Fuzzy Mathematical Morphology approach in Image Processing

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Abstract— Morphological operators transform the original image into another image through the interaction with the other image of certain shape and size which is known as the structure element. Mathematical morphology provides a systematic approach to analyze the geometric characteristics of signals or images, and has been applied widely too many applications such as edge detection, objection segmentation, noise suppression and so on. Fuzzy Mathematical Morphology aims to extend the binary morphological operators to grey-level images. In order to define the basic morphological operations such as fuzzy erosion, dilation, opening and closing, a general method based upon fuzzy implication and inclusion grade operators is introduced. The fuzzy morphological operations extend the ordinary morphological operations by using fuzzy sets where for fuzzy sets, the union operation is replaced by a maximum operation, and the intersection operation is replaced by a minimum operation.

In this work, it consists of two articles. In the first one, fuzzy set theory, fuzzy Mathematical morphology which is based on fuzzy logic and fuzzy set theory; fuzzy Mathematical operations and their properties will be studied in details. As a second part, the application of fuzziness in Mathematical morphology in practical work such as image processing will be discussed with the illustration problems.

Keywords—Binary Morphological, Fuzzy sets, Grayscale morphology, Image processing, Mathematical morphology.

I. INTRODUCTION

MATHEMATICAL MORPHOLOGY is based on set theory. The shapes of objects in a binary image are represented by object membership sets. Objects are connected areas of pixels with value 1, the background pixels have value 0.

Binary mathematical morphology is based on two basic operations, defined in terms of a *structuring element*, a small window that scans the image and alters the pixels in function of its window content: a *dilation* $(A \oplus B)$ of set A with structuring element B enlarges the objects (more 1-pixels will be present in the image), an *erosion* $(A \ominus B)$ shrinks objects (the number of 1-pixels in the image decreases). The basic morphological operators [8] on sets A and B are defined as:

Dilation: $(A \oplus B)(x) = \{x \in X, x = a + b : a \in A, b \in B\}$ (1)

Erosion: $(A \ominus B)(x) = \{x \in X, x + b \in A : b \in B\}$ (2)

The structuring element B can be of any size or shape (square, cross, disk, line…). The choice of this element is based on the content of the image and on the purpose of the morphological operation. Composite operations, like the *opening* $A \circ B = (A \ominus B) \oplus B$ (an erosion followed by a dilation) and the *closing* $A \bullet B = (A \oplus B) \ominus B$ (a dilation followed by an erosion,) are derived from the basic operators. Dilation is expected to produce an image that is brighter than the original and in which small, dark details have been reduced or eliminated. In the other hand, erosion produces darker image and the sizes of small, bright features were reduced [8]. Opening decreases sizes of the small bright detail, with no appreciable effect on the darker gray levels, while the closing decreases sizes of the small darker details, with respectively little effect on bright features.

Grayscale morphological operations are an extension of binary morphological operations to grayscale images. We deal with digital image function of the form $f(x, y)$ and $b(x, y)$ where $f(x, y)$ the input image and $b(x, y)$ is a structuring element. Grayscale dilation, erosion, opening, and closing are defined as follows:

Grayscale Dilation

$$(f \oplus b)(s, t) = \max \left\{ \begin{array}{l} f(s - x, t - y) + b(x, y)/ \\ (s - x), (t - y) \in D_f; (x, y) \in D_b \end{array} \right\} \quad (3)$$

Grayscale Erosion

$$(f \ominus b)(s, t) = \min \left\{ \begin{array}{l} f(s + x, t + y) - b(x, y)/ \\ (s + x), (t + y) \in D_f; (x, y) \in D_b \end{array} \right\} \quad (4)$$

where D_f and D_b are the domain of f and b respectively.

Grayscale Opening and Closing are two other important morphological operations for this process.

Grayscale Opening and Closing

$$f \circ b = (f \ominus b) \oplus b \quad (5)$$

$$f \bullet b = (f \oplus b) \ominus b \quad (6)$$

Generally, the effects of the grayscale opening are to remove small bright details from an image leaving the overall gray levels and larger bright features relatively intact. The grayscale closing tends to remove dark details from an image, while learning bright features relatively undisturbed.

II. IMAGE PROCESSING

A. General Image Processing

Mathematical Morphology is a collection of operations which produces useful outcomes in image processing. It is completely based on set theory [3]. By using set operations, there are many useful operators defined in mathematical morphology. For instance erosion, dilation, opening and closing are these kinds of operations which are beneficial when dealing with the numerous image processing problems. Sets in mathematical morphology represent objects in an image.
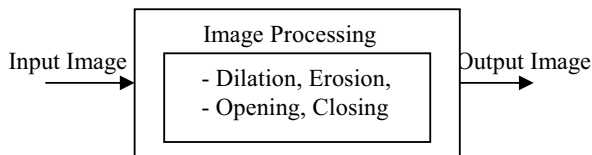
Fig.1. The general structure of image processing

B. Fuzzy Image Processing

Fuzzy image processing has three main stages[3]: Image fuzzification, modification of membership values, and image defuzzification.
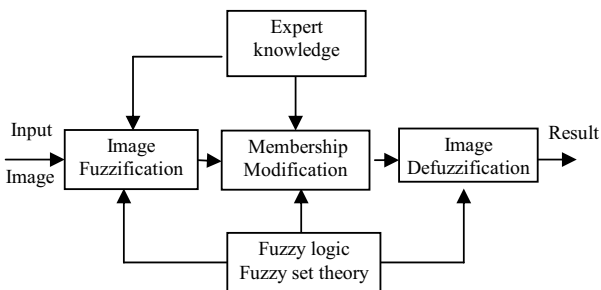
Fig.2. The general structure of fuzzy image processing

The fuzzification and defuzzification steps are due to the fact that we do not possess fuzzy hardware. Therefore, the coding of image data (fuzzification) and decoding of the results (defuzzification) are steps that make possible to process images with fuzzy techniques.

The main power of fuzzy image processing is in the middle step (modification of membership values) as shown in Fig. 3. After the image data are transformed from gray-level plane to the membership plane (fuzzification), appropriate fuzzy techniques modify the membership values. This can be a fuzzy clustering, a fuzzy rule-based approach, a fuzzy integration approach and so on.
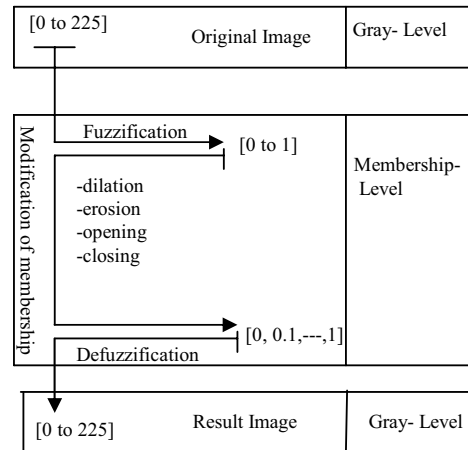
Fig.3. Steps of fuzzy image processing

III. FUZZY MATHEMATICAL MORPHOLOGY

Fuzzy Mathematical Morphology aims to extend the binary morphological operators to gray-level images. In this paper, several points of view are taken to define a Fuzzy Mathematical Morphology (dilation, erosion, opening, closing).

A. Fuzzy Sets

The fuzzy sets theory or the fuzzy logic ([5], [6], [7]) is based on the idea that each element in a certain system can get one value within the interval 0 to 1. Mathematically, it can be expressed as follows. Let X is a classical set which generates a space, and its elements let be marked x. The membership of the set A, which is the subset of the space X, can be described by the membership function $\mu_A(x)$, which gets the values $[0,1]$ as follows:

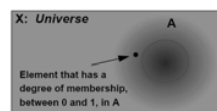$$\mu_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}. \tag{7}$$

Fig.4. Fuzzy set

A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0,1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$. It is clear that A is completely determined by the set of tuples:

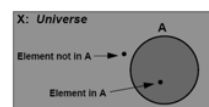$$A = \{(x, \mu_A(x)), x \in X\} \tag{8}$$

Fig.5. Membership function

Define two fuzzy sets A and B on the universe X ([5], [6], [7]). For a given element x of the universe, the following function theoretic operations for the set-theoretic operations of union, intersection, and complement are defined for A and B on X.

$$\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$
$$\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\} \tag{9}$$
$$\mu_{A^c}(x) = 1 - \mu_A(x)$$

Venn diagrams for these operations are shown in Fig. 6,7,8,9.
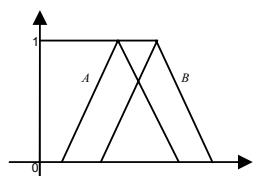

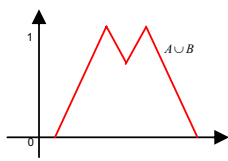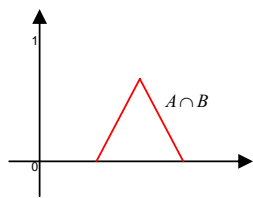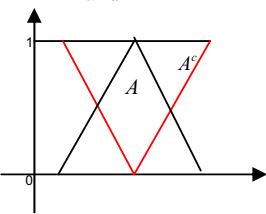Fig.6.Two fuzzy sets

Fig.7. Union of fuzzy sets A and B

Fig.8. Intersection of fuzzy set A and B

Fig.9. Complement of fuzzy sets A and B

B. Fuzzy Logic

The main concepts related to the logic of predicates should be introduced. According to the Boolean logic [4], a predicate is a function p defined in a set X that gets its values from set $\{0, 1\}$. Given two predicates p and q respectively, the basic operations used in this theory are as follows:

Conjunction: $p \wedge q$
Disjunction: $p \vee q$
Negation: $\sim p$
Implication: $\sim p \vee q$

Their definition results from the tables below which determine their values on the basis of p and q corresponding values.

Definition 1. Fuzzy conjunctor

A mapping $C : [0,1] \times [0,1] \rightarrow [0,1]$ is called fuzzy conjunctor. A binary operator C on [0, 1] is a conjunctor on [0, 1] if it is an increasing mapping (i.e., a mapping with increasing partial mappings) that coincides with the Boolean conjunction on $\{0, 1\}^2$, i.e., $C(0,0)= C(0,1)= C(1,0)= 0$ and $C(1,1)= 1$.

TABLE 1
FUZZY CONJUNCTOR

Conjunction		
p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Definition 2. Fuzzy disjunctor

A mapping $D : [0,1] \times [0,1] \rightarrow [0,1]$ is called fuzzy disjunctor. A binary operator D on [0,1] is a disjunctor on [0,1] if it is an increasing mapping (i.e., a mapping with increasing partial mappings) that coincides with the Boolean disjunction on $\{0,1\}^2$, i.e., $D(0,0)= 0$ and $D(0,1)= D(1,0)= D(1,1)= 1$.

TABLE 2
FUZZY DISJUNCTOR

Disjunction		
p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Definition 3. Fuzzy negator

A mapping $N : [0,1] \times [0,1] \rightarrow [0,1]$ is called fuzzy negator. A unary operator N on [0, 1] is a negator if it is a decreasing mapping that coincides with the Boolean negation on $\{0, 1\}$, i.e., $N(0) = 1$ and $N(1) = 0$.

TABLE 3
FUZZY NEGATOR

Negation	
p	$\sim p$
0	1
1	0

Definition 4. Fuzzy implicator

A mapping $I : [0,1] \times [0,1] \rightarrow [0,1]$ is called fuzzy implicator. a binary operator I on [0,1] is a implicator on [0,1] if it is an hybrid monotonous mapping (i.e., a mapping with describing first and increasing second partial mappings) that coincides with the Boolean implication on $\{0,1\}^2$, i.e., $I(0,0)= I(0,1)= I(1,1)= 1$ and $I(1,0)=0$.

TABLE 4
FUZZY IMPLICATOR

Implication		
p	q	$\sim p \vee q$
0	0	1
0	1	1
1	0	0
1	1	1

C. Fuzzy Mathematical Morphology with Membership Function

Fuzzy mathematical morphology is studied in terms of fuzzy fitting. The fuzziness is introduced by the degree to which the structuring element fits into the image.

The operations of erosion $E^\mu(A,B)$ and dilation $D^\mu(A,B)$ of a fuzzy image A by a fuzzy structuring element B having a bounded support are defined in terms of their membership functions. Fuzzy erosion and dilation have membership functions whose values are within the interval [0, 1].

$$E^\mu(A,B) = \mu_{A\ominus B}(x) = \min_{y\in B}\left[\min\left[1, 1+\mu_A(x+y)-\mu_B(y)\right]\right] \quad (10)$$

$$= \min\left[1, \min_{y\in B}\left[1+\mu_A(x+y)-\mu_B(y)\right]\right]$$

$$D^\mu(A,B) = \mu_{A\oplus B}(x) = \max_{y\in B}\left[\max\left[0, \mu_A(x-y)+\mu_B(y)-1\right]\right] \quad (11)$$

$$= \max\left[0, \max_{y\in B}\left[\mu_A(x-y)+\mu_B(y)-1\right]\right]$$

where $x, y \in Z^2$ the spatial are coordinates and μ_A, μ_B are the membership functions of the image and the structuring element, respectively. Fuzzy opening and Fuzzy closing are described as follows:

$$O^\mu(A,B)(x) = \mu_{A\bullet B}(x) = D^\mu\left(E^\mu(A,B)(x), B(x)\right)$$
$$= D^\mu\left(\min_{y\in B}\left[\min\left[1, 1+\mu_A(x+y)-\mu_B(y)\right]\right], B(x)\right) \quad (12)$$

$$C^\mu(A,B)(x) = \mu_{A\circ B}(x) = E^\mu\left(D^\mu(A,B)(x), B(x)\right)$$
$$= E^\mu\left(\max_{y\in B}\left[\max\left[0, \mu_A(x-y)+\mu_B(y)-1\right]\right], B(x)\right) \quad (13)$$

D. Fuzzy Mathematical Morphology with Fuzzy Logical Operators

In view of the fact that traditional morphology relies on the theory of sets, fuzzy morphological operatorscan be defined by means of fuzzy logic[4]. Let A and B belong to the set of images parts, then:

$$A \subseteq B \Leftrightarrow \forall y \in X, y \in A \Rightarrow y \in B$$
$$\Leftrightarrow \forall y \in X, A(y) \Rightarrow B(y)$$
$$\Leftrightarrow \forall y \in X, I(A(y), B(y)) = 1$$

where I denotes the binary implication. The fuzzy erosion of an image A can be defined by a structuring element B, in a point x like:

$$E^I(A,B)(x) = \inf_{y\in A} I\left[B(y), A(y)\right] \quad (14)$$

In a similar way, being A and B part of X set of parts, it is known that:

$$A \cap B \neq \phi \Leftrightarrow \exists y \in X, y \in A \Rightarrow y \in B$$
$$\Leftrightarrow \exists y \in X, C\left(A(y), B(y)\right) = 1$$

where C is the binary conjunction. Then, the fuzzy dilation of an image A can be defined by a structuring element B, in a point x like:

$$D^I(A,B)(x) = \sup_{y\in A} C\left[B(y), A(y)\right]$$
$$= 1 - \inf_{y\in A} C\left[B(y), A(y)\right] \quad (15)$$

Following the steps of the morphological theory, fuzzy opening and fuzzy closing are described as follows:

$$O^I(A,B)(x) = D^I\left[E^I(A,B)(y), B(y)\right]$$
$$= D^F\left[\inf_y I\left[B(y), A(y)\right], B(y)\right] \quad (16)$$
$$= \sup_y C\left[B(y), \inf_y I\left[B(y), A(y)\right]\right]$$

$$C^I(A,B)(x) = E^I\left(D^I(A,B)(y), B(y)\right)$$
$$= E^I\left[\sup_y C\left[B(y), A(y)\right], B(y)\right] \quad (17)$$
$$= \inf_{y\in A} I\left[B(y), \sup_y C\left[B(y), A(y)\right]\right]$$

IV. FUZZY MATHEMATICAL MORPHOLOGY WITH LUKASIEWICZ GENERALIZED OPERATORS

A. Lukasiewicz Generalized Operators

Definition 5: Lukasiewicz generalized operators [1] are the maps $L:[0,1]\times[0,1] \to [01]$ such that

$$L(a,b) = \min\left[1, \lambda(a) + \lambda(1-b)\right] \quad \forall a,b \in [0,1] \quad (18)$$

where function $\lambda:[0,1] \to [0,1]$ verifying $\lambda(0) = 1$ and $\lambda(1) = 0$.

We can prove that the limit conditions required for the function λ is sufficient to assure that the operator L are a generalizing of classical implication.

The Smets-Magrez's axioms [1] for implication operators which are the essential properties for Lukasiewicz implication are as follows:

SM.1- The value $L(a,b)$ depends on values a and b.

SM.2- Opposition law: $L(a,b) = L(1-b, 1-a) \ \forall a,b \in [0,1]$

SM.3-Exchange principle: $L(a, I(b,c)) = L(b, I(a,c)) \ \forall a,b \in [0,1]$

SM.4- $L(.,b)$ is non increasing $\forall b \in [0,1]$ and $L(a,.)$ is non decreasing $\forall a \in [0,1]$

SM.5- $\forall a,b \in [0,1], \ a \leq b \Leftrightarrow L(a,b) = 1$ (implication defines an ordering)

SM.6- $L(1,b) = b \quad \forall b \in [0,1]$

SM.7- $L(a,b)$ is continuous.

Theorem 1: Under the conditions stated in Definition 5, the following properties are verified:

(1) $L(0,b) = 1$ $\forall b \in [0,1]$

(2) $L(a,0) = \lambda(a)$ $\forall a \in [0,1]$

(3) $L(a,1) = 1$ $\forall a \in [0,1]$

(4) $L(1,b) = \lambda(1-b)$ $\forall b \in [0,1]$

(5) $L(a,b) = 1 \Leftrightarrow \lambda(a) + \lambda(1-b) \geq 1$ $\forall a,b \in [0,1]$

(6) $L(a,b) = 0 \Leftrightarrow \lambda(a) \; and \; \lambda(1-b) = 0$

TABLE 5
LUKASIEWICZ IMPLICATION

Lukasiewicz Implication			
	0	b	1
0	1	1	1
a	$\lambda(a)$	$L(a,b)$	1
1	0	$\lambda(1-b)$	1

Theorem 2: In the conditions of Definition 5,
$L(a,b) = \min(1, \lambda(a) + \lambda(1-b)), \forall a,b \in [0,1]$ verifies the axioms SM.2, SM.4, SM.5 and SM.7 iff f and $\lambda : [0,1] \rightarrow [0,1]$ verifies the following conditions:

(1) $\lambda(0) = 1 \; and \; \lambda(1) = 0$

(2) λ non increasing and f non decreasing

(3) $\lambda(p) = f(1-p) \; \forall p \in [0,1]$

(4) $p \leq q \Leftrightarrow \lambda(p) + \lambda(1-q) \geq 1$

(5) λ Continuous

Theorem 3: Let L in the conditions of Definition 5 and λ verifying Theorem 2. Then the following properties are equivalent:

(1) $L[a, L(b,c)] = L[b, L(a,c)]$ $\forall a,b,c \in [0,1]$

(2) $L(1,b) = b$ $\forall b \in [0,1]$

(3) $L[a, L(b,a)] = 1$ $\forall a,b \in [0,1]$

(4) $\lambda(a) = 1 - a$ $\forall a \in [0,1]$

B. The Inclusion Grade

The basic operation of fuzzy mathematical morphology by means of the inclusion grade [1] for fuzzy subsets can be defined. The inclusion grade operator as a fuzzy relation $R : F(x) \times F(x) \rightarrow \{0,1\}$ such that

$$R(A,B) = \inf_{x \in X} L[A(x), B(x)]$$
$$= \inf_{x \in X} \{\min[1, \lambda[A(x)] + \lambda[1 - B(x)]\} \quad \forall A, B \in F(x) \tag{19}$$

verifies the conditions:

(1) $R(A,B) = 1 \Leftrightarrow A \subseteq B$

(2) $R(A,B) = 0 \Leftrightarrow \exists x \in X$ such that $A(x) = 1$ and $B(x) = 0$.

As an inclusion grade for $A, B \in [0,1]$ where $\lambda : [0,1] \rightarrow [0,1]$ is a map that verifies the following conditions:

(1) λ is non increasing

(2) $\lambda(0) = 1$ and $\lambda(1) = 0$

(3) If there exist p and $q \in [0,1]$ such that $\lambda(p) = \lambda(q) \geq 0.5$, then $p = q$.

(4) The equation $\lambda(p) = 0$ has a single solution.

(5) $\lambda(p) + \lambda(1-p) \geq 1$ $\forall p \in [0,1]$.

C. Lukasiewicz Erosion and Dilation

The *Lukasiewicz* erosion (dilation) [1] of an image A by another image B, being A and B fuzzy subset of X, is denoted by $E^L(A,B)$ and $D^L(A,B)$, and it is defined by:

$$E^L(A,B)(x) = R[B(x), A(x)]$$
$$= \inf_{x \in X} L[B(x), A(x)] \tag{20}$$
$$= \inf_{x \in X} \{\min[1, \lambda[B(x)] + \lambda[1 - A(x)]]\} \; \forall x \in X$$

$$D^L(A,B)(x) = 1 - R[B(x), A(x)]$$
$$= 1 - \inf_{x \in X} L[B(x), A(x)] \tag{21}$$
$$= 1 - \inf_{x \in X} \{\min[1, \lambda[B(x)] + \lambda[1 - A(x)]]\} \; \forall x \in X$$

D. Lukasiewicz Opening and Closing

The *Lukasiewicz* opening (closing) of an image A by another image B, being A and B fuzzy subset of X, is denoted by $O^L(A,B)$ and $C^L(A,B)$, and defined by:

$$O^L(A,B)(x) = D^L\left[E^L(A,B)(x), B(x)\right]$$
$$= D^L\left[\inf_{x \in X} L[B(x), A(x)], B(x)\right] \tag{22}$$

$$C^L(A,B)(x) = E^L\left[D^L(A,B)(x), B(x)\right]$$
$$= E^L\left[1 - \inf_{x \in X} L[B(x), A(x)], B(x)\right] \tag{23}$$

V. ILLUSTRATE THE EFFECT OF OPERATORS IN IMAGES

In this section, fuzzy image and fuzzy structuring element, a dilation-erosion operation result in overlapping effects of expands and shrink images will be presented.

A. Effect of Basic Morphological Operators in Images

In this section, we give the examples to show the effect of basic morphological operators such as erosion and dilation with the same original image A and different structuring element B.

Example 1: Let A be the digital image of Shwedagon pagoda which is very famous pagoda in MYANMAR and the structuring element be an array 5x5, $B = \{1,1,1,1,1; \; 1,1,1,1,1; \; 1,1,1,1,1; \; 1,1,1,1,1; \; 1,1,1,1,1\}$.

Fig.10 (a) Original image   Fig.10 (b) Dilation   Fig.10(c) Erosion

Fig. 10(a) is the original image, Fig 10(b) and Fig. 10(c) are the output images, the dilation and erosion images respectively, according to the basic morphological operators and they show the shrink-expand effect of the original image.

Example 2: Let the structuring element be an array 3x3, $B = \{1,1,1; \ 1,1,1; \ 1,1,1; \ 1,1,1; \ 1,1,1\}$.

Fig.11 (a) Original image   Fig.11 (b) Dilation   Fig. 11(c) Erosion

Fig. 11(a) is the same original image as in Example 1, Fig. 11(b) and Fig. 11(c) are output images, the dilation and erosion images respectively, according to the basic morphological operators and they show the shrink-expand effect of the original image.

Example 3: Let A be the one of my digital images and the structuring element be an array 5x5, $B = \{1,1,1,1,1; \ 1,1,1,1,1; \ 1,1,1,1,1; \ 1,1,1,1,1; \ 1,1,1,1,1\}$.

Fig.12 (a) Original image   Fig.12 (b) Dilation   Fig.12(c) Erosion

Fig. 12(a) is the original image, Fig. 12(b) and Fig. 12(c) are output images, the dilation and erosion images respectively, according to the basic morphological operators and they show the shrink-expand effect of the original image.

Example 4: Let the structuring element be an array 3x3, $B = \{1,1,1; \ 1,1,1; \ 1,1,1; \ 1,1,1; \ 1,1,1\}$.

Fig.13 (a) Original image   Fig.13 (b) Dilation   Fig.13(c) Erosion

Fig. 13(a) is the same original image as in Example 3, Fig 13(b) and Fig. 13(c) are output images, the dilation and erosion images respectively, according to the basic morphological operators and they show the shrink-expand effect of the original image.

In examples 1 and 2, we show the erosion-dilation effect with the same digital image and the different structuring element, and it can be clearly seen that the smaller the size of the structuring element is, the clearer the image is. The same effect can also be seen in examples 3 and 4.

B. Morphological Operators by Fuzzy Implication in Images

In this section, we present the analysis of the dilation-erosion operators by basic fuzzy logical operators and Lukasiewicz generalized operators with the same image A and different structuring element B and alternating.

TABLE 6
DILATION-EROSION OPERATORS; E^I AND D^I

A	B	E^I	D^I
0	B	$\begin{cases}0, & B=0 \\ 1, & B=1\end{cases}$	0
1	B	1	$\begin{cases}1, & B=1 \\ 0, & B=0\end{cases}$
A	0	1	0
A	1	$\begin{cases}0, & A=0 \\ 1, & A=1\end{cases}$	$\begin{cases}1, & A=1 \\ 0, & A=0\end{cases}$

TABLE 7
DILATION-EROSION OPERATORS; E^L AND D^L

A	B	E^L	D^L
0	B	$\lambda(B)$	$1-\lambda(B)$
1	B	1	0
A	0	1	0
A	1	$\lambda(1-A)$	$1-\lambda(1-A)$
$0<A<1$	B	$\lambda(B)+\lambda(1-A)$	$1-\left[\begin{matrix}\lambda(B)+ \\ \lambda(1-A)\end{matrix}\right]$
A	$0<B<1$	$\lambda(B)+\lambda(1-A)$	$1-\left[\begin{matrix}\lambda(B)+ \\ \lambda(1-A)\end{matrix}\right]$

The function λ used in the Lukasiewicz Generalized Operator [1] is defined $\lambda:[0,1]\to[0,1]$ given by

$$\lambda(p)=\begin{cases} g(p) & \text{if} \quad p\in[0.0.5] \\ 1-g(1-p) & \text{if} \quad p\in(0.5,1]\end{cases} \qquad (24)$$

satisfies conditions of
(1) $\lambda(1)=0 \ and \ \lambda(0)=1$
(2) $p\le q \Leftrightarrow \lambda(p)+\lambda(1-q)\ge 1$

(3) λ is non increasing and g (p) is defined as $g : [0, 0.5] \rightarrow [0.5, 1]$ strictly non increasing with $g(0) = 1$, and completely continuous.

As an example of λ function, we define $g(p) = -0.5p + 1$. Then we obtain $\lambda(p) = \begin{cases} -0.5p + 1 & if \quad p \in [0, 0.5] \\ -0.5p + 0.5 & if \quad p \in (0.5, 1] \end{cases}$ and it can be illustrated by the following figure, Fig.14.
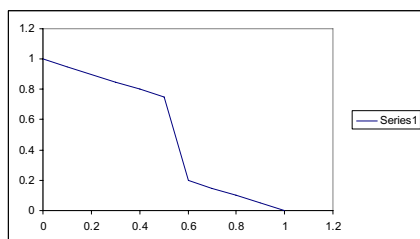

Fig .14. λ - function

In Table 6, we analyze the erosion and dilation operators, E^I and D^I, with respect to the fuzzy logical operators, and formulate these operators for the fuzzy image, A, and structuring element, B, in general. Similarly, in Table 7, we analyze the Lukasiewicz generalized operators, E^L and D^L, and formulate these operators for the fuzzy images and structuring elements, in general. In this case, according to the definitions of erosion and dilation, the λ-function is needed to consider and because of the λ-function, we can define the values of A and B more details. Here it is noticed that the case in Table 6 is the special case of Table 7.

VI. CONCLUSION

We show some digital images to illustrate the effect of dilation-erosion operators in images. It is known that binary mathematical morphology dilation expands the image and erosion shrinks it. Erosion yields a smaller image than the original and dilation in opposite. This idea can be extended to grey level imagery. A point to notice here, is the fact that erosion and dilation, basic operations in binary imagery, can be extended to grey level. If the image and structuring element are binary (0 and 1), binary operators, erosion-dilation hold the same effect that the fuzzy erosion-dilation operation results in overlapping effects of expands/shrink images. Fuzzy logic and fuzzy set theory provide many solutions to the mathematical morphology algorithms. They have extended way of processing the gray scale images by the help of the fuzzy morphological operators. Fuzzy set and fuzzy logic theory is a new research area for defining new algorithms and solutions in mathematical morphology environment.

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