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H_∞ State Estimation of Neural Networks with Discrete and Distributed Delays

Biao Qin, Jin Huang

Abstract—In this paper, together with some improved Lyapunov-Krasovskii functional and effective mathematical techniques, several sufficient conditions are derived to guarantee the error system is globally asymptotically stable with H_∞ performance, in which both the time-delay and its time variation can be fully considered. In order to get less conservative results of the state estimation condition, zero equalities and reciprocally convex approach are employed. The estimator gain matrix can be obtained in terms of the solution to linear matrix inequalities. A numerical example is provided to illustrate the usefulness and effectiveness of the obtained results.

Keywords—H_∞ performance, Neural networks, State estimation.

I. Introduction

In recent years, neural networks have been extensively studied due to its wide application in various areas such as associative memories [1], smart antenna arrays [2], pattern recognition [3], and so on, these applications greatly depend on the dynamic behaviors of the underlying neural networks, therefore, great efforts have been made to analyze the dynamics on NNs and many elegant results have been reported (see e.g.,[4]-[5] and the references therein). In reality, time-delays are frequently encountered in various engineering and scientific fields, time delay, which may results in complex dynamic behaviors, often occurs in neural networks, so, the main focus of attention is on the stability analysis of neural networks with delays and many interesting results have been proposed [7]-[18].

In general, while signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that distributed delays are incorporated into the model [19], on the other hand, a neural network is a highly interconnected network with a large number of neurons, as a result, most neural networks are large-scale and complex networks. In fact, only partial information about the neuron states is available in the network outputs of large-scale neural networks. Therefore, it is important to estimate the neuron states through available measurement outputs, and there are many remarkable attempt to design the state estimator for various types of neural network [19]-[31]. The authors in [19] discussed the problem of H_∞ state estimator for a class of neural networks with mixed time-varying delay. In [20] Lakshmanan investigated the estimation problem for neural networks with leakage, discrete and distributed delays. Further, the state estimation problem for fuzzy cellular neural networks with time delays in leakage term, discrete and unbounded distributed delays is deeply reported in [25]. Moreover, in literature [30], the existence, uniqueness and stability analysis of recurrent neural networks with time delays and leakage term under impulsive perturbations has been investigated. However, some integrals that appeared in the derivative of Lyapunov functional are over bounded, and this leads to conservative results. For example, $-\int_{t-d}^{t} e^T(s)Re(s)ds$ is enlarged as $-\int_{t-d(t)}^{t} e^T(s)Re(s)ds$, the term $-\int_{t-d}^{t-d(t)} e^T(s)Re(s)ds$ is omitted. And, it should be pointed out that under the precondition that the time-derivative of the delay smaller than 1 due to the term $\int_{t-\tau(t)}^{t} e^T(s)Qe(s)ds$ was always chosen in the Lyapunov-Krasovaii functional, the derivation is always positive when $\dot{\tau(t)} < 1$, which is contrary to our ultimate goal $\dot{V}(t, e_t) < 0$. So, removing the above conservation limitations has become urgently necessary.

Based on the above discussions, the H_∞ state estimator problem for a class of neural networks with discrete and distributed delays is considered in this paper. By using a new analysis method based on the lower and upper bound of the time delay and a appropriate Lyapunov-Krasovaii functional with triple integral terms, introducing free-weighting matrices and using reciprocally convex approach, several stability criteria for neural networks with mixed time-varying delays are derived in terms of LMIs. A numerical example is given to illustrate the effectiveness and less conservation of the proposed method.

Notations: Throughout this paper, the superscripts $'-1'$ and $'T'$ stand for the inverse and transpose of a matrix, respectively; $P > 0$, $(P \geq 0, P < 0, P \leq 0)$ means that the matrix P is symmetric positive definite(positive-semi definite, negative definite and negative-semi definite); R^n denotes n-dimensional Euclidean space; $R^{m\times n}$ is the set of $m \times n$ real matrices; I denotes the identity matrix with appropriate dimensions; $*$ denotes the symmetric block in symmetric matrix; $\mathcal{L}_2$ denotes the space of square integrable vector functions on $[0,\infty)$ with norm $\|\cdot\| = (\int_0^\infty \|\cdot\|^2 dt)^{1/2}$.

II. Problem statement and preliminaries

Consider the following neural networks with mixed time-varying delays:

$$\dot{x}(t) = -Cx(t) + B_1 g(x(t)) + B_2 g(x(t-h(t))$$

Biao Qin and Jin Huang are with the School of Mathematics Science, University Electronic Science and Technology of China, Chengdu 611731, PR China.
Email address: qinbiaolixin@163.com.

$$
\begin{aligned}
&+W\int_{t-d(t)}^{t} g(x(s))ds + J + F_1 w(t),\\
&y(t) = Dx(t) + Ex(t-h(t)) + F_2 w(t),\\
&z(t) = Hx(t),\\
&x(s) = \varphi(s), \quad \forall s \in [-\tau, 0], \quad \tau = max\{h, d\} \qquad (1)
\end{aligned}
$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the state vector of the neural network with n neurons, $y(t) \in R^m$ is the network output measurement, $z(t) \in R^p$ to be estimated is a linear combination of the state, $w(t) \in R^q$ is a noise input belonging to $\mathcal{L}_2[0,\infty)$, $C = diag\{c_1, c_2, \ldots, c_n\}$ is a diagonal matrix with positive entries $c_i > 0$, D, E, F_1, F_2 and H are real known constant matrices with appropriate dimensions, the matrices B_1, B_2 and W are connection weight matrix, discrete connection weight matrix and distributed connection weight matrix, respectively, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ denotes the neuron activation function, $J = [J_1, J_2, \ldots, J_n]^T$ is an external input vector, $\varphi(s)$ is the initial condition on $s \in [-\tau, 0]$, $h(t)$ and $d(t)$ is the discrete time-varying delays and distributed time-varying delays, respectively. Satisfying

$$0 \le h(t) \le h, \quad \dot{h}(t) \le \mu, \quad 0 \le d(t) \le d, \qquad (2)$$

where h, μ and d are constants scalars.

Assumption 1. The neuron activation function $g(\cdot)$ is continuous and bounded, and there exist constants ρ_i^- and ρ_i^+ such that

$$\rho_i^- \le \frac{g_i(a) - g_i(b)}{a-b} \le \rho_i^+, \qquad i = 1, 2, \ldots, n \qquad (3)$$

where a, $b \in R$, $a \neq b$.

Here, we denote

$$
\begin{aligned}
&\rho^+ = diag\{\rho_1^+, \ldots, \rho_n^+\}, \quad \rho^- = diag\{\rho_1^-, \ldots, \rho_n^-\},\\
&\rho = diag\{max\{|\rho_1^+|, |\rho_1^-|\}, \ldots, max\{|\rho_n^+|, |\rho_n^-|\}\}. \qquad (4)
\end{aligned}
$$

Remark 1. It is seen from Assumption 1 that ρ_i^- and ρ_i^+ can be positive, negative or zero. when $\rho_i^- = 0$ and $\rho_i^+ > 0$, Assumption 1 describes the monotone nondecreasing activation. Moreover, monotone increasing activation functions can be described when $0 < \rho_i^- < \rho_i^+$.

We consider the following state estimator for the neural networks:

$$
\begin{aligned}
\dot{\hat{x}}(t) &= -C\hat{x}(t) + B_1 g(\hat{x}(t)) + B_2 g(\hat{x}(t-h(t)))\\
&\quad + W\int_{t-d(t)}^{t} g(\hat{x}(s))ds + J + K(y - \hat{y}),\\
\hat{y}(t) &= D\hat{x}(t) + E\hat{x}(t-h(t)),\\
\hat{z}(t) &= H\hat{x}(t),\\
\hat{x}(0) &= \varphi(0), \qquad (5)
\end{aligned}
$$

where $\hat{x}(t) \in R^n$ is the estimated state, $\hat{y}(t) \in R^m$ is the estimated output vector, $\hat{z}(t) \in R^p$ denotes the estimated measurement of $z(t)$ and $K \in R^{n\times m}$ is the state estimator gain matrix to be determined.

denote the errors by $e(t) = x(t) - \hat{x}(t)$ and the output signal as $\bar{z}(t) = z(t) - \hat{z}(t)$, then, based on (1) and (4), we easily obtain the error system of the form

$$
\begin{aligned}
\dot{e}(t) &= -(C + KD)e(t) - KEe(t-h(t)) + B_1 f(t)\\
&\quad + B_2 f(t-h(t)) + W\int_{t-d(t)}^{t} f(s)ds\\
&\quad + (F_1 - KF_2)w(t),\\
\bar{z}(t) &= He(t), \qquad (6)
\end{aligned}
$$

where $f(t) = g(x(t)) - g(\hat{x}(t))$.

From Assumption 1, we can obtain the following inequalities:

$$\rho_i^- \le \frac{f_i(a)}{a} \le \rho_i^+, \qquad i = 1, 2, \ldots, n \qquad (7)$$

where $a \in R$ and $a \neq 0$.

Before proceeding further, the following definition and lemma are introduced.

Definition 1. [29]. Given a prescribed level of noise attenuation $\gamma > 0$, the error system [6] is said to be globally asymptotically stable with H_∞ performance γ, if there is a proper state estimator such that the equilibrium point of the result error system with $w(t) = 0$ is globally asymptotically stable, and

$$\|\bar{z}\|_2 < \gamma \|w\|_2$$

under zero-initial conditions for all nonzero $w(t) \in \mathcal{L}_2[0,\infty)$.

Lemma 1. [15]. For any positive symmetric constant matrix $M \in R^{n\times n}$, a scalar $h > 0$, and a vector function $\omega(s) \in R^n$ such that the integrations concerned are well defined, then

$$
\begin{aligned}
&-\int_{t-h}^{t} \omega^T(s) M \omega(s) ds\\
&\le -\frac{1}{h}\Big(\int_{t-h}^{t} \omega(s)ds\Big)^T M \Big(\int_{t-h}^{t} \omega(s)ds\Big),\\
&-\int_{-h}^{0}\int_{t+\theta}^{t} \omega^T(s) M \omega(s) ds d\theta\\
&\le -\frac{2}{h^2}\Big(\int_{-h}^{0}\int_{t+\theta}^{t} \omega(s) ds d\theta\Big)^T M \Big(\int_{-h}^{0}\int_{t+\theta}^{t} \omega(s) ds d\theta\Big).
\end{aligned}
$$

Lemma 2. [13]. For any constants positive definite matrices $W^T = W \ge 0$, $U \in R^{n\times n}$ are arbitrary matrix with appropriate dimensions and $d \ge 0$, $0 \ge d(t) \ge d$, the following inequalities hold:

$$
\begin{aligned}
&-d\int_{t-d}^{t} f^T(s) W f(s) ds \le\\
&-\begin{bmatrix} \int_{t-d(t)}^{t} f(s)ds \\ \int_{t-d}^{t-d(t)} f(s)ds \end{bmatrix}^T \begin{bmatrix} W & U \\ U^T & W \end{bmatrix} \begin{bmatrix} \int_{t-d(t)}^{t} f(s)ds \\ \int_{t-d}^{t-d(t)} f(s)ds \end{bmatrix}.
\end{aligned}
$$

Lemma 3. [11]. Let the functions $f_1(t), f_2(t), \ldots, f_N(t) : R^m \to R$ have the positive values in an open subset D of R^m and satisfy $\frac{1}{\alpha_1} f_1(t), \frac{1}{\alpha_2} f_2(t), \ldots, \frac{1}{\alpha_N} f_N(t) : D \to R$ with $\alpha_i > 0$ and $\sum_{i=1}^{N} \alpha_i = 1$, then the reciprocal technique of

$f_i(t)$ over the set D satisfies

$$\sum_i \frac{1}{\alpha_i} f_i(t) \geq \sum_i f_i(t) + \sum_{i\neq j} g_{i,j}(t) \quad \forall g_{i,j}(t) : R^m \to R,$$
$$\begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}^T(t) & f_j(t) \end{bmatrix} \geq 0.$$

III. Main Results

In this section, a sufficient condition will be estimated under which the estimation error system (6) is globally asymptotically stable with an H_∞ performance index γ.

Theorem 1. For given scalars $h > 0$, $d > 0$, $\gamma > 0$, $\mu > 0$ and η, the error system (6) is globally asymptotically stable with an H_∞ performance index γ if there exist symmetric positive definite matrices P, R_1, R_2, $Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ * & Q_{i3} \end{bmatrix} > 0 (i = 1, 2, 3, 4)$, diagonal matrices $T = diag\{t_1, t_2, \ldots, t_n\} > 0$, $L = diag\{l_1, l_2, \ldots, l_n\} > 0$, $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, $\tilde{\alpha} = diag\{\alpha_1, \alpha_2, \ldots, \alpha_n\} > 0$, $\tilde{\beta} = diag\{\beta_1, \beta_2, \ldots, \beta_n\} > 0$, $N_i > 0$ $(i = 1, 2, 3)$, any matrices G, M, T_i $(i = 1, 2, 3, 4)$, and S_i $(i = 1, 2, 3, 4)$ such that the following LMIs holds:

$$\begin{bmatrix} \Omega & \tilde{H} \\ * & -I \end{bmatrix} > 0, \tag{8}$$

$$\begin{bmatrix} Q_{11} & Q_{12} & T_1 & T_2 \\ * & Q_{13} & T_3 & T_4 \\ * & * & Q_{11} & Q_{12} \\ * & * & * & Q_{13} \end{bmatrix} > 0, \tag{9}$$

where $\Omega = (\Omega_{l,k})_{17\times 17}$ with

$$\begin{aligned}
\Omega_{1,1} = {} & R_1 + R_2 + 2h^2\tilde{\alpha}(\rho^+ - \rho^-) + \tilde{\beta}h^4(\rho^+ - \rho^-) \\
& + 2h^2\tilde{\alpha}\rho + h^2\tilde{\alpha} + \frac{3h^4}{2}\tilde{\beta} + h^2Q_{11} - Q_{13} + d^2Q_{21} \\
& + \frac{h^4}{2}Q_{31} + \frac{d^4}{2}Q_{41} - h^2Q_{33} - h^2Q_{33}^T - 2\rho^-N_1\rho^+ \\
& - MC - CM^T - GD - D^TG^T,
\end{aligned}$$
$$\Omega_{1,2} = Q_{13} - T_4 - GE, \quad \Omega_{1,3} = T_4,$$
$$\begin{aligned}
\Omega_{1,4} = {} & P - \rho^-T + \rho^+L + \rho\Lambda + h^2Q_{12} + h^2Q_{12} + \frac{h^4}{2}Q_{32} \\
& - M - \eta CM^T - \eta D^TG^T,
\end{aligned}$$
$$\begin{aligned}
\Omega_{1,5} = {} & h^2\tilde{\alpha} + \frac{h^4}{2}\tilde{\beta} + d^2Q_{22} + \frac{d^4}{2}Q_{22} + N_1(\rho^+ + \rho^-) \\
& + MB_1,
\end{aligned}$$
$$\Omega_{1,6} = MB_2, \quad \Omega_{1,8} = -Q_{12}^T + hQ_{33} + hQ_{33}^T,$$
$$\Omega_{1,9} = T_3 + hQ_{33} + hQ_{33}^T, \quad \Omega_{1,12} = MW,$$
$$\Omega_{1,14} = -hQ_{32}^T - hQ_{32}, \quad \Omega_{1,17} = MF_1 - GF_2,$$
$$\Omega_{2,2} = -(1-\mu)R_1 - 2Q_{33} + T_4 + T_4^T - 2\rho^-N_2\rho^+,$$
$$\Omega_{2,3} = Q_{13} - T_4, \quad \Omega_{2,4} = -\eta E^TG^T,$$
$$\Omega_{2,6} = N_2(\rho^+ + \rho^-), \quad \Omega_{2,8} = Q_{12}^T - T_2^T,$$
$$\Omega_{2,9} = -Q_{12}^T + T_3, \quad \Omega_{3,3} = -R_2 - Q_{13} - 2\rho^-N_3\rho^+,$$
$$\Omega_{3,7} = N_3(\rho^+ + \rho^-), \quad \Omega_{3,8} = T_2^T, \quad \Omega_{3,9} = Q_{12}^T,$$
$$\Omega_{4,4} = h^2Q_{13} + \frac{h^4}{2}Q_{33} - \eta M - \eta M^T,$$
$$\Omega_{4,5} = T - L + \Lambda + \eta MB_1, \quad \Omega_{4,6} = \eta MB_2,$$
$$\Omega_{4,12} = \eta MW, \quad \Omega_{4,17} = \eta MF_1 - \eta GF_2,$$
$$\Omega_{5,5} = h^2\tilde{\alpha} + \frac{h^4}{2}\tilde{\beta} + d^2Q_{23} + \frac{d^4}{2}Q_{43} - 2N_1,$$
$$\Omega_{6,6} = -2N_2, \quad \Omega_{7,7} = -2N_3,$$
$$\Omega_{8,8} = -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - Q_{11} - Q_{33} - Q_{33}^T,$$
$$\Omega_{8,9} = -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - T_1 - Q_{33} - Q_{33}^T,$$
$$\Omega_{8,14} = Q_{32} + Q_{32}^T,$$
$$\Omega_{9,9} = -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - Q_{11} - Q_{33} - Q_{33}^T,$$
$$\Omega_{9,14} = Q_{32} + Q_{32}^T, \quad \Omega_{10,10} = -Q_{21}, \quad \Omega_{10,11} = -S_1,$$
$$\Omega_{10,12} = -Q_{22}, \quad \Omega_{10,13} = -S_2, \quad \Omega_{11,11} = -Q_{21},$$
$$\Omega_{11,12} = -S_3^T, \quad \Omega_{11,13} = -Q_{22}, \quad \Omega_{12,12} = -Q_{23}$$
$$\Omega_{12,13} = -S_4, \quad \Omega_{13,13} = -Q_{23},$$
$$\Omega_{14,14} = -4\tilde{\beta}(\rho^+ - \rho^-) - 4\tilde{\beta}\rho - Q_{31} - Q_{31}^T,$$
$$\Omega_{15,15} = -Q_{41} - Q_{41}^T, \quad \Omega_{15,16} = -Q_{42} - Q_{42}^T,$$
$$\Omega_{16,16} = -Q_{43} - Q_{43}^T, \quad \Omega_{17,17} = -\gamma^2 I.$$

Furthermore, the gain matrix K can be designed as $K = M^{-1}G$.

Proof: Introduce the Lyapunov-Krasovskii functional:

$$V(t) = \sum_{i=1}^{6} V_i(t), \tag{10}$$

where

$$\begin{aligned}
V_1(t) = {} & e^T(t)Pe(t), \\
V_2(t) = {} & \int_{t-h(t)}^{t} e^T(s)R_1e(s)ds + \int_{t-h}^{t} e^T(s)R_2e(s)ds, \\
V_3(t) = {} & 2\sum_{i=1}^{n} t_i \int_0^{e_i} [f_i(s) - \rho_i^- s]ds \\
& + 2\sum_{i=1}^{n} l_i \int_0^{e_i} [\rho_i^+ s - f_i(s)]ds \\
& + 2\sum_{i=1}^{n} \lambda_i \int_0^{e_i} [f_i(s) + \rho_i s]ds, \\
V_4(t) = {} & 2h\sum_{i=1}^{n} \int_{-h}^{0}\int_{t+\theta}^{t} \alpha_i e_i(s)[f_i(s) - \rho_i^- e_i(s)]dsd\theta \\
& + 2h\sum_{i=1}^{n} \int_{-h}^{0}\int_{t+\theta}^{t} \alpha_i e_i(s)[\rho_i^+ e_i(s) - f_i(s)]dsd\theta \\
& + 2h\sum_{i=1}^{n} \int_{-h}^{0}\int_{t+\theta}^{t} \alpha_i e_i(s)[f_i(s) + \rho_i e_i(s)]dsd\theta \\
& + 2h^2\sum_{i=1}^{n} \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \beta_i e_i(s)[f_i(s) - \rho_i^- e_i(s)]dsd\lambda d\theta \\
& + 2h^2\sum_{i=1}^{n} \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \beta_i e_i(s)[\rho_i^+ e_i(s) - f_i(s)]dsd\lambda d\theta \\
& + 2h^2\sum_{i=1}^{n} \int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \beta_i e_i(s)[f_i(s) + \rho_i e_i(s)]dsd\lambda d\theta, \\
V_5(t) = {} & h\int_{-h}^{0}\int_{t+\theta}^{t} e^T(s)\tilde{\alpha}e(s)dsd\theta
\end{aligned}$$

$$
\begin{aligned}
&+h\int_{-h}^{0}\int_{t+\theta}^{t} f^T(s)\tilde{\alpha}f(s)dsd\theta\\
&+h^2\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} e^T(s)\tilde{\beta}e(s)dsd\lambda d\theta\\
&+h^2\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} f^T(s)\tilde{\beta}f(s)dsd\lambda d\theta,\\
V_6(t)=&h\int_{-h}^{0}\int_{t+\theta}^{t}\zeta^T(s)Q_1\zeta(s)dsd\theta\\
&+d\int_{-d}^{0}\int_{t+\theta}^{t}\eta^T(s)Q_2\eta(s)dsd\theta\\
&+h^2\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\zeta^T(s)Q_3\zeta(s)dsd\lambda d\theta\\
&+d^2\int_{-d}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\eta^T(s)Q_4\eta(s)dsd\lambda d\theta,
\end{aligned}
$$

where $\zeta(s)=[e^T(s),\dot{e}^T(s)]^T$, $\eta(s)=[e^T(s),f^T(s)]^T$.

Under the zero-initial condition, it is obvious that $V(t)|_{t=0}=0$.

For dealing easily, let

$$
J_\infty=\int_0^t[\bar{z}^T(s)\bar{z}(s)-\gamma^2w^T(s)w(s)]ds,\quad t>0. \tag{11}
$$

Then, we can get

$$
J_\infty\le\int_0^t[\bar{z}^T(s)\bar{z}(s)-\gamma^2w^T(s)w(s)]ds+V(t)-V(t)|_{t=0},\quad t>0. \tag{12}
$$

Then for any $w(t)\in\mathcal{L}_2[0,\infty)$, we can achieve:

$$
J_\infty\le\int_0^t[\bar{z}^T(s)\bar{z}(s)-\gamma^2w^T(s)w(s)+\dot{V}(s)]ds. \tag{13}
$$

Now, calculating the time-derivative of $V(t)$ along the trajectories of (6) yields

$$
\dot{V}_1(t)=2e^T(t)P\dot{e}(t), \tag{14}
$$

$$
\begin{aligned}
\dot{V}_2(t)\le&e^T(t)R_1e(t)-(1-\mu)e^T(t-h(t))R_1e(t-h(t))\\
&+e^T(t)R_2e(t)-e^T(t-h)R_2e(t-h),
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
\dot{V}_3(t)=&2[f^T(t)-e^T(t)\rho^-]T\dot{e}(t)+2[e^T(t)\rho^+-f^T(t)]L\dot{e}(t)\\
&+2[f^T(t)+e^T(t)\rho]\Lambda\dot{e}(t).
\end{aligned} \tag{16}
$$

By using Lemma 1 in $\dot{V}_4(t)$, we get

$$
\begin{aligned}
\dot{V}_4(t)=&2h^2\sum_{i=1}^{n}\alpha_ie_i(t)[f_i(t)-\rho_i^-e_i(t)]\\
&-2h\sum_{i=1}^{n}\int_{t-h}^{t}\alpha_ie_i(s)[f_i(s)-\rho_i^-e_i(s)]ds\\
&+2h^2\sum_{i=1}^{n}\alpha_ie_i(t)[\rho_i^+e_i(t)-f_i(t)]\\
&-2h\sum_{i=1}^{n}\int_{t-h}^{t}\alpha_ie_i(s)[\rho_i^+e_i(s)-f_i(s)]ds\\
&+2h^2\sum_{i=1}^{n}\alpha_ie_i(t)[f_i(t)+\rho_ie_i(t)]\\
&-2h\sum_{i=1}^{n}\int_{t-h}^{t}\alpha_ie_i(s)[f_i(s)+\rho_ie_i(s)]ds\\
&+2\frac{h^4}{2}\sum_{i=1}^{n}\beta_ie_i(t)[f_i(t)-\rho_i^-e_i(t)]\\
&-2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\beta_ie_i(s)[f_i(s)-\rho_i^-e_i(s)]dsd\theta\\
&+2\frac{h^4}{2}\sum_{i=1}^{n}\beta_ie_i(t)[\rho_i^+e_i(t)-f_i(t)]\\
&-2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\beta_ie_i(s)[\rho_i^+e_i(t)-f_i(t)]dsd\theta\\
&+2\frac{h^4}{2}\sum_{i=1}^{n}\beta_ie_i(t)[f_i(t)+\rho_ie_i(t)]\\
&-2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\beta_ie_i(s)[f_i(s)+\rho_ie_i(s)]dsd\theta\\
\le&2e^T(t)[\tilde{\alpha}h^2(\rho^+-\rho^-)]e(t)+2e^T(t)[\frac{\tilde{\beta}h^4}{2}(\rho^+-\rho^-)]e(t)\\
&+2e^T(t)\tilde{\alpha}h^2\rho e(t)+2e^T(t)\tilde{\alpha}h^2f(t)+2e^T(t)\frac{\tilde{\beta}h^4}{2}f(t)\\
&+2e^T(t)\frac{\tilde{\beta}h^4}{2}\rho e(t)-4\int_{-h}^{0}\int_{t+\theta}^{t}e^T(s)dsd\theta\tilde{\beta}(\rho^+-\rho^-)\\
&\int_{-h}^{0}\int_{t+\theta}^{t}e(s)dsd\theta-2h\int_{t-h}^{t}e^T(s)\tilde{\alpha}f(s)ds\\
&-2[\int_{t-h(t)}^{t}e^T(s)ds+\int_{t-h}^{t-h(t)}e^T(s)ds][\tilde{\alpha}(\rho^+-\rho^-)]\\
&[\int_{t-h(t)}^{t}e(s)ds+\int_{t-h}^{t-h(t)}e(s)ds]\\
&-2h\int_{t-h}^{t}e^T(s)\tilde{\alpha}\rho e(s)ds\\
&-2h^2\int_{-h}^{0}\int_{t+\theta}^{t}e^T(s)\tilde{\beta}\rho e(s)dsd\theta\\
&-2h^2\int_{-h}^{0}\int_{t+\theta}^{t}e^T(s)\tilde{\beta}f(s)dsd\theta\\
\le&2e^T(t)[\tilde{\alpha}h^2(\rho^+-\rho^-)]e(t)+2e^T(t)[\frac{\tilde{\beta}h^4}{2}(\rho^+-\rho^-)]e(t)\\
&+2e^T(t)\tilde{\alpha}h^2\rho e(t)+2e^T(t)\tilde{\alpha}h^2f(t)+2e^T(t)\frac{\tilde{\beta}h^4}{2}f(t)\\
&+2e^T(t)\frac{\tilde{\beta}h^4}{2}\rho e(t)-4\int_{-h}^{0}\int_{t+\theta}^{t}e^T(s)dsd\theta\tilde{\beta}(\rho^+-\rho^-)\\
&\int_{-h}^{0}\int_{t+\theta}^{t}e(s)dsd\theta+h\int_{t-h}^{t}e^T(s)\tilde{\alpha}e(s)ds\\
&-2[\int_{t-h(t)}^{t}e^T(s)ds+\int_{t-h}^{t-h(t)}e^T(s)ds][\tilde{\alpha}(\rho^+-\rho^-)]\\
&[\int_{t-h(t)}^{t}e(s)ds+\int_{t-h}^{t-h(t)}e(s)ds]\\
&+h\int_{t-h}^{t}f^T(s)\tilde{\alpha}f(s)ds+h^2\int_{-h}^{0}\int_{t+\theta}^{t}e^T(s)\tilde{\beta}e(s)dsd\theta
\end{aligned}
$$

$$+h^2\int_{-h}^{0}\int_{t+\theta}^{t} f^T(s)\tilde{\beta}f(s)dsd\theta - 4\int_{-h}^{0}\int_{t+\theta}^{t} e^T(s)dsd\theta$$
$$\tilde{\beta}\rho\int_{-h}^{0}\int_{t+\theta}^{t} e(s)dsd\theta - 2[\int_{t-h(t)}^{t} e^T(s)ds$$
$$+\int_{t-h}^{t-h(t)} e^T(s)ds]\tilde{\alpha}\rho[\int_{t-h(t)}^{t} e(s)ds + \int_{t-h}^{t-h(t)} e(s)ds], \tag{17}$$

$$\begin{aligned}\dot{V}_5(t) &= h^2e^T(t)\tilde{\alpha}e(t) - h\int_{t-h}^{t} e^T(s)\tilde{\alpha}e(s)ds\\ &+ h^2f^T(t)\tilde{\alpha}f(t) - h\int_{t-h}^{t} f^T(s)\tilde{\alpha}f(s)ds\\ &+ \frac{h^4}{2}e^T(t)\tilde{\beta}e(t) - h^2\int_{-h}^{0}\int_{t+\theta}^{t} e^T(s)\tilde{\beta}e(s)dsd\theta\\ &+ \frac{h^4}{2}f^T(t)\tilde{\beta}f(t) - h^2\int_{-h}^{0}\int_{t+\theta}^{t} f^T(s)\tilde{\beta}f(s)dsd\theta.\end{aligned} \tag{18}$$

Using Lemma 2 and Lemma 3, one can deduce that

$$\begin{aligned}\dot{V}_6(t) &= h^2\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^T\begin{bmatrix} Q_{11} & Q_{12}\\ * & Q_{13}\end{bmatrix}\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&- h\int_{t-h}^{t}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^T\begin{bmatrix} Q_{11} & Q_{12}\\ * & Q_{13}\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}ds\\
&+ d^2\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}^T\begin{bmatrix} Q_{21} & Q_{22}\\ * & Q_{23}\end{bmatrix}\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}\\
&- d\int_{t-d}^{t}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}^T\begin{bmatrix} Q_{21} & Q_{22}\\ * & Q_{23}\end{bmatrix}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}ds\\
&+ \frac{h^4}{2}\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^T\begin{bmatrix} Q_{31} & Q_{32}\\ * & Q_{33}\end{bmatrix}\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&- h^2\int_{-h}^{0}\int_{t+\theta}^{t}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}^T\begin{bmatrix} Q_{31} & Q_{32}\\ * & Q_{33}\end{bmatrix}\begin{bmatrix} e(s)\\ \dot{e}(s)\end{bmatrix}dsd\theta\\
&+ \frac{d^4}{2}\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}^T\begin{bmatrix} Q_{41} & Q_{42}\\ * & Q_{43}\end{bmatrix}\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}\\
&- d^2\int_{-d}^{0}\int_{t+\theta}^{t}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}^T\begin{bmatrix} Q_{41} & Q_{42}\\ * & Q_{43}\end{bmatrix}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}dsd\theta\\
&\le h^2\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^T\begin{bmatrix} Q_{11} & Q_{12}\\ * & Q_{13}\end{bmatrix}\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&+ d^2\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}^T\begin{bmatrix} Q_{21} & Q_{22}\\ * & Q_{23}\end{bmatrix}\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}\\
&+ \frac{h^4}{2}\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}^T\begin{bmatrix} Q_{31} & Q_{32}\\ * & Q_{33}\end{bmatrix}\begin{bmatrix} e(t)\\ \dot{e}(t)\end{bmatrix}\\
&+ \frac{d^4}{2}\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}^T\begin{bmatrix} Q_{41} & Q_{42}\\ * & Q_{43}\end{bmatrix}\begin{bmatrix} e(t)\\ f(t)\end{bmatrix}\\
&- \begin{bmatrix} \int_{t-h(t)}^{t} e(s)ds\\ e(t)-e(t-h(t))\end{bmatrix}^T\begin{bmatrix} Q_{11} & Q_{12}\\ * & Q_{13}\end{bmatrix}\begin{bmatrix} \int_{t-h(t)}^{t} e(s)ds\\ e(t)-e(t-h(t))\end{bmatrix}\\
&- \begin{bmatrix} \int_{t-h}^{t-h(t)} e(s)ds\\ e(t-h(t))-e(t-h)\end{bmatrix}^T\begin{bmatrix} Q_{11} & Q_{12}\\ * & Q_{13}\end{bmatrix}\begin{bmatrix} \int_{t-h}^{t-h(t)} e(s)ds\\ e(t-h(t))-e(t-h)\end{bmatrix}\\
&- 2\begin{bmatrix} \int_{t-h(t)}^{t} e(s)ds\\ e(t)-e(t-h(t))\end{bmatrix}^T\begin{bmatrix} T_1 & T_2\\ T_3 & T_4\end{bmatrix}\begin{bmatrix} \int_{t-h}^{t-h(t)} e(s)ds\\ e(t-h(t))-e(t-h)\end{bmatrix}\\
&- \begin{bmatrix} \int_{t-d(t)}^{t}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}ds\\ \int_{t-d}^{t-d(t)}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}ds\end{bmatrix}^T\begin{bmatrix} Q_{21} & Q_{22} & S_1 & S_2\\ * & Q_{23} & S_3 & S_4\\ * & * & Q_{21} & Q_{22}\\ * & * & * & Q_{23}\end{bmatrix}\begin{bmatrix} \int_{t-d(t)}^{t}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}ds\\ \int_{t-d}^{t-d(t)}\begin{bmatrix} e(s)\\ f(s)\end{bmatrix}ds\end{bmatrix}\\
&- 2\begin{bmatrix} \int_{-h}^{0}\int_{t+\theta}^{t} e(s)dsd\theta\\ he(t)-\int_{t-h(t}^{t} e(s)ds-\int_{t-h}^{t-h(t)} e(s)ds\end{bmatrix}^T\begin{bmatrix} Q_{31} & Q_{32}\\ * & Q_{33}\end{bmatrix}\begin{bmatrix} \int_{-h}^{0}\int_{t+\theta}^{t} e(s)dsd\theta\\ he(t)-\int_{t-h(t)}^{t} e(s)ds-\int_{t-h}^{t-h(t)} e(s)ds\end{bmatrix}\\
&- 2\begin{bmatrix} \int_{-d}^{0}\int_{t+\theta}^{t} e(s)dsd\theta\\ \int_{-d}^{0}\int_{t+\theta}^{t} f(s)dsd\theta\end{bmatrix}^T\begin{bmatrix} Q_{41} & Q_{42}\\ * & Q_{43}\end{bmatrix}\begin{bmatrix} \int_{-d}^{0}\int_{t+\theta}^{t} e(s)dsd\theta\\ \int_{-d}^{0}\int_{t+\theta}^{t} f(s)dsd\theta\end{bmatrix}.\end{aligned} \tag{19}$$

Noting that, for positive diagonal matrices N_1, N_2, N_3 and using Assumption 1, we can achieve the following inequalities:

$$\begin{aligned}0 \le &-2f^T(t)N_1f(t) + 2e^T(t)N_1(\rho^+ + \rho^-)f(t)\\ &- 2e^T(t)\rho^-N_1\rho^+e(t),\end{aligned} \tag{20}$$
$$\begin{aligned}0 \le &-2f^T(t-h(t)N_2f(t-h(t))\\ &+ 2e^T(t-h(t))N_2(\rho^+ + \rho^-)f(t-h(t))\\ &- 2e^T(t-h(t))\rho^-N_2\rho^+e(t-h(t)),\end{aligned} \tag{21}$$
$$\begin{aligned}0 \le &-2f^T(t-h)N_3f(t-h)\\ &+ 2e^T(t-h)N_3(\rho^+ + \rho^-)f(t-h)\\ &- 2e^T(t-h)\rho^-N_3\rho^+e(t-h).\end{aligned} \tag{22}$$

Moreover, for any matrix M with appropriate dimension and scalar η, one has

$$\begin{aligned}&2[e^T(t)M + \eta\dot{e}^T(t)M][-(C+KD)e(t) - KEe(t-h(t))\\ &+ B_1f(t) + B_2f(t-h(t)) + W\int_{t-d(t)}^{t} f(s)ds\\ &+ (F_1 - KF_2)w(t) - \dot{e}(t)] = 0.\end{aligned} \tag{23}$$

From (13)-(24) and using $G = MK$, one can deduce that

$$\bar{z}^T(t)\bar{z}(t) - \gamma^2w^T(t)w(t) + \dot{V}(t) \le \theta^T(t)\Lambda_1\theta(t), \tag{24}$$

where $\Lambda_1 = \Omega + \tilde{H}^TH$, $\tilde{H} = [H, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$

$0, 0, 0, 0]^T$ and

$$\theta^T(t) =[e^T(t), e^T(t-h(t)), e^T(t-h), \dot{e}^T(t), f^T(t), f^T(t-h(t)), f^T(t-h), \int_{t-h(t)}^{t} e^T(s)ds, \int_{t-h}^{t-h(t)} e^T(s)ds, \int_{t-d(t)}^{t} e^T(s)ds, \int_{t-d}^{t-d(t)} e^T(s)ds, \int_{t-d(t)}^{t} f^T(s)ds, \int_{t-d}^{t-d(t)} f^T(s)ds, \int_{-h}^{0}\int_{t+\theta}^{t} e^T(s)dsd\theta, \int_{-d}^{0}\int_{t+\theta}^{t} e^T(s)dsd\theta, \int_{-d}^{0}\int_{t+\theta}^{t} f^T(s)dsd\theta, w^T(t)]. \tag{25}$$

By using Schur complement, the LMIs (8) and (9) can guarantee $\Lambda_1 < 0$. In this case, when $\Lambda_1 < 0$, we can ensure the error system (6) with the guaranteed H_∞ performance defined by Definition 2. Next, we will show that the equilibrium point of (6) with $w(t) = 0$ is globally asymptotically stable if $\Lambda_1 < 0$ holds. When $w(t) = 0$, the error system (6) becomes

$$\dot{e}(t) = -(C + KD)e(t) - KEe(t-h(t)) + B_1 f(t) + B_2 f(t-\tau(t)) + W\int_{t-d(t)}^{t} f(s)ds,$$
$$\bar{z}(t) = He(t), \tag{26}$$

We still consider the Lyapunov-Krasovskii functional in (10) and calculate its time-derivative along the trajectories of (26), then we can easily get

$$\dot{V}(t) \le \theta_1^T(t)\Lambda_2\theta_1(t), \tag{27}$$

where

$$\theta_1^T(t) =[e^T(t), e^T(t-h(t)), e^T(t-h), \dot{e}^T(t), f^T(t), f^T(t-h(t)), f^T(t-h), \int_{t-h(t)}^{t} e^T(s)ds, \int_{t-h}^{t-h(t)} e^T(s)ds, \int_{t-d(t)}^{t} e^T(s)ds, \int_{t-d}^{t-d(t)} e^T(s)ds, \int_{t-d(t)}^{t} f^T(s)ds, \int_{t-d}^{t-d(t)} f^T(s)ds, \int_{-h}^{0}\int_{t+\theta}^{t} e^T(s)dsd\theta, \int_{-d}^{0}\int_{t+\theta}^{t} e^T(s)dsd\theta, \int_{-d}^{0}\int_{t+\theta}^{t} f^T(s)dsd\theta],$$

and $\Lambda_2 = (\Lambda_{j,k})_{16\times16}$ with

$$\Lambda_{1,1} = R_1 + R_2 + 2h^2\tilde{\alpha}(\rho^+ - \rho^-) + \tilde{\beta}h^4(\rho^+ - \rho^-) + 2h^2\tilde{\alpha}\rho + h^2\tilde{\alpha} + \frac{3h^4}{2}\tilde{\beta} + h^2Q_{11} - Q_{13} + d^2Q_{21} + \frac{h^4}{2}Q_{31} + \frac{d^4}{2}Q_{41} - h^2Q_{33} - h^2Q_{33}^T - 2\rho^-N_1\rho^+ - MC - CM^T - GD - D^TG^T,$$

$\Lambda_{1,2} = Q_{13} - T_4 - GE, \quad \Lambda_{1,3} = T_4,$

$$\Lambda_{1,4} = P - \rho^-T + \rho^+L + \rho\Lambda + h^2Q_{12} + h^2Q_{12} + \frac{h^4}{2}Q_{32} - M - \eta CM^T - \eta D^TG^T,$$

$$\Lambda_{1,5} = h^2\tilde{\alpha} + \frac{h^4}{2}\tilde{\beta} + d^2Q_{22} + \frac{d^4}{2}Q_{22} + N_1(\rho^+ + \rho^-) + MB_1,$$

$\Lambda_{1,6} = MB_2, \quad \Lambda_{1,8} = -Q_{12}^T + hQ_{33} + hQ_{33}^T,$
$\Lambda_{1,9} = T_3 + hQ_{33} + hQ_{33}^T, \quad \Lambda_{1,12} = MW,$
$\Lambda_{1,14} = -hQ_{32}^T - hQ_{32},$
$\Lambda_{2,2} = -(1-\mu)R_1 - 2Q_{33} + T_4 + T_4^T - 2\rho^-N_2\rho^+,$
$\Lambda_{2,3} = Q_{13} - T_4, \quad \Lambda_{2,4} = -\eta E^TG^T,$
$\Lambda_{2,6} = N_2(\rho^+ + \rho^-), \quad \Lambda_{2,8} = Q_{12}^T - T_2^T,$
$\Lambda_{2,9} = -Q_{12}^T + T_3, \quad \Lambda_{3,3} = -R_2 - Q_{13} - 2\rho^-N_3\rho^+,$
$\Lambda_{3,7} = N_3(\rho^+ + \rho^-), \quad \Lambda_{3,8} = T_2^T, \quad \Lambda_{3,9} = Q_{12}^T,$

$$\Lambda_{4,4} = h^2Q_{13} + \frac{h^4}{2}Q_{33} - \eta M - \eta M^T,$$

$\Lambda_{4,5} = T - L + \Lambda + \eta MB_1, \quad \Lambda_{4,6} = \eta MB_2,$
$\Lambda_{4,12} = \eta MW,$

$$\Lambda_{5,5} = h^2\tilde{\alpha} + \frac{h^4}{2}\tilde{\beta} + d^2Q_{23} + \frac{d^4}{2}Q_{43} - 2N_1,$$

$\Lambda_{6,6} = -2N_2, \quad \Lambda_{7,7} = -2N_3,$
$\Lambda_{8,8} = -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - Q_{11} - Q_{33} - Q_{33}^T,$
$\Lambda_{8,9} = -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - T_1 - Q_{33} - Q_{33}^T,$
$\Lambda_{8,14} = Q_{32} + Q_{32}^T,$
$\Lambda_{9,9} = -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - Q_{11} - Q_{33} - Q_{33}^T,$
$\Lambda_{9,14} = Q_{32} + Q_{32}^T, \quad \Lambda_{10,10} = -Q_{21}, \quad \Lambda_{10,11} = -S_1,$
$\Lambda_{10,12} = -Q_{22}, \quad \Lambda_{10,13} = -S_2, \quad \Lambda_{11,11} = -Q_{21},$
$\Lambda_{11,12} = -S_3^T, \quad \Lambda_{11,13} = -Q_{22}, \quad \Lambda_{12,12} = -Q_{23},$
$\Lambda_{12,13} = -S_4, \quad \Lambda_{13,13} = -Q_{23},$
$\Lambda_{14,14} = -4\tilde{\beta}(\rho^+ - \rho^-) - 4\tilde{\beta}\rho - Q_{31} - Q_{31}^T,$
$\Lambda_{15,15} = -Q_{41} - Q_{41}^T, \quad \Lambda_{15,16} = -Q_{42} - Q_{42}^T,$
$\Lambda_{16,16} = -Q_{43} - Q_{43}^T.$

Let $G = MK$, it is obvious that if $\Lambda_1 < 0$, then $\Lambda_2 < 0$. Consequently, the error system (26) is globally asymptotically stable. On the other hand, if $\Lambda_1 < 0$, then the state estimator (5) for the neural network (1) guarantees H_∞ performance and also guarantees that the error system (6) is globally asymptotically stable. This completes the proof. ∎

Remark 2. Compared to [20], the constructed Lyapunov-Krasovskii functional in Theorem 1 is more general and desirable, the time-delay and its time variation can be fully considered, we introduced the following integral terms:

$$2\sum_{i=1}^{n}\lambda_i\int_0^{e_i}[f_i(s) + \rho_i s]ds$$
$$2h\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\alpha_i e_i(s)[f_i(s) + \rho_i e_i(s)]dsd\theta$$

$$2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\beta_i e_i(s)[f_i(s)+\rho_i e_i(s)]dsd\lambda d\theta$$

Remark 3. In this paper, the constraint on derivative of time-varying delay less than one is removed. In addition, the time-varying delay is assumed to be bounded and differentiable, these results become non-differential when $R_1 = 0$ in Lyapunov-Krasovskii functional (10).

Remark 4. Using Lemma 2, the term $-\int_{t-d}^{t-d(t)} e^T(s) Re(s)ds$ is not omitted. Moveover, Lemma 3 is used, more information on cross terms $\int_{t-h(t)}^{t} e(s)ds$, $\int_{t-h}^{t-h(t)} e(s)ds$, $e(t)$, $e(t-h(t))$ and $e(t-h)$ are employed. Through the numerical examples, the effectiveness of this method was demonstrated.

When $W = 0$, the system (1) can be described as follows:

$$\begin{aligned}\dot{x}(t) &= -Cx(t) + B_1 g(x(t)) + B_2 g(x(t-h(t)) \\ &\quad + J + F_1 w(t), \\ y(t) &= Dx(t) + Ex(t-h(t)) + F_2 w(t), \\ z(t) &= Hx(t), \\ x(s) &= \varphi(s), \quad \forall s \in [-\tau, 0], \quad \tau = max\{h, d\} \end{aligned} \tag{28}$$

then, the error system becomes

$$\begin{aligned}\dot{e}(t) &= -(C+KD)e(t) - KEe(t-h(t)) + B_1 f(t) \\ &\quad + B_2 f(t-h(t)) + (F_1 - KF_2)w(t) \\ \bar{z}(t) &= He(t). \end{aligned} \tag{29}$$

We consider the Lyapunov-Krasovskii functional for system (29) as follows:

$$V(t) = \sum_{i=1}^{5} V_k(t) + \tilde{V}_6(t), \tag{30}$$

where

$$\begin{aligned}\tilde{V}_6(t) =& h\int_{-h}^{0}\int_{t+\theta}^{t}\zeta^T(s)Q_1\zeta(s)dsd\theta \\ &+ h^2\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\zeta^T(s)Q_2\zeta(s)dsd\lambda d\theta.\end{aligned}$$

By a similar method to that employed in Theorem 1, it is easy to have the following results.

Corollary 1. For given scalars $h > 0$, $\gamma > 0$, $\mu > 0$ and η, the error system (29) is globally asymptotically stable with an H_∞ performance index γ if there exist symmetric positive definite matrices P, R_1, R_2, $Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ * & Q_{i3} \end{bmatrix} > 0 (i = 1, 2)$, diagonal matrices $T = diag\{t_1, t_2, \ldots, t_n\} > 0$, $L = diag\{l_1, l_2, \ldots, l_n\} > 0$, $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, $\tilde{\alpha} = diag\{\alpha_1, \alpha_2, \ldots, \alpha_n\} > 0$, $\tilde{\beta} = diag\{\beta_1, \beta_2, \ldots, \beta_n\} > 0$, $N_i > 0$ $(i = 1, 2, 3)$, any matrices G, M and T_i $(i = 1, 2, 3, 4)$ such that the following LMI holds:

$$\begin{bmatrix} \Omega & \tilde{H} \\ * & -I \end{bmatrix} > 0, \tag{31}$$

where $\tilde{H} = [H, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T$ and $\Omega = (\Omega_{l,k})_{11\times 11}$ with

$$\begin{aligned}
\Omega_{1,1} &= R_1 + R_2 + 2h^2\tilde{\alpha}(\rho^+ - \rho^-) + \tilde{\beta}h^4(\rho^+ - \rho^-) \\
&\quad + 2h^2\tilde{\alpha}\rho + h^2\tilde{\alpha} + \frac{3h^4}{2}\tilde{\beta} + h^2 Q_{11} - Q_{13} \\
&\quad + \frac{h^4}{2}Q_{21} - h^2 Q_{23} - h^2 Q_{23}^T - 2\rho^- N_1\rho^+ \\
&\quad - MC - CM^T - GD - D^T G^T, \\
\Omega_{1,2} &= Q_{13} - T_4 - GE, \quad \Omega_{1,3} = T_4, \\
\Omega_{1,4} &= P - \rho^- T + \rho^+ L + \rho\Lambda + h^2 Q_{12} + h^2 Q_{12} + \frac{h^4}{2}Q_{22} \\
&\quad - M - \eta CM^T - \eta D^T G^T, \\
\Omega_{1,5} &= h^2\tilde{\alpha} + \frac{h^4}{2}\tilde{\beta} + N_1(\rho^+ + \rho^-) + MB_1, \\
\Omega_{1,6} &= MB_2, \quad \Omega_{1,8} = -Q_{12}^T + hQ_{23} + hQ_{23}^T, \\
\Omega_{1,9} &= T_3 + hQ_{23} + hQ_{23}^T, \quad \Omega_{1,10} = -hQ_{22}^T - hQ_{22}, \\
\Omega_{1,11} &= MF_1 - GF_2, \\
\Omega_{2,2} &= -(1-\mu)R_1 - 2Q_{23} + T_4 + T_4^T - 2\rho^- N_2\rho^+, \\
\Omega_{2,3} &= Q_{13} - T_4, \quad \Omega_{2,4} = -\eta E^T G^T, \\
\Omega_{2,6} &= N_2(\rho^+ + \rho^-), \quad \Omega_{2,8} = Q_{12}^T - T_2^T, \\
\Omega_{2,9} &= -Q_{12}^T + T_3, \quad \Omega_{3,3} = -R_2 - Q_{13} - 2\rho^- N_3\rho^+, \\
\Omega_{3,7} &= N_3(\rho^+ + \rho^-), \quad \Omega_{3,8} = T_2^T, \quad \Omega_{3,9} = Q_{12}^T, \\
\Omega_{4,4} &= h^2 Q_{13} + \frac{h^4}{2}Q_{23} - \eta M - \eta M^T, \\
\Omega_{4,5} &= T - L + \Lambda + \eta MB_1, \quad \Omega_{4,6} = \eta MB_2, \\
\Omega_{4,11} &= \eta MF_1 - \eta GF_2, \quad \Omega_{5,5} = h^2\tilde{\alpha} + \frac{h^4}{2}\tilde{\beta} - 2N_1, \\
\Omega_{6,6} &= -2N_2, \quad \Omega_{7,7} = -2N_3, \\
\Omega_{8,8} &= -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - Q_{11} - Q_{23} - Q_{23}^T, \\
\Omega_{8,9} &= -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - T_1 - Q_{23} - Q_{23}^T, \\
\Omega_{8,10} &= Q_{22} + Q_{22}^T, \\
\Omega_{9,9} &= -2\tilde{\alpha}(\rho^+ - \rho^-) - 2\tilde{\alpha}\rho - Q_{11} - Q_{23} - Q_{23}^T, \\
\Omega_{9,10} &= Q_{22} + Q_{22}^T, \\
\Omega_{10,10} &= -4\tilde{\beta}(\rho^+ - \rho^-) - 4\tilde{\beta}\rho - Q_{21} - Q_{21}^T, \\
\Omega_{11,11} &= -\gamma^2 I.
\end{aligned}$$

Furthermore, the gain matrix K can be designed as $K = M^{-1}G$.

Remark 5. In this paper, when the term $V_4(t)$ changed as

$$\begin{aligned}
\tilde{V}_4(t) =& 2h\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\alpha_{1i}e_i(s)[f_i(s) - \rho_i^- e_i(s)]dsd\theta \\
&+ 2h\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\alpha_{2i}e_i(s)[\rho_i^+ e_i(s) - f_i(s)]dsd\theta \\
&+ 2h\sum_{i=1}^{n}\int_{-h}^{0}\int_{t+\theta}^{t}\alpha_{3i}e_i(s)[f_i(s) + \rho_i e_i(s)]dsd\theta \\
&+ 2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\beta_{1i}e_i(s)[f_i(s) - \rho_i^- e_i(s)]dsd\lambda d\theta \\
&+ 2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\beta_{2i}e_i(s)[\rho_i^+ e_i(s) - f_i(s)]dsd\lambda d\theta
\end{aligned}$$

$$+2h^2\sum_{i=1}^{n}\int_{-h}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\beta_{3i}e_i(s)[f_i(s)+\rho_i e_i(s)]dsd\lambda d\theta,$$

the constructed Lyapunov-Krasovskii functional in Theorem 1 and in Corollary 1 can more general and desirable. In the future, we will do some further research on state estimation problems about this generalized Lyapunov-Krasovskii functional.

IV. Numerical Examples and Simulation

In this section, a numerical example and its simulations are given to show the effectiveness of the result.

Example 1. Consider the error-state system (6) with the following parameters

$$C=\begin{bmatrix}1.4 & 0\\ 0 & 1.2\end{bmatrix},\quad B_1=\begin{bmatrix}1 & 0.73\\ -0.4 & 1.2\end{bmatrix},$$
$$B_2=\begin{bmatrix}-0.7 & 0.4\\ -0.5 & 0.15\end{bmatrix},\quad W=\begin{bmatrix}-0.3 & 0.2\\ -0.3 & 0.2\end{bmatrix},$$
$$F_1=\begin{bmatrix}-0.2\\ 0.2\end{bmatrix},\quad F_2=0.2,\quad D=[-1.1\quad 0],$$
$$E=[0.4\quad 0]\quad H=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix},$$

here the activation functions are assumed to be $g(x(t)) = tanh(x(t))$ with $\rho_1^- = \rho_2^- = 0$ and $\rho_1^+ = \rho_2^+ = 1$. We consider the time-varying delay as $h(t) = 0.5 + 0.5cost$, which means $h = 1$ and $\mu = 0.5$, the distributed delay is chosen as $0 \le d(t) \le d = 0.5$. Furthermore, the noise distraction is taken as $w(t) = 0.01e^{-0.0005t}sin(0.02t)$, $t \ge 0$. By applying the MATLAB LMI Tool box to solve the problem, the estimator gain matrix is obtained as:

$$K = M^{-1}G = \begin{bmatrix}-12.2869\\ -6.7245\end{bmatrix}$$

with the optimal H_∞ index $\gamma_{min} = 1.5969$ and $\eta = 0.15$. In addition, we can derive the minimum H_∞ performance index for different h and μ with fixed $d = 0.5$, which are summarized in Table I. Figs. 1 and 2 show that the trajectories of true state $x_1(t)$ and $x_2(t)$ and their estimations $\hat{x}_1(t)$ and $\hat{x}_2(t)$ with initial conditions $[30, -25]^T$ and $[-17, 5]^T$, respectively. The response of the error $e_1(t)$ and $e_2(t)$ are given in Fig. 3.

TABLE I
Minimum H_∞ Performance Index γ With Different (h, μ) and Fixed $d = 0.5$

Method	(1, 0.3)	(0.9, 0.3)	(0.8, 0.5)	(0.8, 1.2)
$\eta = 0.15$	1.4517	1.3320	1.3212	1.6923
$\eta = 0.20$	1.1348	2.2576	0.9565	1.1240

V. Conclusion

In this paper, the problem of H_∞ state estimation for neural networks with discrete and distributed delays has been investigated, The presented sufficient conditions are based on the appropriated Lyapunove-Krasovskii functional with triple

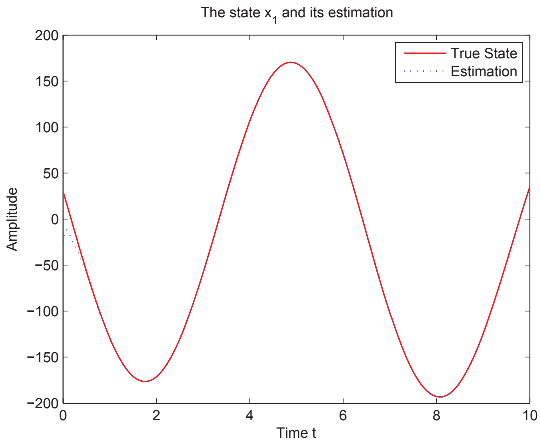

Fig. 1. Trajectories of state $x_1(t)$ and its estimation

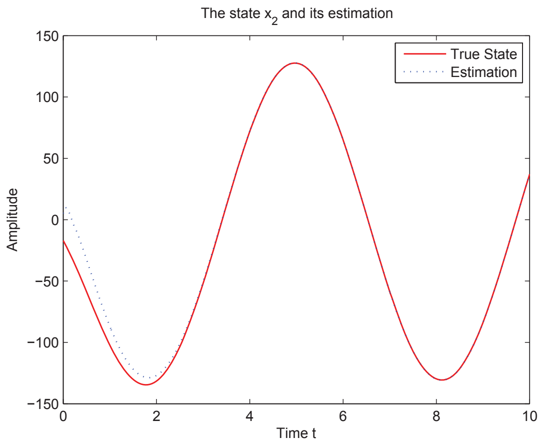

Fig. 2. Trajectories of state $x_2(t)$ and its estimation

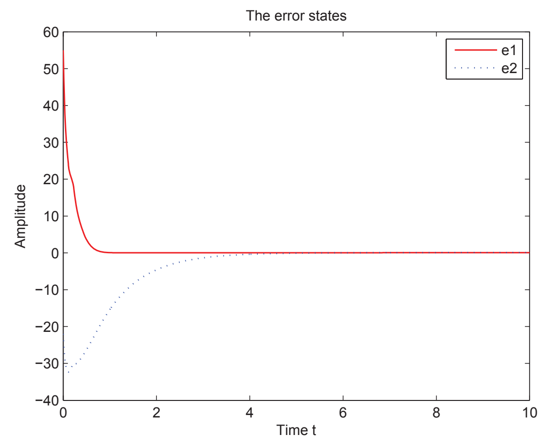

Fig. 3. The state responses of the error system with different initial conditions

integrals, appropriate free-weighting matrices, zero equalities and reciprocally convex approach. Moreover, the time-delay and its time variation can be fully considered. New delay-dependent stability criteria are established in terms of LMIs. A numerical example and its simulations are given to demonstrate the usefulness and effectiveness of the proposed results.

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