Fuzzy EOQ Models for Deteriorating Items with Stock Dependent Demand & Non-Linear Holding Costs

G. C. Mahata and A. Goswami

Abstract—This paper deals with infinite time horizon fuzzy Economic Order Quantity (EOQ) models for deteriorating items with stock dependent demand rate and nonlinear holding costs by taking deterioration rate θ_0 as a triangular fuzzy number $(\theta_0 - \delta_1, \theta_0, \theta_0 + \delta_2)$, where $0 < \delta_1, \delta_2 < \theta_0$ are fixed real numbers. The traditional parameters such as unit cost and ordering cost have been kept constant but holding cost is considered to vary. Two possibilities of variations in the holding cost function namely, a non-linear function of the length of time for which the item is held in stock and a non-linear function of the amount of on-hand inventory have been used in the models. The approximate optimal solution for the fuzzy cost functions in both these cases have been obtained and the effect of non-linearity in holding costs is studied with the help of a numerical example.

Keywords—Inventory Model, Deterioration, Holding Cost, Fuzzy Total Cost, Extension Principle.

I. INTRODUCTION

DETERMINISTIC inventory models have been developed in the literature based on the demand rate to be either constant, time dependent, or stock dependent. Since, the assumption of static demand over the entire time horizon is unrealistic, so Donaldson [8] was the first to solve analytically the no shortage EOQ model where the demand is assumed to be a linearly increasing function of time. Two types of time varying demand patterns considered are linear positive or negative trend in demand and exponentially increasing or decreasing demand. It is also experienced that for consumer goods, sales increase with the increase of the inventory. Basically, large inventory attracts a large number of people to buy more and more of the inventory. This requires consideration of the demand to be a function of the on-hand inventory. As a result, many papers appeared in literature to deal with inventory models using some form of functional

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dependencies between the demand rate and on-hand inventory. Gupta & Vrate [1] had assumed demand rate to be a function of the initial stock. Mondal & Phaujder [2] considered demand rate to be linear and non-linear functions of the on-hand inventory. However, they do not consider the deterioration of inventory in their models. T.K.Dutta & A.K.Pal [3] extended the models of [2] by allowing deterioration effect and shortages, which are completely backlogged, for finite as well as for infinite time horizon. But they considered demand rate to be a linear function of the on-hand inventory. M. Goh [4] considered EOQ models with general demand and holding cost functions. B.C. Giri & K.S. Chaudhuri [5] extended Goh's [4] model by allowing deterioration effect. They considered the deterioration rate, unit cost and ordering cost as fixed, but all of them probably will have some little disturbances for each cycle in the real situation. Hence these parameters should be treated as fuzzy variables.

This paper has used fuzzy concepts to develop a fuzzy EOQ model with stock-dependent demand rate and non-linear holding cost by taking rate of deterioration to be a triangular fuzzy number. Two fuzzy based models are provided for both the cases of non-linear time dependent holding cost and stock dependent holding cost. The solutions for minimizing the fuzzy cost functions have been derived by using the techniques of Zimmermann [6] and Kaufmann & Gupta [7]. The approximate optimal solutions for both the models have been obtained for a numerical example. Finally, the effect of non-linearity in holding cost is studied for the same numerical example.

This paper is organized as follows. In section 2, we describe in brief the notations and assumptions used in the developed models. Section 3 gives the mathematical formulations of the models, both when the holding cost is a nonlinear function of the on hand stock and when it is nonlinear function of the time for which item is held in stock. In section 4, the fuzzy model of model-1 of section 3 are given. The fuzzy model of model-2 of section 3 follows similar to that of fuzzy model of model-1 and hence its formulation is not given. Section 5 illustrates the developed models on a numerical example and study the effect of non linearity on the holding cost. Finally, conclusions are given in section 6.

II. ASSUMPTIONS AND NOTATION

The inventory model is developed on the basis of the following assumptions and notations

A. Assumptions:

1. Item cost does not vary with order size.
2. Lead time is zero.
3. Replenishment is instantaneous i.e., replenishment rate is infinite.
4. Replenishment cost is known and constant.
5. There is only one stocking point in each cycle.
6. The time horizon of the inventory system is infinite. Only a typical planning schedule of length is considered, all remaining cycles are identical
7. The demand rate is deterministic and is known function of the instantaneous level of inventory q. The functional relationship between the demand rate $R(q)$ and the instantaneous inventory level q(t) is given by the following expression:

   $R(q) = Dq^{\beta}$, $D > 0$, $0 < \beta < 1$, $q \geq 0$, where

   denotes the on-hand inventory level at time t
8. A constant fraction θ, assumed to be small, of the on-hand inventory gets deteriorated per unit time after a time μ from the instant of arrival of a fresh replenishment in stock.
9. $\tilde{\theta}$ is the fuzzy deterioration rate of the on-hand inventory per unit time.

B. Notation:

Q: Order quantity of the item.

TCU: Total relevant inventory cost per unit time.

T: Cycle time.

$q_1(t)$:   On-hand inventory level at time t, where $0 \leq t \leq \mu$.

$q_2(t)$:   On-hand inventory level at time t, where $\mu \leq t \leq T$.

K: Ordering cost per order.

C: Cost per unit item.

HC: Holding cost per cycle.

DC: Deterioration cost per cycle.

III. MATHEMATICAL MODEL

In this section we shall describe the mathematical models developed in this paper. It is assumed that the inventory level initially at time $t = 0$ is Q. At the beginning of each cycle, the inventory level decreases rapidly because the quantity demanded is greater at a higher level of inventory. The on-hand inventory level gradually falls to the level Q_1 at time $t = \mu$ due to demand. After time $t = \mu$, the stock level further decreases due to demand and deterioration. Ultimately, the inventory reaches to the zero level at the end of the cycle time T. The graphical representation of the model is given in figure1.

The differential equations governing the instantaneous

states of $q(t)$ over the cycle T can be written as follows:

$$\frac{dq_1(t)}{dt} = -D(q_1(t))^{\beta}, \text{ for } 0 \leq t \leq \mu, \tag{1}$$

$$\frac{dq_2(t)}{dt} + \theta q_2(t) = -D(q_2(t))^{\beta}, \text{ for } \mu \leq t \leq T. \tag{2}$$

With the conditions

$$q_1(0) = Q, \ q_1(\mu) = q_2(\mu) \text{ and } q_2(T) = 0. \tag{3}$$
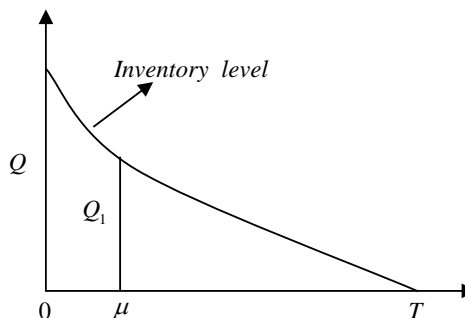

Fig. 1 The inventory system with inventory level-dependent demand rate.

Solving (2), we get

$$t = \frac{Q^{\alpha} - q_1^{\alpha}}{\alpha D}, \text{ where } \alpha = 1-\beta, \ 0 < \alpha < 1. \tag{4}$$

At $t = \mu$, let $q_1(\mu) = Q_1$ then

$$Q_1 = (Q^{\alpha} - \mu \alpha D)^{1/\alpha} \tag{5}$$

In a similar manner solving (2), we get

$$(T - t)\alpha\theta = \ln\left(1 + \frac{\theta}{D} q_2^{\alpha}\right).$$

On expansion of the right hand side, the first order approximation of θ gives,

$$t = T - \frac{q_2^{\alpha}}{\alpha D}(1 - \frac{\theta}{2D} q_2^{\alpha}) \tag{6}$$

Since $q_2(\mu) = Q_1$, the cycle time T is given by

$$T = \mu + \frac{Q_1^{\alpha}}{\alpha D}(1 - \frac{\theta}{2D} Q_1^{\alpha}), \text{ for } 0 < \alpha < 1. \tag{7}$$

A. Model 1 for non- linear time dependent holding cost

For this model, we assume that the cost of holding an amount dq of the item up to and including time t is given by $ht^n dq$, where $n \in Z^+\backslash\{1\}$, $h > 0$. This leads to

$$HC = \int_0^{Q_1} ht^n dq_1 + \int_{Q_1}^{Q} ht^n dq_2 \tag{8}$$

Substitution of (4) and (6) in (8) and evaluation of the above integrals in the right hand side, we get

$$HC = A(Q) + T^n B(Q) + \theta T^{n-1} C(Q) \tag{9}$$

where $A(Q) = \frac{hQ_1 Q^{n\alpha}}{\alpha^n D^n} \sum_{r=0}^{n}\left\{(-1)^r \frac{n!}{r!(n-r)!}\frac{1}{1+r\alpha}\left(\frac{Q_1}{Q}\right)^{r\alpha}\right\}$,

$$B(Q) = h \sum_{r=0}^{n} \left\{ (-1)^r \frac{n!}{r!(n-r)!} \frac{1}{(\alpha TD)^r} \frac{Q^{1+r\alpha} - Q_1^{1+r\alpha}}{1+r\alpha} \right\} \quad \text{and}$$

$$C(Q) = \frac{nh}{2\alpha D^2} \sum_{r=0}^{n-1} \left\{ (-1)^r \frac{(n-1)!}{r!(n-1-r)!} \frac{1}{(\alpha TD)^{\alpha}} \frac{Q^{1+(r+2)\alpha} - Q_1^{1+(r+2)\alpha}}{1+(r+2)\alpha} \right\}.$$

The deterioration cost in $(0, T)$ is given by,

$$DC = C\left(Q - \int_0^T Dq^{\beta} dt \right) \qquad (10)$$

Using (4) and (6) in (10) one can easily find

$$DC = \frac{C\theta}{(\alpha+1)D} \left(Q^{\alpha} - \mu\alpha D \right)^{\frac{\alpha+1}{\alpha}} \qquad (11)$$

The total relevant cost per unit time is, therefore, given by

$$TCU = \frac{K + HC + DC}{T} \qquad (12)$$

Our problem is to determine the order quantity Q^* which minimizes the TCU.

$$\text{Minimize } TCU(Q) = f(Q) + \theta\,g(Q) \qquad (13)$$
$$Q > 0.$$

Where

$$f(Q) = \frac{\alpha D}{Q^{\alpha}} (K + A(Q)) + \left(\frac{Q^{\alpha}}{\alpha D} \right)^{n-1} B(Q) \text{ and}$$

$$g(Q) = \frac{\alpha}{2} \left(\frac{Q_1}{Q} \right)^{2\alpha} (K + A(Q)) - \frac{(n-1)Q_1^{2\alpha}}{2\alpha D^2} \left(\frac{Q^{\alpha}}{\alpha D} \right)^{n-2} B(Q)$$

$$+ \left(\frac{Q^{\alpha}}{\alpha D} \right)^{n-2} C(Q) + \frac{\alpha D}{Q^{\alpha}} D(Q).$$

B. Model 2 for Non-linear stock-dependent holding cost

In this formulation, we treat the holding cost rate as a power function of the on-hand inventory; namely,

$$\frac{d(HC)}{dt} = hq^n, \text{ n > 1.} \qquad (14)$$

To derive the holding cost per order, we integrate (14) over time t between the limits $t = 0$ and $t = T$. Hence,

$$HC = \int_0^T hq^n dt \qquad (15)$$

Using (4) and (6) into (15) yields,

$$HC = h \left\{ \frac{Q^{n+\alpha}}{(n+\alpha)D} - \theta \frac{Q_1^{n+2\alpha}}{(n+2\alpha)D^2} \right\} \qquad (16)$$

Replacing this expression for HC in (12), the total relevant inventory cost per unit time is, therefore, given by

$$TCU(Q) = \frac{\alpha D}{Q^{\alpha}} \left(K + \frac{hQ^{n+\alpha}}{(n+\alpha)D} \right) + \theta$$

$$\left[\frac{\alpha D}{Q^{\alpha}} \left\{ \frac{CQ_1^{\alpha+1}}{(\alpha+1)D} - \frac{hQ_1^{n+2\alpha}}{(n+2\alpha)D^2} \right\} + \frac{\alpha}{2} \left(\frac{Q_1}{Q} \right)^{2\alpha} \left\{ K + \frac{hQ^{n+\alpha}}{(n+\alpha)D} \right\} \right]$$

IV. FUZZY MODEL AND SOLUTION PROCEDURE

When the deterioration rate becomes fuzzy, the function of

(13) can be redefined as

$$\text{Mini}\widetilde{m}\text{ize } TCU(Q) = f(Q) + \tilde{\theta}\,g(Q)$$
$$\text{where } Q > 0.$$

(wavy bar (~) denotes the fuzzification of the parameters.)

We express the fuzzy deterioration rate $\tilde{\theta}$ as the triangular fuzzy number $(\theta_0 - \delta_1, \theta_0, \theta_0 + \delta_2)$. Suppose, the membership function of the fuzzy deterioration rate $\tilde{\theta}$ is as follows:

$$\mu_{\tilde{\theta}}(\theta) = \begin{cases} \dfrac{\theta - \theta_0 + \delta_1}{\delta_1}, & \theta_0 - \delta_1 \leq \theta \leq \theta_0 \\[2mm] \dfrac{\theta_0 + \delta_2 - \theta}{\delta_2}, & \theta_0 \leq \theta \leq \theta_0 + \delta_2 \\[2mm] 0, & elsewhere \end{cases} \qquad (18)$$

Here, $0 < \delta_1 < \theta_0$, $0 < \delta_2$ and θ_0 are given fixed numbers. δ_1 and δ_2 are determined by the decision maker based on the given uncertainty. From equation (13), for each Q, let $G_Q(\theta) = f(Q) + \theta\,g(Q)$, $0 < \theta < 1$ and $y = G_Q(\theta)$. We then have $\theta = \dfrac{y - f(Q)}{g(Q)} \geq 0$ if $y \geq f(Q)$. By the Extension Principle [6], we have the following:

If $y < f(Q)$, then $\mu_{G_Q(\tilde{\theta})}(y) = 0$. If $y \geq f(Q)$ then

$$\mu_{G_Q(\tilde{\theta})}(y) = \sup_{\theta \in G_Q^{-1}(y)} \mu_{\tilde{\theta}}(\theta) = \mu_{\tilde{\theta}} \left(\frac{y - f(Q)}{g(Q)} \right)$$

$$= \begin{cases} \dfrac{y - f(Q) - (\theta_0 - \delta_1)g(Q)}{\delta_1 g(Q)}, & u_1 \leq y \leq u_0 \\[2mm] \dfrac{(\theta_0 + \delta_2)g(Q) - y + f(Q)}{\delta_2 g(Q)}, & u_0 \leq y \leq u_2 \\[2mm] 0, & elsewhere \end{cases}$$

where

$$\left. \begin{array}{l} u_1 = f(Q) + (\theta_0 - \delta_1)g(Q) \\ u_0 = f(Q) + \theta_0 g(Q) \\ u_2 = f(Q) + (\theta_0 + \delta_2)g(Q) \end{array} \right\} \qquad (19)$$

This leads to the following properties.

Property1. For each Q and δ_1, δ_2 which satisfy $0 < \delta_1 < \theta_0$, $0 < \delta_2$, the membership function of the fuzzy total cost $G_Q(\tilde{\theta})$ is

$$\mu_{G_Q(\tilde{\theta})}(y) = \begin{cases} \dfrac{y - f(Q) - (\theta_0 - \delta_1)g(Q)}{\delta_1 g(Q)}, & u_1 \leq y \leq u_0 \\[2mm] \dfrac{(\theta_0 + \delta_2)g(Q) - y + f(Q)}{\delta_2 g(Q)}, & u_0 \leq y \leq u_2 \\[2mm] 0, & elsewhere \end{cases} \qquad (20)$$

where $u_j, j = 1, 0, 2$ are given in equation (19).

We find the centroid of $\mu_{G_Q(\tilde{\theta})}(y)$ as

$$\int_{-\infty}^{\infty} \mu_{G_Q(\tilde{\theta})}(y)dy = \frac{1}{\delta_1 g(Q)} \int_{u_1}^{u_0} \left[y - f(Q) - (\theta_0 - \delta_1)g(Q)\right] dy$$

$$+ \frac{1}{\delta_2 g(Q)} \int_{u_0}^{u_2} \left[(\theta_0 + \delta_1)g(Q) - y + f(Q)\right] dy$$

$$= \frac{\delta_1 + \delta_2}{2} g(Q) = P \quad \text{(say)}, \quad (21)$$

and $\int_{-\infty}^{\infty} y\mu_{G_Q(\tilde{\theta})}(y)dy = \frac{\delta_1 + \delta_2}{2} g(Q).$

$$\times \left\{f(Q) + \theta_0 g(Q) + \frac{1}{3}(\delta_2 - \delta_1)g(Q)\right\}$$

$$= R, \quad \text{(say)}. \quad (22)$$

Property 2. The centroid of $\mu_{G_Q(\tilde{\theta})}(y)$ is

$$M(Q, \delta_1, \delta_2) = \frac{R}{P} = f(Q) + \theta_0 g(Q) + \frac{1}{3}(\delta_2 - \delta_1)g(Q), \quad (23)$$

where $Q > 0$, $0 < \delta_1 < \theta_0$, $0 < \delta_2$.

Remark 1. $M(Q, \delta_1, \delta_2)$ is the estimate of total cost in fuzzy sense. If $\delta_1 = \delta_2$, then for $\theta = \theta_0$ equation (13) is equal to $M(Q, \delta_1, \delta_2)$.

Proceeding in the same way as in the fuzzy Model – 1 given above, the value of Q^* and the corresponding values of T^* and TCU^* can be determined numerically for fuzzy Model – 2 corresponding to Model - 2 of section 3 also. Its derivation is not given here as it follows in a similar manner.

V. NUMERICAL EXAMPLE

In this section, a numerical example is considered to illustrate the models. The following values of parameters are used in the example:

$D = 2.0$, $C = \$10.0$ per unit, $h = \$0.5$ per unit, $K = \$200$ per order, $\theta_0 = 0.03$ $\delta_1 = 0.01$ and $\delta_2 = 0.015$. Tables 1 and 2 present the effects of the parameter β and non-linearity factor n on the approximate optimal solution (Q^*, T^*, TCU^*).

It is observed from Table - 1 and Table - 2 that,

(1) For a particular β, Q^* and T^* generally decrease but TCU^* increases as the degree of non-linearity in the holding cost increases.

(2) For a particular n, Q^* and T^* of Model - 1 are larger than those of Model - 2 as β increases;

(3) TCU^* is much higher when the non-linear stock-dependent holding cost is used in the inventory models.

VI. CONCLUSIONS

In this paper, fuzzy EOQ models in infinite time horizon by taking stock dependent demand rate, fuzzy deterioration rate and nonlinear holding cost are developed. The approximate optimal solutions of fuzzy models for two variations of holding cost namely a nonlinear function of the length of time

TABLE I EFFECTS OF β AND N ON (Q^*, T^*, TCU^*) FOR MODEL -1

n		β				
		0.1	0.3	0.5	0.7	0.9
2	Q^*	9.55	11.82	14.94	18.30	14.16
	T^*	6.53	6.35	6.21	6.49	10.71
	TCU^*	43.22	43.25	42.19	37.47	20.22
3	Q^*	5.78	6.85	8.33	10.02	8.24
	T^*	4.32	4.44	4.70	5.45	10.15
	TCU^*	60.22	57.57	52.87	43.44	21.31
4	Q^*	4.17	4.69	5.29	5.68	3.67
	T^*	3.27	3.44	3.77	4.61	9.38
	TCU^*	75.36	70.77	63.36	50.08	22.94
5	Q^*	3.35	3.60	3.78	3.56	1.63
	T^*	2.70	2.88	3.20	4.02	8.66
	TCU^*	87.93	82.01	72.65	56.40	24.72
6	Q^*	2.86	2.96	2.92	2.42	0.75
	T^*	2.36	2.52	2.82	3.59	8.01
	TCU^*	98.36	91.54	80.84	62.33	26.59
7	Q^*	2.54	2.55	2.38	1.75	0.35
	T^*	2.12	2.27	2.55	3.26	7.45
	TCU^*	107.10	99.70	88.06	67.84	28.51
8	Q^*	2.21	2.06	1.63	0.66	0.03
	T^*	1.88	1.97	2.12	2.45	4.62
	TCU^*	119.09	113.12	104.30	89.06	45.68
9	Q^*	2.19	2.10	1.81	1.10	0.10
	T^*	1.87	1.99	2.23	2.84	6.56
	TCU^*	118.71	110.92	98.53	76.57	32.12
10	Q^*	2.02	1.89	1.56	0.85	0.05
	T^*	1.74	1.85	2.07	2.63	6.13
	TCU^*	126.29	118.19	105.24	82.09	30.19

TABLE II EFFECTS OF β AND N ON (Q^*, T^*, TCU^*) FOR MODEL -2

n		β				
		0.1	0.3	0.5	0.7	0.9
2	Q^*	9.04	9.43	9.57	9.02	6.33
	T^*	6.25	5.48	5.02	5.28	9.89
	TCU^*	45.43	48.78	49.58	43.47	21.23
3	Q^*	5.01	5.04	4.95	4.60	3.49
	T^*	3.82	3.61	3.65	4.34	9.33
	TCU^*	67.33	67.89	63.75	50.70	22.16
4	Q^*	3.56	3.54	3.45	3.21	2.57
	T^*	2.85	2.84	3.06	3.90	9.05
	TCU^*	85.28	82.28	73.40	55.08	22.65
5	Q^*	2.86	2.82	2.74	2.57	2.13
	T^*	2.36	2.44	2.74	3.65	8.89
	TCU^*	99.74	93.24	80.29	58.00	22.95
6	Q^*	2.45	2.41	2.34	2.21	1.88
	T^*	2.06	2.19	2.53	3.49	8.78
	TCU^*	111.50	101.80	85.45	60.08	23.16
7	Q^*	2.18	2.15	2.09	1.98	1.72
	T^*	1.86	2.02	2.39	3.38	8.70
	TCU^*	121.20	108.65	89.43	61.64	23.31
8	Q^*	2.00	1.97	1.91	1.82	1.61
	T^*	1.72	1.90	2.29	3.30	8.64
	TCU^*	129.30	114.23	92.61	62.84	23.43
9	Q^*	1.86	1.83	1.79	1.71	1.53
	T^*	1.61	1.81	2.22	3.24	8.60
	TCU^*	136.17	118.87	95.20	63.81	23.52
10	Q^*	1.76	1.73	1.69	1.62	1.47
	T^*	1.53	1.74	2.16	3.19	8.56
	TCU^*	142.05	122.78	97.34	64.60	23.58

for which the item is held in stock and a non linear function of the amount of on-hand inventory are obtained and effect of non linearity is studied for a numerical example.

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