Hall Coefficient in the Presence of Strong Electromagnetic Waves Caused by Confined Electrons and Phonons in a Rectangular Quantum Wire

Nguyen Quang Bau, Nguyen Thu Huong, Dang Thi Thanh Thuy

***Abstract*—**The analytic expression for the Hall Coefficient (HC) caused by the confined electrons in the presence of a strong electromagnetic wave (EMW) including the effect of phonon confinement in rectangular quantum wires (RQWs) is calculated by using the quantum kinetic equation for electrons in the case of electron - optical phonon scattering. It is because the expression of the HC for the confined phonon case contains indexes m, m' which are specific to the phonon confinement. The expression in a RQW is different from that for the case of unconfined phonons in a RQW or in 2D. The results are numerically calculated and discussed for a GaAs/GaAsAl RQW. The numerical results show that HC in a RQW can have both negative and positive values. This is different from the case of the absence of EMW and the case presence of EMW including the effect of phonon unconfinement in a RQW. These results are also compared with those in the case of unconfined phonons in a RQW and confined phonons in a quantum well. The conductivity in the case of confined phonon has more resonance peaks compared with that in case of unconfined phonons in a RQW. This new property is the same in quantum well. All results are compared with the case of unconfined phonons to see differences.

***Keywords*—**Hall coefficient, rectangular quantum wires, electron-optical phonon interaction, quantum kinetic equation, confined phonons.

I. Introduction

THE size-reduced effect dramatically altered the physical properties of low-dimensional semiconductor materials. Objects having a direct impact on the effect are semiconductor electronics, of which movements are restricted according to the size-reduced dimensions. In comparison with the bulk semiconductor, the physical properties of low-dimensional systems are preeminent due to this motion restriction. Apart from electronic confinement, in low-dimensional systems, phonons can be detained by this effect as well. Recently, there have been numerous studies on the physical properties of semiconductor taking the phonon confinement into account [1]-[7]. Phonon confinement in low dimensional systems has significantly changed the physical effects compared with those in the case of bulk phonons such as the electronic - phonon scattering in doped superlattices [8], in the quantum wire [9], and so on. The HC has been done by taking account of confined elements for some research in the two-dimensional system. The impact of confined phonon on the Hall effect in these low dimensional systems is still opened to research. To obtain a better assessment of the HC when phonons are confined in quantum wires, in this work, we study the HC in a RQW considering the phonon confinement. A RQW with a cross-section $L_x \times L_y$ in the plane (x, y), the length L_z are selected as a model. Then, we estimate numerical values for the specific GaAs/GaAsAl RQW to show the dependence of the HC and magnetoresistance (MR) by the confined optical phonons in a RQW on the temperature T of the system and the frequency. The effect of phonon confinement creates new property of the conductivity tensor which has more resonance peaks in case confined phonons in a RWQ compared with that in a quantum well. Both the conductivity tensors in RQW of phonon confinement and in quantum wells have more resonance peaks compared to those in the confined phonon case.

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II. Hamiltonian of the Confined Electron – Phonon System in a RQW with Infinitely High Potential in the Presence of a Laser Radiation

A confined phonon model is similar to an electronic one. Each state of the phonon is also described by two quantum numbers m, m' corresponding to the confinement in two dimensions of that phonon *Ox, Oy*. The phonon wave vector is defined as $\vec{q} = (q_x, q_y, q_z)$ in which $q_x = \frac{m\pi}{L_x}, q_y = \frac{m'\pi}{L_y}$. The quantum wire is set in the laser field $\vec{E}(t) = \vec{E}_0 \sin(\Omega t)$ and vector potential $\vec{A}(t) = \frac{c}{\Omega}\vec{E}_0 \cos(\Omega t)$ driven to Oz, dc electric field $\vec{E} = (0,0,E)$ and magnetic field $\vec{B} = (B,0,0)$. Under these conditions, the wave function and energy spectrum can be written as:

$$\psi_{\gamma,\vec{k}}(x,y,z) = \sqrt{\frac{1}{L_z}} e^{ikz} \sqrt{\frac{2}{L_x}} \sin\left(\frac{n\pi x}{L_x}\right) \sqrt{\frac{2}{L_y}} \sin\left(\frac{l\pi y}{L_y}\right)$$

$$\text{When} \begin{cases} 0 \le y \le L_y \\ 0 \le x \le L_x \end{cases} \qquad (1)$$

and $\psi_{\gamma,\vec{k}}(x,y,z) = 0$ if else.

$$\varepsilon_\gamma(k) = \frac{\hbar^2 k^2}{2m^*} + \frac{\pi^2\hbar^2}{2m^*}\left(\frac{n^2}{L_x^2} + \frac{l^2}{L_y^2}\right) + \omega_c(N + \frac{1}{2}) - \frac{1}{2m^*}\left(\frac{eE}{\omega_c}\right)^2 \quad (2)$$

The Hamiltonian of confined optical electron-phonon system is expressed as a particle number operator.

$$H(t) = \sum_{n,\ell,\vec{k}} \varepsilon_{n,\ell}(\vec{k} - \frac{e}{c}\vec{A}(t)) a^+_{n,\ell,\vec{k}} a_{n,\ell,\vec{k}} + \sum_{m,m',\vec{q}} \omega_{\vec{q}} b^+_{m,m',\vec{q}} b_{m,m',\vec{q}} + \sum_{\vec{q}} \varphi(\vec{q}) a^+_{n,l,\vec{q}+\vec{k}} a_{n,l,\vec{k}} + \sum_{\substack{n,l,\vec{k},n',l' \\ \vec{q},m,m'}} C^{m,m'}_{\vec{q}} I^{m,m'}_{n,l,n',l'} a^+_{n,l,\vec{k}+\vec{q}} a_{n',l',\vec{k}} (b^+_{m,m',-\vec{q}} + b_{m,m',\vec{q}}) \quad (3)$$

where $a^+_{n,l,\vec{k}}$ and $a_{n,l,\vec{k}}$ ($b^+_{m,m',\vec{q}}$ and $b_{m,m',\vec{q}}$) are the creation and annihilation operators of electron (confined optical phonon), respectively, $\vec{k}$ is the electron wave momentum, $\vec{q}$ is the phonon wave vector, $\omega_{\vec{q}}$ is optical phonon frequency, γ and γ' are the quantum numbers (n,ℓ) and (n',ℓ') of electron. N, N' are the Landau levels $(N = 0,1,2,...)$. The quantities in the equation (3) vary from those in RQW [14] since it contains specific indexes denoted by m, m' for the phonon confinement.

Electron – confined optical phonon interaction constant is:

$$|C^{m,m'}_{\vec{q}}|^2 = \frac{\pi e^2 \omega_o}{2V\varepsilon_0}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{\vec{q}_z^{\,2} + (\frac{m\pi}{L_x})^2 + (\frac{m'\pi}{L_y})^2} \quad (4)$$

in which, V is the normalized volume, and ε_0 is the vacuum permittivity.

The electron form factor $I_{\gamma,\gamma'}(\vec{q})$ can be written as [9]:

$$I^{m,m'}_{n,\ell}(q_z) = (2\pi)^2 \sum_{m,m'=1,3,5,...} 16 P^2_{m,m'} / \vec{q} \quad (5)$$

where

$$P_{m,m'} = \int_0^{L_x} dx \frac{2}{L_x} \int_0^{L_y} dy \frac{2}{L_y} \cos(\frac{n\pi x}{L_x}) \cos(\frac{n'\pi x}{L_x}) \times \cos(\frac{\ell\pi x}{L_x}) \cos(\frac{\ell'\pi y}{L_y}) \cos(\frac{m\pi y}{L_y}) \cos(\frac{m'\pi y}{L_y}) \quad (6)$$

$\vec{A}(t)$ is the vector potential of the electromagnetic field which is defined as:

$$-\frac{d\vec{A}(t)}{cdt} = \vec{E}_0 \sin(\Omega t) \quad (7)$$

$V, \rho, \xi, \upsilon_S, \chi_O, \chi_\infty$ are the volume, the density, the deformation potential, the sound velocity, the static dielectric constant, and the high frequency dielectric constant, respectively. $\varphi(\vec{q})$ is the potential undirected:

$$\varphi(\vec{q}) = (2\pi i)^3 (e\vec{E} + \omega_c[\vec{q}, \vec{h}]) \frac{\partial}{\partial \vec{q}} \delta(\vec{q}) \quad (8)$$

III. The Quantum Kinetic Equation of Electrons in RQWs when Phonons Are Confined

The quantum kinetic equation of electrons in a RQW considering the phonon confinement, which is based on the general quantum kinetic equation of the particle number operators, is formulated as:

$$i\frac{\partial n_{n,\ell,\vec{k}}}{\partial t}(t) = i\frac{\partial}{\partial t}\langle a^+_{n,\ell,\vec{k}} a_{n,\ell,\vec{k}} \rangle_t = \langle [a^+_{n,\ell,\vec{k}} a_{n,\ell,\vec{k}}, H] \rangle_t \quad (9)$$

Using the Haniltonian (3), transformations of operators and setting:

$$F_{\gamma_1,\vec{k}_1,\gamma_2,\vec{k}_2,m,m',\vec{q}}(t) = \langle a^+_{\gamma_1,\vec{k}_1} a_{\gamma_2,\vec{k}_2} b_{m,m',\vec{q}} \rangle_t$$

One obtains:

$$i\frac{\partial n_{\gamma,\vec{k}}}{\partial t} = \sum_{m,m',\vec{q}} \sum_{\gamma_1} C^{m,m'}_{\vec{q}} I^{m,m'}_{\gamma,\gamma_1}(\vec{q}) (F_{\gamma_1,\vec{k}_1,\gamma_2,\vec{k}_2,m,m',\vec{q}}(t) + F^*_{\gamma_1,\vec{k}-\vec{q},\gamma,\vec{k},m,m',-\vec{q}}(t) - F_{\gamma_1,\vec{k}-\vec{q},\gamma,\vec{k},m,m',\vec{q}}(t) - F^*_{\gamma,\vec{k},\gamma_1,\vec{k}+\vec{q},m,m',-\vec{q}}(t)) \quad (10)$$

We formulate an expression for operator $F_{\gamma_1,\vec{k}_1,\gamma_2,\vec{k}_2,m,m',\vec{q}}(t)$ to obtain the formula for the quantum kinetic equation

$$i\frac{\partial}{\partial t} F_{\gamma_1,\vec{k}_1,\gamma_2,\vec{k}_2,m,m',\vec{q}}(t) = \langle [a^+_{\gamma_1,\vec{k}_1} a_{\gamma_2,\vec{k}_2} b_{m,m',\vec{q}}, H] \rangle_t \quad (11)$$

Doing some necessary calculations, one acquires:

$$\frac{\partial}{\partial t}F(t)=i[\varepsilon_{\gamma_1}(\vec{k}_1-\frac{e}{c}\vec{A}(t))-\varepsilon_{\gamma_2}(\vec{k}_2-\frac{e}{c}\vec{A}(t))-\omega_o]F(t)+ \\ +i\sum_{m_1,m'_1,\vec{q}_1}\sum_{\gamma_3}C_{\vec{q}_1}^{m_1,m_1'}[\langle I_{\gamma_3,\gamma_1}^{m_1,m_1'}a^+_{\gamma_3,\vec{k}_1+\vec{q}_1}a_{\gamma_2,\vec{k}_2}(b_{m_1,m_1',\vec{q}_1}+b^+_{m_1,m_1',-\vec{q}_1})b_{m,m',\vec{q}}\rangle_t \\ -\langle I_{\gamma_3,\gamma_2}^{m_1,m_1'}a^+_{\gamma_1,\vec{k}_1}a_{\gamma_3,\vec{k}_2-\vec{q}_1}b_{m,m',\vec{q}}(b_{m_1,m_1',\vec{q}_1}+b^+_{m_1,m_1',-\vec{q}_1})\rangle_t] \tag{12}$$

in which $F(t)=F_{\gamma_1,\vec{k}_1,\gamma_2,\vec{k}_2,m,m',\vec{q}}(t)$ (11) is the non-linear differential equation. Solve this equation using variational method and boundary conditions under adiabatic hypothesis $F(t)|_{t=-\infty}=0$ to give us:

$$F(t)=i\int_{-\infty}^{t}dt_2\sum_{m_1,m_1',\vec{q}_1}\sum_{\gamma_3}C_{\vec{q}_1}^{m_1,m_1'}[\langle I_{\gamma_1,\gamma_3}^{m_1,m_1'}a^+_{\gamma_3,\vec{k}_1+\vec{q}_1}a_{\gamma_2,\vec{k}_2}(b_{m_1,m_1',\vec{q}_1}+b^+_{m_1,m_1',-\vec{q}_1})b_{m,m',\vec{q}}\rangle_{t_2} \\ -\langle I_{\gamma_3,\gamma_2}^{m_1,m_1'}a^+_{\gamma_1,\vec{k}_1}a_{\gamma_3,\vec{k}_2-\vec{q}_1}b_{m,m',\vec{q}}(b_{m_1,m_1',\vec{q}_1}+b^+_{m_1,m_1',-\vec{q}_1})\rangle_{t_2}]\times \\ \times\exp\{i[\varepsilon_{\gamma_1}(\vec{k}_1)-\varepsilon_{\gamma_2}(\vec{k}_2)-\omega_o](t-t_2)-\frac{ie}{m^*c}\int_{t_2}^{t}(\vec{k}_1-\vec{k}_2)\vec{A}(t_1)dt_1\} \tag{13}$$

We substitute (12) into (9) with the corresponding terms, using the (7) and $\exp(\pm iz\sin x)=\sum_{n=-\infty}^{\infty}J_n(z)\exp(\pm inx)$ ($J_n(z)$ is the Bessel function of real arguments), associated with changes based on the symmetry of the statistical average quantity to obtain:

$$\frac{\sum_{\gamma,\vec{k}}\frac{e}{m^*}\vec{k}n_{\gamma,\vec{k}}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})}{\tau(\varepsilon_{\gamma,\vec{k}})}+\omega_c[\text{h},\sum e\frac{\vec{k}}{m^*}n_{\gamma,\vec{k}}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})]= \\ =-\frac{e}{m^*}\sum_{\gamma,\vec{k}}\vec{k}(F\frac{\partial n_{\gamma,\vec{k}}}{\partial\vec{k}})\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})+\frac{2\pi e}{m^*}\sum_{m,m',\vec{q}}\sum_{\gamma_1}\left|C_{\vec{q}}^{m,m'}\right|^2\left|I_{\gamma,\gamma_1}^{m,m'}\right|^2N_{m,m',\vec{q},\vec{k}}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}}) \\ \times\left\{\left[\bar{n}_{\gamma_1,\vec{q}+\vec{k}}-\bar{n}_{\gamma,\vec{k}}\right]\left[\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q})-\varepsilon_{\gamma}(\vec{k})-\omega_o)+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q})-\varepsilon_{\gamma}(\vec{k})-\omega_o+\Omega)+\right.\right. \\ \left.+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q})-\varepsilon_{\gamma}(\vec{k})-\omega_o-\Omega)\right]+\left[\bar{n}_{\gamma_1,\vec{k}-\vec{q}}-\bar{n}_{\gamma,\vec{k}}\right]\left[\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma_1}(\vec{k}-\vec{q})-\varepsilon_{\gamma}(\vec{k})+\omega_o-\Omega)+\right. \\ \left.\left.+\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q})-\varepsilon_{\gamma}(\vec{k})-\omega_o)+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q})-\varepsilon_{\gamma}(\vec{k})-\omega_o+\Omega)\right]\right\} \tag{14}$$

in which δ is the infinitesimal parameter inserted to ensure the adiabatic hypothesis. (14) is the quantum kinetic equation for unbalanced distribution function of the confined electrons in RQW when the phonons are confined. From (14), we can see that the coefficient of the interaction between electron and phonon has changed by the phonon confinement in a RQW, and the HC also has changed. The expression of the HC includes indexes m, m' which are particular to the phonon confinement.

Set: $$R(\varepsilon)=\sum_{\gamma,\vec{k}}\frac{e}{m^*}\vec{k}n_{\gamma,\vec{k}}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})$$

$$Q(\varepsilon)=-\frac{e}{m^*}\sum_{\gamma,\vec{k}}\vec{k}(F\frac{\partial n_{\gamma,\vec{k}}}{\partial\vec{k}})\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})$$

$$
\begin{aligned}
S(\varepsilon) &= \frac{2\pi e}{m^*} \sum_{m,m',\vec{q}} \sum_{\gamma_1} \left| C_{\vec{q}}^{m,m'} \right|^2 \left| I_{\gamma,\gamma_1}^{m,m'} \right|^2 N_{m,m',\vec{q},\vec{k}} \delta(\varepsilon - \varepsilon_{\gamma,\vec{k}}) \\
&\times \Bigg\{ \left[\bar{n}_{\gamma_1,\vec{q}+\vec{k}} - \bar{n}_{\gamma,\vec{k}} \right] \Bigg[\left(1 - \frac{\lambda^2}{2\Omega^2} \right) \delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q}) - \varepsilon_\gamma(\vec{k}) - \omega_o) + \frac{\lambda^2}{4\Omega^2} \delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q}) - \varepsilon_\gamma(\vec{k}) - \omega_o + \Omega) + \\
&+ \frac{\lambda^2}{4\Omega^2} \delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q}) - \varepsilon_\gamma(\vec{k}) - \omega_o - \Omega) \Bigg] + \left[\bar{n}_{\gamma_1,\vec{k}-\vec{q}} - \bar{n}_{\gamma,\vec{k}} \right] \Bigg[\frac{\lambda^2}{4\Omega^2} \delta(\varepsilon_{\gamma_1}(\vec{k}-\vec{q}) - \varepsilon_\gamma(\vec{k}) + \omega_o - \Omega) + \\
&+ \left(1 - \frac{\lambda^2}{2\Omega^2} \right) \delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q}) - \varepsilon_\gamma(\vec{k}) - \omega_o) + \frac{\lambda^2}{4\Omega^2} \delta(\varepsilon_{\gamma_1}(\vec{k}+\vec{q}) - \varepsilon_\gamma(\vec{k}) - \omega_o + \Omega) \Bigg] \Bigg\}
\end{aligned}
$$

$$
\frac{\vec{R}_{(\varepsilon)}}{\tau_{(\varepsilon)}} + \omega_c \left[\vec{h} \wedge \vec{R} \right] = \overrightarrow{Q_{(\varepsilon)}} + \overrightarrow{S_{(\varepsilon)}}
$$

After making several calculations, the expression for the conductivity tensor and the HC are obtained:

$$
\begin{aligned}
\sigma_{ij} &= \frac{ea\tau}{1+\omega_c^2\tau^2} \left(\delta_{ij} - \omega_c \tau \varepsilon_{ijk} h_k + \omega_c^2 \tau h_i h_j \right) + \frac{b}{m} \frac{\tau^2}{\left(1+\omega_c^2\tau^2\right)^2} \times \\
&\times \left[\left(1 - \omega_c^2 \tau^2\right) \delta_{ij} + \left(3\omega_c^2\tau^2 + \omega_c^4\tau^4\right) h_i h_j - \omega_c \tau \varepsilon_{ijk} h_k \right]
\end{aligned} \tag{15}
$$

$$
R_H = \frac{1}{B} \frac{\dfrac{\omega_c \tau}{1+\omega_c^2\tau^2} \left[\dfrac{ea}{\omega_c} + \dfrac{b}{m} \dfrac{\tau^2}{1+\omega_c^2\tau^2} \right]}{\left\{ \dfrac{-\omega_c\tau}{1+\omega_c^2\tau^2} \left[\dfrac{ea}{\omega_c} + \dfrac{b}{m} \dfrac{\tau^2}{1+\omega_c^2\tau^2} \right] \right\}^2 + \left\{ \dfrac{\tau}{1+\omega_c^2\tau^2} \left[ea + \dfrac{b}{m} \dfrac{\tau}{1+\omega_c^2\tau^2} (1-\omega_c^2\tau^2) \right] \right\}^2} \tag{16}
$$

in which

$$
a = \frac{e\beta L_x}{4m\sqrt{\pi}} \left(\frac{2m}{\beta\hbar^2} \right)^{1/2} \exp\left\{ \beta \left[\varepsilon_F - \frac{1}{2m} \left(\frac{eE_1}{\omega_c} \right)^2 - \frac{\pi^2\hbar^2}{2m} \left(\frac{n^2}{L_x^2} + \frac{l^2}{L_y^2} \right) - \omega_c \left(N + \frac{1}{2} \right) \right] \right\}
$$

$$
\begin{aligned}
b &= \frac{2\pi e}{m^*} I_{\gamma,\gamma'}^{m,m'} \sum_{\gamma,\gamma'} (M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7 + M_8) \times \\
&\times \exp\left\{ \beta \left[\varepsilon_F - \frac{1}{2m} \left(\frac{eE_1}{\omega_c} \right)^2 - \frac{\pi^2\hbar^2}{2m} \left(\frac{n^2}{L_x^2} + \frac{l^2}{L_y^2} \right) - \omega_c \left(N + \frac{1}{2} \right) \right] \right\}
\end{aligned}
$$

$$
M_1 = \frac{\beta L_x k_B T e^2}{4\hbar\pi^3 V \varepsilon_o} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \left[1 - \phi(B\sqrt{\beta m} B_1) \right] \frac{\pi}{2B} e^{B^2 \beta B_1^2 m}
$$

$$
M_2 = \frac{B_1 \beta L_x k_B T e^4 E_o}{8\sqrt{2}\hbar\Omega^4 V \varepsilon_o} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \left[\frac{\sqrt{\pi}}{2\sqrt{\dfrac{\beta B_1^2}{2} m^*}} - \frac{\pi B}{2} e^{\frac{\beta B_1^2 m^* B^2}{2}} \right]
$$

$$
M_3 = \frac{B_3 \beta L_x k_B T e^4 E_o}{8\sqrt{2}\hbar\Omega^4 V \varepsilon_o} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \left[\frac{\sqrt{\pi}}{2\sqrt{\dfrac{\beta B_3^2}{2} m^*}} - \frac{\pi B}{2} e^{\frac{\beta B_3^2 m^* B^2}{2}} \right]
$$

$$
M_4 = \frac{B_4 \beta L_x k_B T e^4 E_o}{8\sqrt{2}\hbar\Omega^4 V \varepsilon_o} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \left[\frac{\sqrt{\pi}}{2\sqrt{\dfrac{\beta B_4^2}{2} m^*}} - \frac{\pi B}{2} e^{\frac{\beta B_4^2 m^* B^2}{2}} \right]
$$

$$
M_5 = \frac{\beta L_x k_B T e^2}{4\hbar\pi^3 V \varepsilon_o} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o} \right) \left[1 - \phi(B\sqrt{\beta m} B_5) \right] \frac{\pi}{2B} e^{B^2 \beta B_5^2 m}
$$

$$M_6 = \frac{B_5 \beta L_x k_B T e^4 E_o}{8\sqrt{2}\hbar\Omega^4 V \varepsilon_o}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\left[\frac{\sqrt{\pi}}{2\sqrt{\frac{\beta B_5^2}{2} m^*}} - \frac{\pi B}{2} e^{\frac{\beta B_5^2 m^* B^2}{2}}\right]$$

$$M_7 = \frac{B_7 \beta L_x k_B T e^4 E_o}{8\sqrt{2}\hbar\Omega^4 V \varepsilon_o}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\left[\frac{\sqrt{\pi}}{2\sqrt{\frac{\beta B_7^2}{2} m^*}} - \frac{\pi B}{2} e^{\frac{\beta B_7^2 m^* B^2}{2}}\right]$$

$$M_8 = \frac{B_8 \beta L_x k_B T e^4 E_o}{8\sqrt{2}\hbar\Omega^4 V \varepsilon_o}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\left[\frac{\sqrt{\pi}}{2\sqrt{\frac{\beta B_8^2}{2} m^*}} - \frac{\pi B}{2} e^{\frac{\beta B_8^2 m^* B^2}{2}}\right]$$

$$B_1 = \frac{q^2}{2m^*} + \frac{\pi}{2m^*}\left(\frac{n'^2 - n^2}{L_x^2} + \frac{l'^2 - l^2}{L_y^2} + (N' - N)\omega_c - \omega_o\right)$$

$$B^2 = (\frac{m\pi}{L_x})^2 + (\frac{m'\pi}{L_y})^2;\ B_3 = B_1 + \Omega;\ B_4 = B_1 - \Omega;\ B_5 = B_1 + 2\omega_o;\ B_7 = B_5 + \Omega;\ B_8 = B_5 - \Omega$$

where δ_{ij} is the Kronecker delta, ε_{ijk} the anti - symmetrical Levi–Civita tensor, and the letters h_k, h_l, h_j stand for the components x, y, z of the Cartesian coordinate system. The quantities in (16) vary from those in RQWs [14]. Therefore, the expression for the HC for the unconfined phonon case in a RQW is different from that for the case of confined phonons in a RQW or in 2D. Because the expression of the HC for the unconfined phonon case in RQW does not contain the indexes m, m' which are specific to phonon confinement. When the indexes m, m' go to zero, we obtain results as the case of unconfined phonon in a RWQ [14]. When the length L_x (L_y) approaches infinity, at this time the properties of the wire are similar to those of bulk semiconductor. It means that HC is no longer dependent on the length of quantum wires at all different temperatures, which is the characteristic of bulk semiconductor.

IV. Numerical Results and Discussions

To get a better assessment of the effect of the confined phonons on HC caused by the confined electrons in a RQW, we will survey and plot the dependence of the HC on the characteristic quantities of the GaAs/GaAsAl RQW for both cases – confined phonons and unconfined phonons – on the same graph. The parameters used for the computation are as follows [10]-[13]:

$$\chi_\infty = 10.9, \chi_0 = 12.9, \varepsilon_0 = \frac{10^{-9}}{36\pi}, m = 0.067m_0, \varepsilon_F = 50meV, \hbar\omega_0 = 36.25meV,$$
$$\Omega = 3\times10^{13}s^{-1}, n = 0,1; n' = 1, l = 0,1; l' = 1, \tau = 10^{-12}s, \rho = 5320kgm^{-3}, N' - N = 1$$

Figs. 1 and 2 show that the dependence of the HC on frequency EMW at different values of the magnetic field for both cases – confined phonons and unconfined phonons. The figures show that the HC depends strongly on frequency since there are appearing two resonance peaks. Differing from the case of unconfined phonons [14], the curve has only one resonance peak. This is due to the fact that the confined phonon has quantum wave number following the confined axis. Because the expression of the HC for the confined phonon case contains the indexes m, m'.

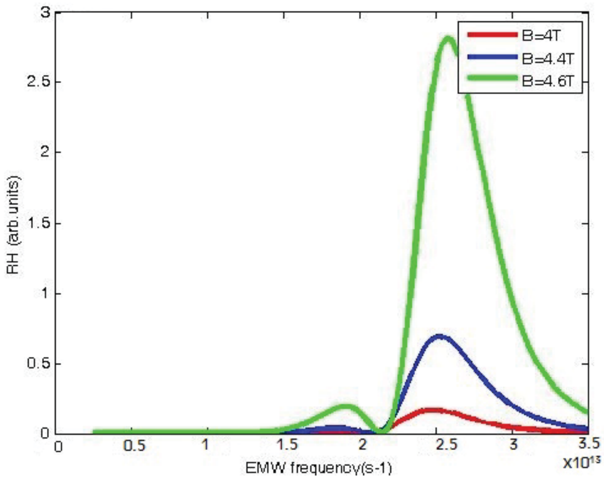

Fig. 1 The dependence of the HC on frequency EMW at different values of the magnetic field

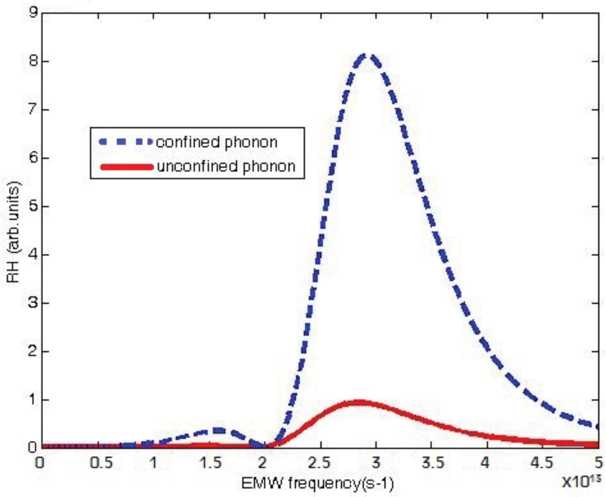

Fig. 2 The dependence of the HC on frequency EMW at different values of the magnetic field for both cases – confined phonons and unconfined phonons – on the same graph

Fig. 3 shows that the HC is a function of temperature in both cases of confined phonons and unconfined phonons. The graph also shows that the confined phonons increase the intensity of the HC in comparison with the case of unconfined phonons.

At the same temperature $T \approx 118K$, the HC in the case of confined phonons, which is approximately 7 (arb. units) with $m = m' = 2$ and 4 (arb. units) with $m = m' = 1$ is higher than that in the case of unconfined phonons, which is approximately 1.8 (arb. units). As the temperature increases, the HC decreases. However, when the phonons confinement as well as the indexes of the phonons confinement increase, the HC is higher than that in the case of unconfined phonons. The higher the indexes of the phonons confinement increase, the higher the HC increases.

Fig. 4 shows dependencies of the MR on the ratio Ω / ω_c at $B = 6T$ for different values of E_o of both cases – confined phonons and unconfined phonons. The dashed curve has one maximum and one minimum in the confined phonon case, but one minimum in the case of unconfined phonons [14]. In addition, the increase of quantum number m, m' leads the MR to increase. Consequently, this is a new behavior of the HC due to the effect of phonon confinement. We can see very clearly that the minima in both cases of confined phonons and unconfined phonons are at $\Omega / \omega_c = 1{,}85$. The MR in the confined phonons case which is approximately 11 (arb. units) is lower than that in the case of unconfined phonons, which is approximately 2 (arb. units).

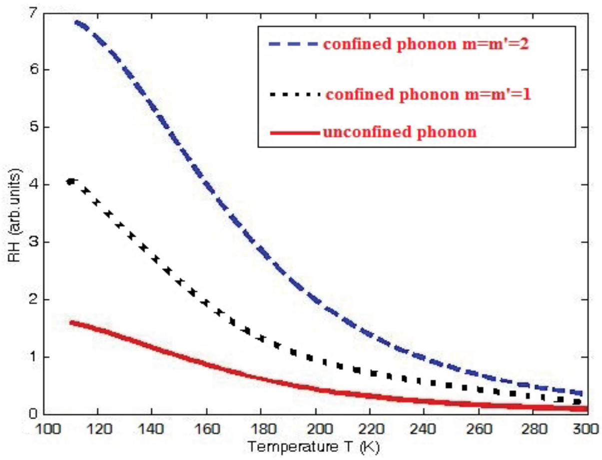

Fig. 3 The dependence of the HC on temperature for both cases– confined phonons and unconfined phonons

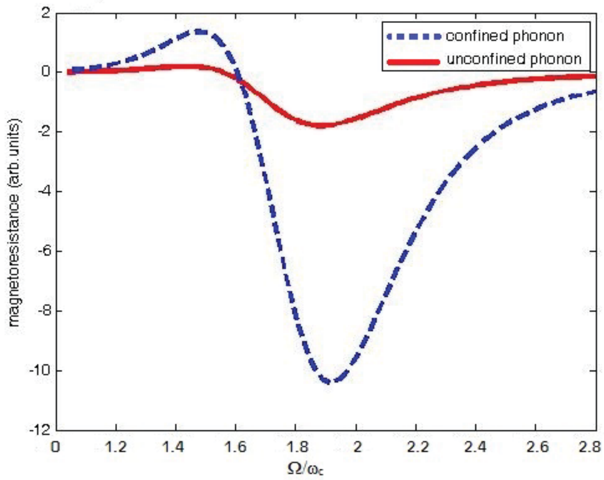

Fig. 4 Dependencies of the MR on the ratio Ω / ω_c at $B = 6T$ for different values of E_o for both cases – confined phonons and unconfined phonons

In Fig. 5, the dashed curve describes the dependence of the conductivity on the cyclotron energy in the case of unconfined phonon. This curve has two maximum peaks. As we can see in figure, from the left to the right, resonance peaks of the conductivity tensor in the case of unconfined phonon correspond to the conditions:

$$\omega_c = \omega_o + \Omega - \frac{\pi}{2m^* L_y^2},\quad \omega_c = \omega_o - \Omega + \frac{\pi}{2m^* L_x^2}.$$

The solid curve in Fig. 5 shows the dependence of the conductivity on the cyclotron energy in the case of confined phonon. It seems that, besides the main resonant peaks, as in the case of unconfined phonon, the subordinate peaks appear. When the cyclotron energy increases further, the conductivity increases continuously and reaches saturation at the high cyclotron energy. There are multiple resonance peaks of the conductivity tensor. These peaks correspond to the following conditions

$$(N'-N)\omega_c = \omega_o \pm \Omega \pm \frac{\pi}{2m^*}\left(\frac{n'^2 - n^2}{L_x^2} - \frac{l'^2 - l^2}{L_y^2}\right)$$

in which ω_c is the cyclotron frequency (eB/m^*), and ω_o is the optical phonon frequency. This condition is generally called the intersubband magnetophonon resonance (MPR) condition. In Fig 5, from the left to the right, this curve has seven maximum peaks which correspond to the conditions:

$$\omega_c = \omega_o - \Omega,\ \omega_c = \omega_o - \Omega - \frac{\pi}{2m^* L_y^2},$$

$$\omega_c = \omega_o + \Omega - \frac{\pi}{2m^* L_y^2},\quad \omega_c = \omega_o,$$

$$\omega_c = \omega_o - \Omega + \frac{\pi}{2m^* L_x^2},\ \omega_c = \omega_o - \Omega + \frac{\pi}{2m^* L_x^2},$$

$$\omega_c = \omega_o + \Omega.$$

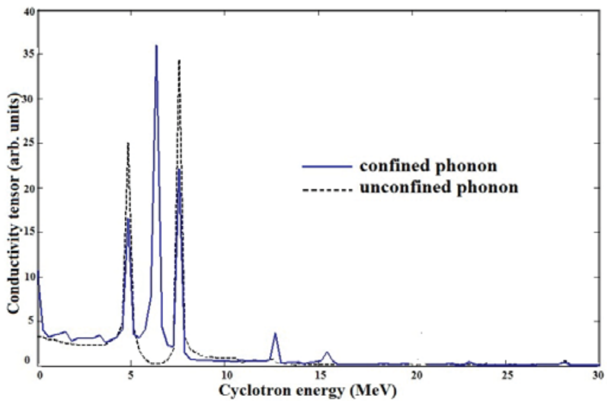

Fig. 5 The dependence of the conductivity tensor σ_{xx} on the cyclotron energy for both cases – confined phonons and unconfined phonons

When the phonons are confined, the conductivity has more resonance peaks compared to that in case of unconfined phonons in a RQW [14]. This behavior of conductivity tensor is similar in quantum wells, the increase of quantum number m, m' characterizing the effect of phonon confinement leads to the increase of conductivity tensor in quantum wells.

V. Conclusion

In this paper, we analytically investigated the possibility of parametric resonance of confined optical phonons in a RQW. We have obtained a set of quantum kinetic equations for transformation of phonons. We numerically calculated and graphed the intensity of the HC for a GaAs/GaAsAl RQW. The results show that the confined phonons cause some unusual effects. The HC and MR depend strongly on the temperature T and the frequency Ω. The confined phonons will increase the values of the HC and MR. When the temperature is low, the HC gets large. In a RQW, for the unconfined phonon case, the HC does not depend on m, m' [14], whereas the HC in a RQW for the unconfined phonon case depends on m, m' which is particular to phonon confinement. The curve has two resonance peaks in the unconfined phonon case, but in unconfined phonons, the curve has only one resonance peak [14]. The fact was not seen in bulk semiconductor [4] as well as in quantum wells [3] and RQWs [14]. The analytical expressions for the HC tensor are obtained. These theoretical results are very different from the previous ones because of the effect of optical phonon confinement. When the phonons are not confined, we have results similar to the case of unconfined phonon in a RWQ [14]. The conductivity tensor has more resonance peaks which correspond to the conditions:

$$(N'-N)\omega_c = \omega_o \pm \Omega \pm \frac{\pi}{2m^*}\left(\frac{n'^2 - n^2}{L_x^2} - \frac{l'^2 - l^2}{L_y^2}\right)$$

in the case of confined optical phonons compared with that in the case of unconfined phonons. The effect of phonon confinement creates new property of the conductivity tensor which has more resonance peaks in the case confined phonons in a RWQ in comparison with the case of a quantum well. Both the conductivity tensors in a RQW with the phonon confinement and in a quantum well have more resonance peaks than those in the confined phonon case.

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