Gravitational and Centrifugal Forces in the Nut-Kerr-Newman Space-Time

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***Abstract*—**Nayak et al have discussed in detail the inertial forces such as Gravitational, Coriolis-Lense-Thirring and Centrifugal forces in the Kerr-Newman Space-time in the Kerr-Newman Space-time. The main theme of this paper is to study the Gravitational and Centrifugal forces in the NUT-Kerr-Newman Space-time.

***Keywords*—**Gravitational Forces, Centrifugal Forces, Nut-Kerr-Newman, Space time.

I. Introduction

CHAKRABARTI and Prasanna [1] have described centrifugal forces in the Kerr space-time. They have used the formalism developed by Abramowicz et al[2], which considers the forces in the quotient space orthogonal to the time like killing vector ξ^a. Nayak et al[3] have discussed in detail the inertial forces in the Kerr-Newman Space-time. In this paper, the Gravitational and Centrifugal forces in the NUT- Kerr-Newman Space-time has studied. The NUT- Kerr-Newman Space-time is the Kerr-Newman black-hole Space-time with extra magnetic mass parameter. Here, the expressions for the Gravitational and Centrifugal forces in the NUT- Kerr-Newman Space-time are obtained.

II. The NUT- Kerr-Newman Space-Time Procedure

The NUT- Kerr-Newman Space-time can be written in the form:

$$ds^2 = \left(1-\frac{\mu}{p}\right)dt^2 + 2\frac{\mu h}{\rho}\sin^2\theta dt d\Phi - \left(h^2+r^2+\frac{\mu h^2}{\rho}\sin^2\theta\right)\sin^2\theta d\Phi^2 - \rho^2\left(\frac{dr^2}{\Delta}+d\theta^2\right) \quad (1)$$

Where

$$\mu = 2Mr - e^2 + 2nh\cos\theta$$
$$\Delta = r^2 - 2Mr + h^2 + e^2 - n^2 \quad (2)$$
$$\rho = r^2 + (n + h\cos\theta)^2$$

Where M,e ,h and n are the mass, electric charge, angular momentum per unit mass and NUT(magnetic mass) parameters respectively.

The NUT- Kerr-Newman Space-time gives the following results with certain conditions:

(i) Kerr-Newman black-hole Space-time when $n = 0$

(ii) Kerr black-hole Space-time $n = e = 0$

(iii) Reissner-Nordstrom black-hole Space-time if $n = h = 0$

(iv) Schwarzschild black-hole Space-time when $n = h = e = 0$

(v) NUT-Kerr Space-time when $e = 0$

(vi) NUT Space-time when $e = h = 0$

So the NUT- Kerr-Newman Space-time includes all the black-hole space-times which are asymptotically flat are observed.

III. Inertial Forces

The particle 4-velocity is decomposed as

$$\mu^i = \gamma(n^i + v\tau^i) \quad (3)$$

In the above, n^i is a globally hyper surface orthogonal time like unit vector τ^i is the unit vector orthogonal to it along which the spatial 3-velocity 'v' of the particle is aligned and γ is the normalization factor that makes $u^i u_i = 1$. Then the forces acting on the particle can be written as

Gravitational Force

$$G_k = \phi,_k \quad (4)$$

Centrifugal force

$$Z_k = -(\gamma v)^2 \bar{\tau}^j \overline{\nabla}_j \bar{\tau}_k \quad (5)$$

Where

$$\phi,_k = -n^j n_{k;j} \quad (6)$$

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Here $\bar{\tau}^j$ is the unit vector along τ^j in the conformal space orthogonal to n^j with the metric

$$\bar{h}_{jk} = e^{-2\Phi}\left(g_{jk} - n_j n_k\right) \tag{7}$$

One can show that the covariant derivatives in the two spaces are related by

$$\bar{\tau}^j \bar{\nabla}_j \bar{\tau}_k = \tau^j \nabla_j \tau_k - \tau^j \tau_k \nabla_j \phi - \nabla_k \phi \tag{8}$$

IV. Gravitational and Centrifugal Forces in the NUT-Kerr- Newman Space-Time

Inertial forces in the NUT-Kerr-Newman space-time for circular orbits with fixed but arbitrary values of r, θ and ω can be computed by using the formalism summarized in the previous section. The forces have the following expressions:

Gravitational force

$$G_k = \frac{1}{\Delta}\left(0, g_1, g_2, 0\right) \tag{9}$$

Where

$$g_1 = (r-M) - \frac{\Delta\left[r - \left(\frac{\lambda}{\rho^2}\right)\left\{h^2 - (n+h\cos\theta)^2\right\}\right]}{\left[\left(r^2+h^2+n^2\right) + \left(\mu/\rho\right)\left\{h^2-(n+h\cos\theta)^2\right\}\right]}$$

$$g_2 = -\frac{\mu\Delta\left(r^2+h^2+n^2\right)(n+h\cos\theta)\sqrt{\left\{h^2-(n+h\cos\theta)^2\right\}}}{\rho\left\{\left(r^2+h^2+n^2\right)+\mu h\sin^2\theta\right\}}$$

$$\lambda = M\left\{r^2 - (n+h\cos\theta)^2\right\} - \left(e^2-n^2\right)r \tag{10}$$

Centrifugal force

$$Z_k = \frac{W^2}{A}\left(0, z_1, z_2, 0\right) \tag{11}$$

Where

$$W = \omega - \frac{\mu h}{\left\{\rho\left(r^2+h^2+n^2\right)+\mu(n+h\cos\theta)^2\right\}}$$

$$A = 1 - \omega^2\sin^2\theta\left(r^2+h^2+n^2\right) - \frac{\mu}{\rho}\left\{1-\omega\left(h - \frac{(n+h\cos\theta)^2}{h}\right)\right\}$$

$$z_1 = \frac{\sin^2\theta}{\Delta}\left[\begin{array}{l}(r-M)\left\{\left(r^2+h^2+n^2\right)+\left(\mu/\rho\right)\left\{h^2-(n+h\cos\theta)^2\right\}\right\} - \\ 2\Delta\left\{r - \left(\mu/\rho^2\right)\left\{h^2-(n+h\cos\theta)^2\right\}\right\}\end{array}\right]$$

$$z_2 = -\sin\theta\cos\theta\left\{\begin{array}{l}\left(r^2+h^2+n^2\right)+\left(\mu/\rho\right)\left\{h^2-(n+h\cos\theta)^2\right\} \\ \left(2\left(r^2+h^2+n^2\right)+\rho\right)\end{array}\right\} \tag{12}$$

For the equatorial plane, take $\theta = \frac{\pi}{2}$ so that the expressions of the inertial forces in NUT-Kerr-Newman space-time become

Gravitational force

$$G_k = \frac{1}{\Delta}\left(0, g_1, g_2, 0\right) \tag{13}$$

Where

$$g_1 = (r-M) - \frac{\Delta\left[r - \left(\frac{\lambda}{\left(r^2+n^2\right)^2}\right)\left(h^2-n^2\right)\right]}{\left[\left(r^2+h^2+n^2\right)+\left(\mu/\left(r^2+n^2\right)\right)\left(h^2-n^2\right)\right]} \tag{14}$$

$$g_2 = -\frac{\mu\Delta n\left(r^2+h^2+n^2\right)\sqrt{\left(h^2-n^2\right)}}{\left(r^2+n^2\right)\left\{\left(r^2+h^2+n^2\right)+\mu h\right\}}$$

$$\lambda = M\left(r^2-n^2\right) - \left(e^2-n^2\right)r$$

Centrifugal force

$$Z_k = \frac{W^2}{A}\left(0, z_1, 0, 0\right) \tag{15}$$

Where

$$W = \omega - \frac{\left(2Mr-e^2\right)h}{\left\{\left(r^2+h^2+n^2\right)\left(r^2+n^2\right)+\left(2Mr-e^2\right)n^2\right\}}$$

$$A = 1 - \omega^2\left(r^2+h^2+n^2\right) - \frac{\left(2Mr-e^2\right)}{r^2+n^2}\left\{1-\omega\left(h-\frac{n^2}{h}\right)\right\}$$

$$z_1 = \frac{1}{\Delta}\left[\begin{array}{l}(r-M)\left\{\left(r^2+h^2+n^2\right)+\left(\left(2Mr-e^2\right)/\left(r^2+n^2\right)\right)\left(h^2-n^2\right)\right\} - \\ 2\Delta\left\{r - \left(\left(2Mr-e^2\right)/\left(r^2+n^2\right)^2\right)\left(h^2-n^2\right)\right\}\end{array}\right] \tag{16}$$

On the axis of symmetry take $\theta = 0$. So that the expressions of the inertial forces in the NUT-Kerr-Newman Space-time become

Gravitational force

$$G_k = \frac{1}{\Delta}\left(0, g_1, g_2, 0\right) \tag{17}$$

Where

$$g_1 = (r-M) - \frac{\Delta\left[r + \left(\frac{\lambda}{\rho^2}\right)\left(n^2+2nh\right)\right]}{\left[\left(r^2+h^2+n^2\right) - \left(\mu/\rho\right)\left(n^2+2nh\right)\right]}$$

$$g_2 = -\frac{i\mu\Delta\left(r^2+h^2+n^2\right)(n+h)\sqrt{n^2+2nh}}{\rho\left\{\left(r^2+h^2+n^2\right)+\mu h\sin^2\theta\right\}}$$

$$\lambda = M\left\{r^2-(n+h)^2\right\} - \left(e^2-n^2\right)r \tag{18}$$

Centrifugal force

$$Z_k = 0 \tag{19}$$

V. Gravitational and Centrifugal Forces in the Kerr-Newman Space-Time

To find the inertial forces in Kerr-Newman space-time, substitute in the above expressions $n = 0$ and then find the following expressions:

Gravitational force

$$G_k = \frac{1}{\Delta}\left(0, g_1, g_2, 0\right) \tag{20}$$

Where

$$g_1 = \left[(r-m) - \frac{\Delta\left[r - \left(\frac{\lambda}{\rho^2}\right) h^2 \sin^2\theta \right]}{\left[\left(r^2+h^2\right) + \left(\mu/\rho\right) h^2 \sin^2\theta \right]} \right]$$

$$g_2 = -\frac{\mu\Delta a^2\left(r^2+h^2\right)\sin\theta\cos\theta}{\rho\left\{\left(r^2+h^2\right)+\mu h\sin^2\theta\right\}} \tag{21}$$

$$\lambda = M\left(r^2 - h^2\cos^2\theta\right) - e^2 r$$

Centrifugal force

$$Z_k = \frac{W}{A}\left(0, z_1, z_2, 0\right) \tag{22}$$

Where

$$W = \omega - \frac{\mu h}{\left\{\rho\left(r^2+h^2\right)+\mu h^2\cos\theta\right\}}$$

$$A = 1 - \omega^2\sin^2\theta\left(r^2+h^2\right) - \frac{\mu}{\rho}\left\{1-\omega h\sin^2\theta\right\}^2$$

$$z_1 = \frac{\sin^2\theta}{\Delta}\left[\begin{array}{l} (r-M)\left\{\left(r^2+h^2\right)+\left(\mu/\rho\right)h^2\sin^2\theta\right\} - \\ 2\Delta\left\{r^2 - \left(\mu/\rho^2\right)h^2\sin^2\theta\right\} \end{array} \right]$$

$$z_1 = -\sin\theta\cos\theta\left\{\left(r^2+h^2\right)+\left(\mu/\rho^2\right)h^2\sin^2\theta\left(2\left(r^2+h^2\right)+\rho\right)\right\} \tag{23}$$

For the equatorial plane, take $\theta = \frac{\pi}{2}$ so that the expressions of the inertial forces in Kerr-Newman space-time become:

Gravitational force

$$G_k = \frac{(r-M)\left\{\left(r^2+h^2\right)r^2+\left(2Mr-e^2\right)h^2\right\} - \frac{\Delta}{r}\left\{r^4-\left(Mr-e^2\right)h\right\}}{\left[\Delta\left\{\left(r^2+h^2\right)r^2+\left(2Mr-e^2\right)h^2\right\}\right]}(0,1,0,0) \tag{24}$$

Centrifugal force

$$Z_k = \frac{W}{Ar^2\Delta}\left[\begin{array}{l} (r-M)r\left\{\left(r^2+h^2\right)r^2+\left(2Mr-e^2\right)h^2\right\} \\ -2\Delta\left\{r^4-\left(Mr-e^2\right)h\right\} \end{array} \right](0,1,0,0) \tag{25}$$

Where

$$W = \omega - \frac{\left(2Mr-e^2\right)h}{\left\{\left(r^2+h^2\right)r^2+\left(2Mr-e^2\right)h^2\right\}}$$

$$A = 1 - \omega^2\left(r^2+h^2\right) - \frac{\left(2Mr-e^2\right)}{r^2}\left(1-\omega h\right)^2 \tag{26}$$

$$\Delta = r^2 - 2Mr + h^2 + e^2$$

On the axis of symmetry take $\theta = 0$ .so that the expressions of the inertial forces in the NUT-Kerr-Newman Space-time become:

Gravitational force

$$G_k = \frac{r-m}{r^2-2Mr+h^2+e^2}\left(0,1,0,0\right) \tag{27}$$

Centrifugal force

$$Z_k = 0 \tag{28}$$

VI. Conclusion

It has derived the expressions for the general relativistic analogues of inertial forces such as Gravitational and Centrifugal forces in the NUT-Kerr-Newman Space-time. For circular orbits the r components and z components of the two forces are active. On the equatorial plane $\left(\theta = \frac{\pi}{2}\right)$ r components and z components of the two forces are active where as on the axis of symmetry $(\theta = 0)$ the gravitational force is non-zero but the Centrifugal force is zero. When $n = 0$, and get the known result for the case of the Kerr-Newman Space-time which is obtained by Neyak et al [3]. In this paper it is observed that the expressions of the Gravitational and Centrifugal forces for the NUT-Kerr-Newman Space-time becomes to the expressions for the Kerr-Newman Space-time when $n = 0$. Due to this observation, it is claimed that this study encompasses the known result of Neyak et al [3] in the context of Kerr-Newman Space-time.

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