)_{nf}} \left(\frac{\partial u}{\partial y} \right)^2, \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + D_1 \frac{\partial^2 T}{\partial y^2} - K_0 (C_w - C_\infty), \quad (4)$$

where u and v are the velocity components in the x and y directions respectively, T is the temperature, C is the concentration of the nanofluid, C_∞ is the concentration of the nanofluid far from the sheet, B_0 is the uniform magnetic field strength, σ is the electrical conductivity, c_p is the specific heat at constant pressure, D is the species diffusivity, D_1 is the coefficient that signifies the contribution to mass flux through temperature gradient, K is the permeability of the porous medium, K_0 is a chemical reaction parameter; and μ_{nf} , ρ_{nf} , α_{nf} , and $(\rho c_p)_{nf}$ are the dynamic viscosity, density, thermal diffusivity, and heat capacitance of the nanofluid, respectively, which are given as [46], [47]

$$\begin{aligned} \nu_f &= \frac{\mu_f}{\rho_f}, \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \\ \mu_{nf} &= \frac{\mu_f}{(1 - \phi)^{2.5}}, (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s, \\ \frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f - 2\phi(k_f - k_s))}{(k_s + 2k_f + \phi(k_f - k_s))} \end{aligned} \quad (5)$$

In which ν_f , μ_f , ρ_f and k_f are the kinematic viscosity, dynamic viscosity, density, and thermal conductivity of the base fluid respectively; ρ_s , k_s , $(\rho c_p)_s$ are the density, thermal conductivity and heat capacitance of the nanoparticle respectively; ϕ is the solid volume fraction of nanoparticles.

The associated boundary conditions to the flow problem can be written as

$$\begin{aligned} u &= u_w = bx, v = v_w, T = T_w = T_\infty + A\left(\frac{x}{\ell}\right)^2, C = C_w = C_\infty + \\ &\quad B\left(\frac{x}{\ell}\right)^2 \text{ at } y = 0, \\ u &\rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty. \end{aligned} \quad (6)$$

In which A , B and b are constants, ℓ is the characteristic length T_w and C_w are the temperatures and concentration of the sheet, respectively T_∞ and C_∞ are the temperature and concentration of the nanofluid far away from the sheet, respectively, and v_w is the wall mass transfer velocity.

The continuity equation is satisfied by introducing a stream function $\psi(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (7)$$

Introducing the following similarity transformations

$$\eta = y \sqrt{\frac{b}{\nu_f}}, u = bx f'(\eta), v = -\sqrt{b \nu_f} f(\eta), g(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, h(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (8)$$

Making use of (8), the continuity equation (1) is automatically satisfied and (2), (3), (4) and (6) reduce to

$$f'' + \phi_1 (f f'' - (f')^2 - \frac{M}{\phi_2} f') - K_1 f' = 0 \quad (9)$$

$$g'' + \phi_3 \frac{k_f}{k_{nf}} Pr [f g' - 2f' g + \frac{Ec}{\phi_4} (f')^2] = 0, \quad (10)$$

$$h'' - Sc [2f' h - f h' + \gamma h] + Srg'' = 0 \quad (11)$$

with boundary condition

$$f(0) = f_w, \quad f'(0) = 1, \quad g(0) = 1, \quad h(0) = 1, \\ f(\eta) \rightarrow 0, \quad g(\eta) \rightarrow 0, \quad h(\eta) \rightarrow 0, \quad \text{as } \eta \rightarrow \infty \quad (12)$$

where $f(\eta)$, $g(\eta)$, and $h(\eta)$ are the dimensionless velocity, temperature and nanoparticle concentration, respectively, primes denote differentiation with respect to the similarity variable η , and $M = \frac{\sigma B_0^2}{b \rho_{nf}}$, (Magnetic parameter), $K_1 = \frac{\nu_f}{bK}$, (Porous medium parameter), $Pr = \frac{\nu_f}{\alpha_f}$, (Prandtl number), $Ec = \frac{U_w^2}{(c_p)_f(T_w - T_\infty)}$, (Eckert number), $Sc = \frac{\nu_f}{D}$, (Schmidt number), $\gamma = \frac{K_0}{b}$, (Scaled chemical reaction parameter), and $Sr = \frac{D_1(T_w - T_\infty)}{D(C_w - C_\infty)}$, (Soret number)

The permeability or suction/injection parameter $f_w = -\frac{v_w}{\sqrt{b\nu_f}}$ which shows the strength of the mass transfer at the sheet is negative for mass injection, positive in the presence of suction along the surface and has a zero value for an impermeable surface. The physical quantities of interest in this problem are the local skin friction coefficient C_f , the Nusselt number Nu_x , which represents the rate of heat transfer at the surface of the plate and the local Sherwood number Sh_x , which represents the rate of mass transfer at the surface of the plate, which are defined as

$$C_f = \frac{2\tau_w}{\rho_f u_w^2}, \quad Nu_x = \frac{x q_w}{k_f(T_w - T_\infty)}, \quad Sh_x = \frac{x J_w}{D(C_w - C_\infty)} \quad (13)$$

where τ_w is the skin friction, q_w is the heat flux and J_w is the mass flux through the plate, which are given by

$$\tau_w = -\mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad J_w = -D \left(\frac{\partial C}{\partial y} \right)_{y=0} \quad (14)$$

Making use of (8) and (5) in (13), the dimensionless skin friction coefficient, wall heat and mass transfer rates are obtained as

$$C_f(1 - \phi)^{2.5} \sqrt{Re_x} = -2f''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} \frac{k_f}{k_{nf}} = -g'(0), \quad \frac{Sh_x}{\sqrt{Re_x}} = -h'(0) \quad (15)$$

where $Re_x = \frac{u_w x}{\nu_f}$ is the local Reynolds number.

III. NUMERICAL SOLUTION

The system of non linear ordinary differential equations (9)-(11) together with the boundary condition (12) were solved numerically using a very efficient finite difference scheme known as **Keller box method**. The scheme employed is the box method developed by Keller [42]. This method has been shown to be particularly accurate for parabolic problems. It is much faster, easier to program and it is chosen because it seems to be the most flexible of the common methods, being easily adaptable to solving equations of any order. The Keller-box method is essentially an implicit finite difference scheme, which has been found to be very suitable in dealing with nonlinear problems. Details of the method may be found in

many recent publications, and here we have used the procedure outlined in [47]. One of the basic ideas of the box method is to write the governing system of equations in the form of a first order system. First derivative of u and other quantities with respect to η must therefore be introduced as new unknown functions. With the resulting first order equations, the "centered-difference" derivatives and averages at the midpoints of net rectangles and net segments are used, as they are required to get accurate finite difference equations. The resulting finite difference equations are implicit and nonlinear. Newton's method is first introduced to linearize the nonlinear system of equations before a block-tri-diagonal factorization scheme is employed on the coefficient matrix of the finite difference equations. The solution of the linearized system of difference equations can be obtained in a very efficient manner by using the block-elimination method [47]. In this study a uniform grid of size $\Delta\eta = 0.01$ is chosen to satisfy the convergence criteria of 10^{-6} which gives about four decimal places accuracy for most of the prescribed quantities.

IV. RESULTS AND DISCUSSION

We have studied heat and mass transfer in nanofluids flow due to a permeable stretching sheet in porous medium with magnetic field, viscous dissipation, chemical reaction and Soret effects. We considered two different types of nanoparticles, namely, copper and silver, with water as the base fluid (i.e. with a constant Prandtl number $Pr = 6.2$). The transformed non linear equations (9)-(11) subjected to the boundary condition (12) was solved numerically using Keller box method, which is described in [47]. The velocity, temperature, and concentration profiles were obtained and utilized to compute the skin-friction coefficient, the local Nusselt number, and local Sherwood number in (17). The numerical results for different values of the governing parameters viz., nanoparticle concentration ϕ , magnetic parameter M , porous medium parameter K_1 , viscous dissipation parameter (Eckert number) Ec , Schmidt number Sc , Soret number Sr , scaled chemical reaction parameter γ and suction/injection parameter f_w are presented in graphs. In the absence of magnetic field, porous media, chemical reaction and with impermeable sheet, to validate the accuracy of our results a comparison has been made with previously reported work by [48] and [3]. The comparisons are found to be in an excellent agreement (see Tables II and III).

The skin friction coefficients for different values of the magnetic parameter M and nanoparticle volume fraction ϕ are given in Table II. Increasing the value of M results in a considerable increase in the skin friction coefficient; this is due to the opposition to the flow caused by the induced Lorentz force. The results show a good agreement with [48].

The heat transfer coefficients are shown in Table III for different Prandtl number Pr . It is clear that the heat transfer coefficient increases with Prandtl numbers. The present results are in good agreement with the earlier findings by [3].

TABLE I

THERMOPHYSICAL PROPERTIES OF WATER, COPPER AND SILVER [43]

Physical Properties	Fluid Phase (water)	Cu	Ag
ρ (kg/m ³)	997.1	8933	10500
c_p (J/kgK)	4179	385	235
k (W/mK)	0.613	401	429

TABLE II

COMPARISON OF THE SKIN FRICTION COEFFICIENT $-f''(0)$ FOR DIFFERENT VALUES OF MAGNETIC PARAMETER (M) AND NANOPARTICLE VOLUME FRACTION (ϕ) WHEN PRANDTL NUMBER = 6.2 WITH [48]

M	ϕ	Cu-water		Ag-water	
		[48]	Present	[48]	Present
0	0.05	1.10892	1.1089	1.13966	1.1397
	0.1	1.17475	1.1747	1.22507	1.2251
	0.15	1.20886	1.2089	1.27215	1.2722
	0.2	1.21804	1.2180	1.28979	1.2898
0.5	0.05	1.29210	1.2921	1.31858	1.3186
	0.1	1.32825	1.3282	1.37296	1.3730
	0.15	1.33955	1.3396	1.39694	1.3969
	0.2	1.33036	1.3304	1.39634	1.3963
1	0.05	1.45236	1.4524	1.47597	1.4760
	0.1	1.46576	1.4658	1.50640	1.5064
	0.15	1.45858	1.4586	1.51145	1.5115
	0.2	1.43390	1.4339	1.49532	1.4953
2	0.05	1.72887	1.7289	1.74875	1.7487
	0.1	1.70789	1.7079	1.74289	1.7429
	0.15	1.67140	1.6714	1.71773	1.7177
	0.2	1.62126	1.6213	1.67583	1.6758

TABLE III

COMPARISON THE WALL HEAT TRANSFER RATE $-G'(0)$ FOR VARIOUS VALUES OF PRANDTL NUMBER PR, WHEN $\phi = M = K_1 = Ec = f_w = 0$

Pr	0.72	1	3	10	100
[40]	1.0885	1.3333	2.5097	4.7969	15.7120
Present	1.0886	1.3333	2.5097	4.7970	15.7198

Fig. 1 shows the influence of magnetic field parameter M on the velocity distribution $f'(\eta)$ for cu-water and Ag-water nanofluids. The presence of transverse magnetic field sets Lorentz force effects, which results in the retarding effect on the velocity field. As the values of magnetic parameter M increases, the retarding force increases and consequently the velocity decreases. It is also noted that the boundary layer thickness reduces as magnetic parameter M increases, and the Cu-water has higher value of velocity distribution than Ag-water nanofluid.

Fig. 2 shows, the effects of nanoparticle volume fraction parameter ϕ on the nanofluid temperature in case of Cu-water and Ag-water nanofluids. It is observed that, as the nanoparticles volume fraction increases, the temperature of the nanofluid increases. It is also observed that, the temperature distribution is higher in Ag-water nanofluid than in Cu-water nanofluid. In addition to this, when the volume fraction of the nanoparticle increases, the thermal conductivity of the fluid increases as a result the thermal boundary layer increases.

The effect of the viscous dissipation parameter Ec on the temperature profile in the case of Cu-water nanofluid is shown in Fig. 3. It is clear that the temperature distribution increases

with an increase in the viscous dissipation parameter Ec. The combined effect of the magnetic field and the viscous dissipation parameter (see Figs. 3 and 9) is to generate more heat in the boundary layer region and hence to reduce the wall heat transfer rate.

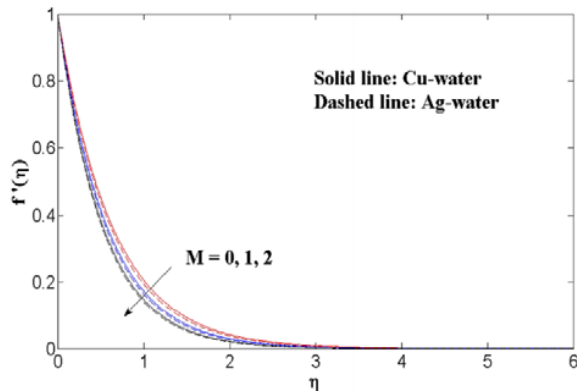


Fig. 1 Effects of M on the velocity distribution $f'(\eta)$, when $\phi = Sr = 0.2, Pr = 6.2, K_1 = Ec = Sc = 1, f_w = 0$ In case of Cu-water and Ag-water nanofluids

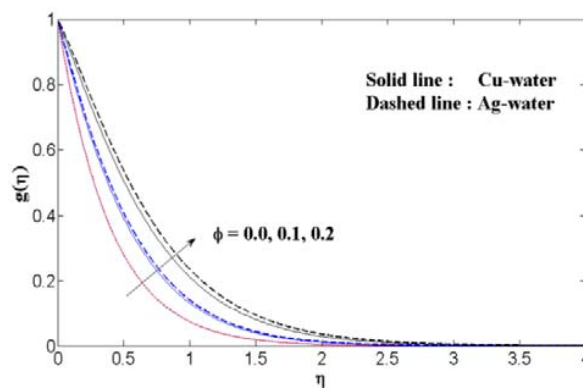


Fig. 2 Effects of ϕ on the temperature profile $G(\eta)$, when $M = 0, K_1 = 1, Pr = 6.2, Ec = 1, Sc = 0, \gamma = 0.08, Sr = 0.2, f_w = 0$ in case of Cu-water and Ag-water nanofluids

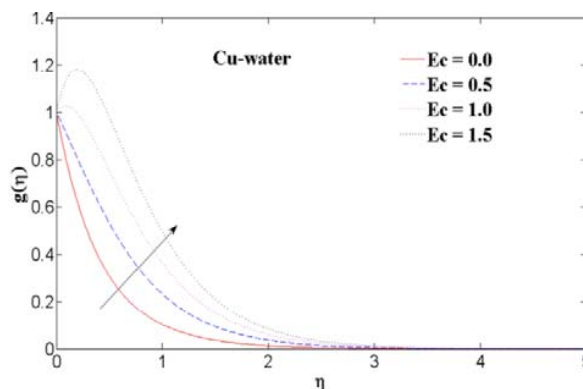


Fig. 3 Effects of Ec on the temperature profile $g(\eta)$, when $\phi = 0.2, M = 0, K_1 = 1, Pr = 6.2, \gamma = 0.08, Sr = 0.2, f_w = 0$ in case of Cu-water and Ag-water nanofluids

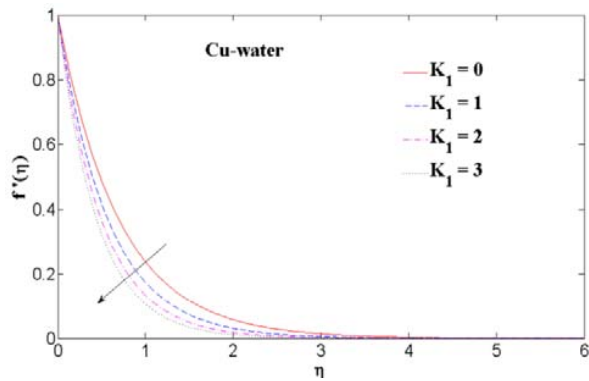


Fig. 4 Effects of K_1 on the Velocity profile $f'(\eta)$, when $\phi = 0.2$, $M = 1$, $Pr = 6.2$, $Ec = 1$, $Sc = 1$, $\gamma = 0.08$, $Sr = 0.2$, $f_w = 0$ in case of Cu-water and Ag-water nanofluids

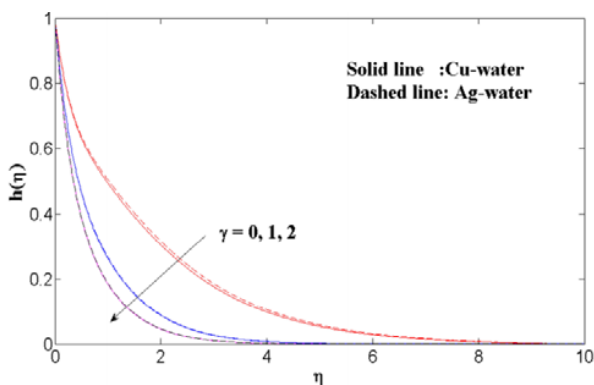


Fig. 5 Effects of γ on the concentration profile $h(\eta)$, when $\phi = 0.2$, $M = 1$, $K_1 = 1$, $Pr = 6.2$, $Ec = 1$, $Sc = 1$, $Sr = 0.2$, $f_w = 0$ in case of Cu-water and Ag-water nanofluids

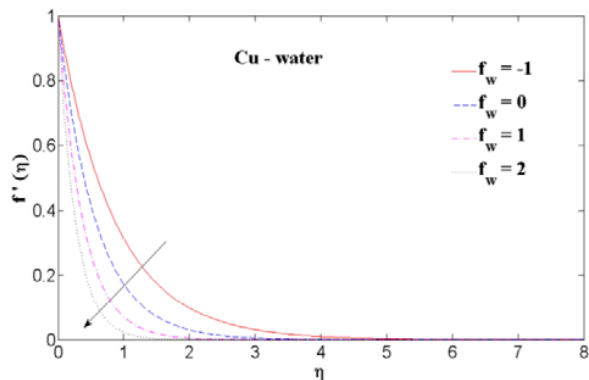


Fig. 6 Effects of f_w on the velocity profile $f'(\eta)$, when $\phi = 0.2$, $M = 1$, $K_1 = 1$, $Pr = 6.2$, $Ec = 1$, $\gamma = 0.08$, $Sc = 1$, $Sr = 0.2$ in case of Cu-water nanofluid

One can see from Fig. 5 that, temperature distribution, $g(\eta)$ decreases with an increase of the chemical reaction parameter γ in the case of Cu-water nanofluid.

Figs. 6 and 7, show the effects of suction/injection parameter f_w on the velocity and temperature distributions in the Cu-water nanofluid. It is illustrated that the velocity and

temperature distributions decrease with an increase in the suction/injection parameter f_w .

Fig. 4 shows, the effect of porosity parameter K_1 on the velocity distribution of the Cu-water nanofluid. It is observed that the velocity distribution decreases with increasing the porosity parameter. This is because the presence of a porous medium increases the resistance to the flow causing a decrease in the fluid velocity.

The effects of various parameters on the wall skin friction, heat and mass transfer rates are shown in Figs. 8-10. Fig. 8 illustrates the effects of the porosity parameter K_1 and nanoparticle volume fraction ϕ on the wall skin friction. It is observed that the magnitude of the skin friction coefficient $-f''(0)$ increases with increasing in the porosity parameter and nanoparticle volume fraction for both the Cu-water and Ag-water nanofluids. The wall skin friction is higher in the Ag-water nanofluid compared to the Cu-water nanofluid.

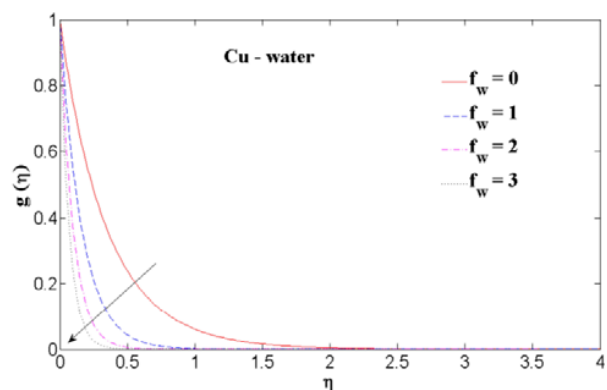


Fig. 7 Effects of f_w on the temperature profile $g(\eta)$, when $\phi = 0.2$, $M = 1$, $K_1 = 1$, $Pr = 6.2$, $Ec = 1$, $\gamma = 0.08$, $Sc = 1$, $Sr = 0.2$ in case of Cu-water nanofluid

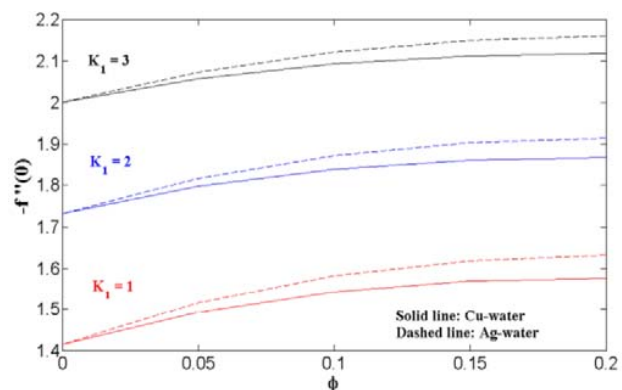


Fig. 8 Effects of ϕ on the skin friction coefficient $-f''(0)$ for different values of K_1 , when $M = f_w = 0$, $Pr = 6.2$, $Ec = Sc = 1$, $\gamma = 0.08$, $Sr = 0.2$ in case of Cu-water and Ag-water nanofluids

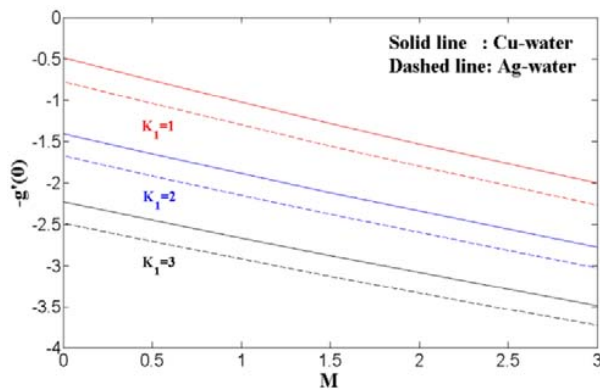


Fig. 9 Effects of M on the rate of heat transfer $-g'(0)$ for different values of K_1 , when $\phi = 0.2$, $Pr = 6.2$, $Ec = 1$, $Sc = 1$, $\gamma = 0.08$, $Sr = 0.2$, $f_w = 0$ in case of Cu-water and Ag-water nanofluids

Thus, the Ag-water nanofluid gives a higher drag force in opposition to the flow as compared to the Cu-water nanofluid. The effects of magnetic field parameter M and porous media parameter K_1 on the wall heat transfer rate $-g'(0)$ are shown in Fig. 9 in the case of Cu-water and Ag-water nanofluids. The influence of magnetic field is to reduce the wall heat transfer rates. The porous media effect reduces the wall heat transfer rate.

Moreover, the rate of heat transfer at the wall is less in case of the Ag-water nanofluid compared to the Cu-water nanofluid.

The combined effects of the Soret number and Schmidt number on the wall mass transfer rate are shown in Fig. 10 in case of Cu-water and Ag-water nanofluids. It is observed that, the rate of mass transfer in the wall $-h'(0)$ is an increasing function of Soret number and Schmidt number.

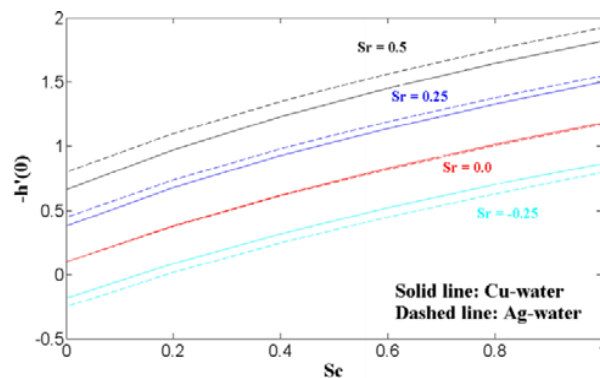


Fig. 10 Effects of Sc on the rate of mass transfer $-h'(0)$ for different values of Sr , when $\phi = 0.2$, $M = 0$, $K_1 = 1$, $Pr = 6.2$, $Ec = 1$, $\gamma = 0.08$, $f_w = 0$ in case of Cu-water and Ag-water nanofluids

V. CONCLUSIONS

This paper presents the problem MHD flow, heat and mass transfer of nanofluid through a porous medium due a stretching sheet in the presence of viscous dissipation, chemical reaction and Soret effects. The governing nonlinear partial differential equations were transformed into ordinary

differential equations using the similarity approach and solved numerically using the Keller box method. Two types of nanofluids were considered, Cu-water and Ag-water, and our results revealed, among others, the following.

1. Cu-water shows a thicker velocity boundary than Ag-water nanofluid. The velocity boundary layer thickness decreases with increasing the nanoparticle volume fraction and porous medium parameter.
2. Ag-water nanofluid shows a thicker thermal boundary layer than Cu-water nanofluid. The thermal boundary layer thickness increases with increasing the values of ϕ and Ec .
3. Ag-water nanofluid shows a slightly thicker concentration boundary layer than Cu-water nanofluid. The concentration boundary layer thickness increases with increasing the values of ϕ while it decreases with increasing the values of the Sc and γ .
4. The velocity distribution and temperature profiles of the Cu-water nanofluid decrease with an increase in the suction/injection parameter f_w .
5. The skin friction increases with increasing the nanoparticle volume fraction and porous medium parameter; the Ag-water nanofluid shows a higher skin friction than Cu-water nanofluid.
6. The heat transfer rate at the plate surface decreases with increasing the nanoparticle volume fraction, magnetic field parameter, porous medium parameter and Eckert number. The Cu-water nanofluid has a higher rate of heat transfer than the Ag-water nanofluid.
7. The mass transfer rate at the plate surface increases with increasing the nanoparticle volume fraction, Schmidt number, Chemical reaction parameter and Soret number. The Ag-water nanofluid has higher rate of mass transfer than the Cu-water nanofluid.

A comparison with previously reported data is made and an excellent agreement is noted.

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