

Fuzzy Decision Making via Multiple Attribute

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Abstract—In this paper, a method for decision making in fuzzy environment is presented. A new subjective and objective integrated approach is introduced that used to assign weight attributes in fuzzy multiple attribute decision making (FMADM) problems and alternatives and finally ranked by proposed method.

Keywords—Multiple Attribute Decision Making, Triangular fuzzy numbers, ranking index, Fuzzy Entropy.

I. INTRODUCTION

IN Multiple Attribute Decision Making (MADM) problems, decision makers often confront a problem of electing among alternatives that have disagree attributes. Since human judgments and preferences are often vague and complex, and decision makers cannot appraise their preferences with an exact scale, we can only give linguistic evaluations instead of exact evaluations.

Multiple criteria decision making was introduced as a favorable and important area of study in the early 1970's. Since then the number of theories and models, which could be used as a basis for more methodical and reasoning decision making with multiple criteria, has continued to extend at a fixed rate. The number of reviews shows the dynamism of the area and as a result, the throng of methods which have been extended [3].

Weights of attributes assume the relative weightiness of the attributes must be assigned. There are many approaches to assign the weights of attributes. Criteria weights are assigned after numerous value tradeoff operations. Saaty [21] made the analytic hierarchy process (AHP) method by using pairwise comparison. Then a corresponding pairwise comparison matrix was established. Criteria weights are obtained by combining various evaluations in a methodical manner. The uncertainty and imprecision of the weighting operation are indirectly modeled. Takeda [23] further generalize this method to indicate the DM's uncertainty about the appraisals in the corresponding matrix. Laarhoven and Pedrycz [16], Buckley [8] extend this method to directly regard the uncertainty and inaccuracy of the pairwise comparison operation using fuzzy set theory. Some researchers think these methods may cause the rank inversion occurrence, and the computation involved can be absolutely complex and intricate when fuzzy numbers are used in the pairwise comparison operations. So, Von Winterfeldt and Edwards [26] propose a direct ranking and rating method. DMs first rank all criteria in the order of their

weightiness, and then give each criterion an appraised numerical value to indicate its relative weightiness. Criteria weights are obtained by normalizing these appraised values. Mareschal [18] and Fischer [13] use a mathematical programming model with sensitivity analysis to assign the intervals of weights, inside which the identical ranking result is produced. This method gives DMs flexibility in assessing criteria weights and helps them to understand much better how criteria weights influence the decision consequences, and reducing their cognitive burden in determining accurate weights. However, this operation may become boring and difficult to organize the number of criteria increasing. When Bellman and Zadeh [30], and a few years later Zimmermann [31], introduced fuzzy sets into a field, they introduced a way for a new kind of method to deal with problems which was impenetrable, and remote with standard MADM techniques [9].

MADM final step is ranking, where multitudes of fuzzy set ranking methods exist (Bortolan and Degani [7], Prodanovic [20]). Due to the complexity of the problem, a lot of attempts have been made to suggest a more acceptable approach for ranking of various alternatives in fuzzy environment. Because of the intricacy of notable and utilizing methods, a simple ranking method will be propounded. Therefore, lastly, alternatives are ranked by final method.

II. BACKGROUND INFORMATION

The fuzzy sets theory, introduced by Zadeh (1968) to deal with vague, imprecise and uncertain problems, has been applied as a modeling tool for complex systems that are hard to define precisely. Some basic definitions of fuzzy sets, fuzzy numbers and linguistic variables which are presented by Buckley (1985) and Kaufmann and Gupta (1991) will be reviewed.

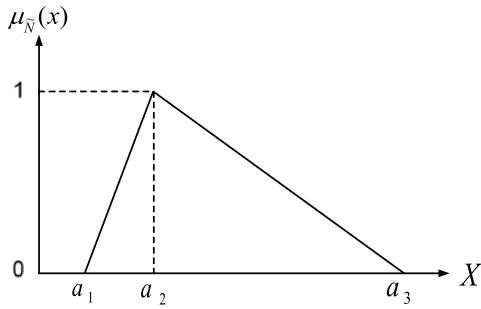
Definition 1: A fuzzy set \tilde{N} in a universe of discourse X is characterized by a membership function $\mu_{\tilde{N}}(x)$ which associates with each element x in X , a real number in the interval $[0,1]$. The function value $\mu_{\tilde{N}}(x)$ is termed the grade of membership of x in \tilde{N} . (Bellman and Zadeh (1970)).

Definition 2: A fuzzy number is a fuzzy subset of the universe of discourse X that is both convex and normal. Figure (1) shows a fuzzy number \tilde{N} . In the universe of discourse X that conforms to this definition (Zadeh (1965)).

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Fig. 1 a triangular fuzzy number \tilde{N}

We also use triangular fuzzy numbers. A triangular fuzzy number \tilde{N} can be defined by a triplet (a_1, a_2, a_3) . Its conceptual schema and mathematical form is shown by (1).

$$\mu_{\tilde{N}}(x) = \begin{cases} 0 & x \leq a_1; \\ \frac{x-a_1}{a_2-a_1} & a_1 < x \leq a_2; \\ \frac{a_3-x}{a_3-a_2} & a_2 < x \leq a_3; \\ 0 & x > a_3; \end{cases} \quad (1)$$

Where (a_1, a_2, a_3) denote as left hand number, middle number and right hand number of \tilde{N} respectively.

Definition 3: Assuming that both $\tilde{N} = (a_1, a_2, a_3)$ and $\tilde{M} = (b_1, b_2, b_3)$ are fuzzy numbers and c is positive real number, then the basic operations such as multiplication, addition, distance, maximum and minimum on fuzzy triangular numbers are defined as follows respectively (Zadeh (1965)).

$$c \times \tilde{N} = (c \times a_1, c \times a_2, c \times a_3)$$

$$\tilde{N} + \tilde{M} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$d(\tilde{N}, \tilde{M}) = \frac{a_1 + 2a_2 + a_3}{4} - \frac{b_1 + 2b_2 + b_3}{4}$$

$$\text{Max}\{(a_i, b_i, c_i)_{i=1, \dots, n}\} = (\text{max}(a_i), \text{max}(b_i), \text{max}(c_i))$$

$$\text{Min}\{(a_i, b_i, c_i)_{i=1, \dots, n}\} = (\text{min}(a_i), \text{min}(b_i), \text{min}(c_i)) \quad (2)$$

Definition 4: when we consider a variable, in general, it takes numbers as its value. If the variable takes linguistic terms, it is called linguistic variable (Zadeh (1975)). The concept of a linguistic variable is very useful to describe situations that are too complex or not well defined in conventional quantitative expressions. For example, "temperature" is a linguistic variable which contains the values like freeze, cold, cool, hot, very hot and etc, where it is defined as linguistic term.

III. PROPOSED METHOD

Assume m alternatives of $A_i, i = 1, \dots, m$ be evaluated against n criteria $C_j, j = 1, \dots, n$. All elements of decision

matrix are fuzzy numbers and demonstrate by $(y_{ij}^l, y_{ij}^m, y_{ij}^r)$. These elements are achieved by the brainstorming techniques by decision makers. Because all values of decision matrix have not same scale measurement, we have to normalize them.

TABLE I DECISION MATRIX FILLED BY TFNS

Criteria	C_1	...	C_n
Alternatives			
A_1	$(y_{11}^l, y_{11}^m, y_{11}^r)$...	$(y_{1n}^l, y_{1n}^m, y_{1n}^r)$
\vdots	\vdots		\vdots
A_m	$(y_{m1}^l, y_{m1}^m, y_{m1}^r)$...	$(y_{mn}^l, y_{mn}^m, y_{mn}^r)$

Implementation of the proposed method is dependent on the following steps:

Step 1: Some of the criteria have positive concepts, thus decision makers (DM) want to increase them (e.g. productivity). In contrast some of criteria have negative concept where DM would like to decrease them (e.g. cost). We normalize any columns separately. If j^{th} criterion of decision matrix has positive concept, then i^{th} row at the j^{th} column element of decision matrix is normalized by below equation :

$$\left(\frac{y_{ij}^l}{\text{Max}_{i \in (1, 2, \dots, m)} y_{ij}^l}, \frac{y_{ij}^m}{\text{Max}_{i \in (1, 2, \dots, m)} y_{ij}^m}, \frac{y_{ij}^r}{\text{Max}_{i \in (1, 2, \dots, m)} y_{ij}^r} \right) \quad (3)$$

Conversely, if j^{th} criterion of decision matrix has positive concept, then i^{th} row at the j^{th} column element of decision matrix is normalized by following relation:

$$\left(\frac{\text{Min}_{i \in (1, 2, \dots, m)} y_{ij}^l}{y_{ij}^l}, \frac{\text{Min}_{i \in (1, 2, \dots, m)} y_{ij}^m}{y_{ij}^m}, \frac{\text{Min}_{i \in (1, 2, \dots, m)} y_{ij}^r}{y_{ij}^r} \right) \quad (4)$$

Step 2: this step tries to give a new weight determination approach to retain the merits of both subjective and objective approaches (Tien 2009, Hobbs 1980): to assign weights by solving mathematical models automatically and at the same time taking into account the decision maker's preferences:

Step 2.1: objective weights (w_j^o): The objective modes select weights through mathematical calculations, which quit subjective judgment information of decision makers. Entropy theory is another important theory to study the problem of uncertainty. Entropy weight is a parameter that clarifies how much diverse alternatives approach one to another in respect to a certain attribute. The greater the value of the entropy, the smaller the entropy weight. then the smaller the differences of diverse alternatives in this specific attribute, and the less information the specific attribute affords, and the less important this attribute becomes in the decision making operation. In this paper we give a Fuzzy Entropy Weight, while for fuzzy numbers could not use the crisp formula to calculate the entropies of fuzzy numbers directly. Generally, we first transform the fuzzy numbers into crisp numbers, and

then calculate their respective entropies. So, we transform all elements of normalized matrix in crisp to obtain x_{ij} (as elements). Thereafter, ratio of x_{ij} is computed according to the following equation and notified by f_{ij} :

$$f_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad (5)$$

Then, the fuzzy entropies of the j^{th} attributes can be calculated with the following:

$$\bar{E}_j = -K \cdot \sum_{i=1}^m f_{ij} \times \ln(f_{ij}) = -\frac{1}{\ln(m)} \sum_{i=1}^m f_{ij} \times \ln(f_{ij}) \quad (6)$$

Now, the objective weight of j^{th} attribute is calculated with the following equation:

$$W_j^o = \frac{1 - \bar{E}_j}{\sum_{j=1}^n (1 - \bar{E}_j)} \quad (7)$$

Step 2.2 : subjective weights (w_j^s) : Weights assigned by subjective modes can specify the subjective judgments of decision makers, thus makes the rankings of alternatives in Fuzzy MADM problem have more discretionary factors. For calculating the subjective weights, it is needed a linguistic value to each criterion. Note that, they are values of importance linguistic variable. The corresponding linguistic values of the i^{th} criterion are denoted simply as MFC_i . Reciprocal subtraction of each criterion defined by:

$$RS_i = \sum_{j=1}^n d(MFC_i, MFC_j) \quad (8)$$

As defined in definition 3, $d(MFC_i, MFC_j)$ states the fuzzy distance relation. Therefore, subjective weight of i^{th} criterion achieve by relation (9):

$$w_i^s = \frac{a^{RS_i}}{\sum_{j=1}^n a^{RS_j}} \quad (9)$$

The parameter a is greater and not equal than 1. If it is equal 1, subjective weights achieve in same value.

Step 2.3: Calculation of the combined weights of attributes: Derive the combined weight of j^{th} attribute by geometric average according to:

$$w_j = (w_j^o)^\alpha \times (w_j^s)^\gamma \quad (10)$$

Where α and γ represent the relative weightiness of the objective weights and the subjective weights to decision makers respectively, such $\alpha + \gamma = 1$. Combined fuzzy weight is such as marker that not only shows how much important an attribute is to the decision maker, but also shows how much various of attribute are in various alternatives.

Step 3: Weighting the normalized matrix: At this stage, we multiplied normalized matrix in weight vector. M-times TFNs are resulted by this multiplication. In fact, the results show the value of each alternative.

Step 4: Ranking: when the values of each alternative are obtained, we must rank them. Several techniques exist in literature to rank the fuzzy numbers. To do this, this section proposes a novel method for ranking of fuzzy numbers based on the distance of numbers value to their minimum and maximum (see details in (3)).

Let (a_i, b_i, c_i) , $i = 1, \dots, n$ be the fuzzy numbers. To define the index of each alternative first we obtain the distance of each alternative value from maximum and minimum of all alternatives. The unique point of this method is that the number is more important, if its distance is higher than minimum and is lower than maximum value, simultaneously. In contrast, we must obtain the average of per fuzzy number where it is acquired by (11):

$$\tilde{x}_{(a_i, b_i, c_i)} = \frac{(a_i + b_i + c_i)}{3} \quad (11)$$

As mentioned above, Equation (12) will be considered as the ranking index in which the larger value of index is the better ranking of each fuzzy number.

$$Index_{(a_i, b_i, c_i)} = \frac{d((a_i, b_i, c_i), (a_{\min}, b_{\min}, c_{\min}))}{d((a_{\max}, b_{\max}, c_{\max}), (a_i, b_i, c_i))} \times \tilde{x}_{(a_i, b_i, c_i)} \quad (12)$$

In next stage, we use the proposed method in a condensation case study.

IV. NUMERICAL EXAMPLE

In an effort to study the tourist attraction, a regular survey is usually done in the four famous place of the Iran: TakhteJamshid in Shiraz (A_1), GhareAlisadr in Hamadan (A_2), Sio'sepol in Esfahan (A_3) and Bis'toon in Kermanshah (A_4). They are several of Iran's greatest tourist attractions, so we are now going to select the best case for tourist attraction (A_i : $i = 1, 2, 3, 4$) based on three criteria (C_j : $j = 1, 2, 3$) as popularity (C_1), climate (C_2) and cost (C_3). Figure 2 shows each fuzzy linguistic term with its correspondent fuzzy number for each criterion. Note that, either Positive or negative concept of each criterion is included in following figures.

TABLE II A

FUZZY LINGUISTIC TERMS FOR CRITERION C_1 AND C_2

Importance	Abbreviation	Fuzzy Number
Very low	VL	(0,0,1)
Low	L	(0,1,3)
Medium low	ML	(1,3,5)
Medium	M	(3,5,7)
Medium high	MH	(5,7,9)
High	H	(7,9,10)
Very high	VH	(9,9,10)

TABLE II B
FUZZY LINGUISTIC TERMS FOR CRITERION C_3

Importance	Abbreviation	Fuzzy Number
Very low	VL	(0,0,50)
Low	L	(50,150,200)
Medium low	ML	(50,150,200)
Medium	M	(150,200,250)
Medium high	MH	(200,250,350)
High	H	(250,350,400)
Very high	VH	(350,350,400)

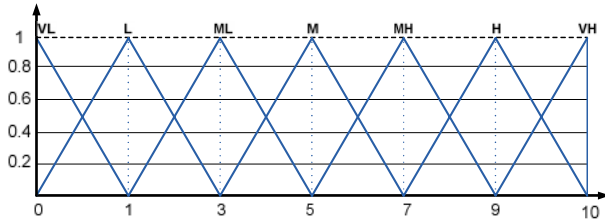


Fig. (2.A).Fuzzy linguistic terms for criterion C_1 and C_2

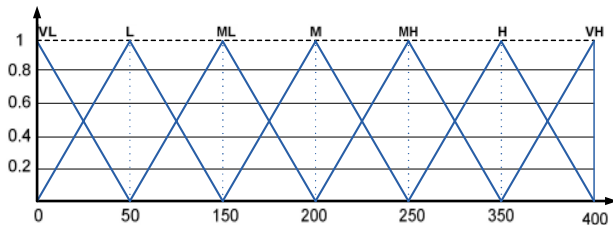


Fig. (2.B).Fuzzy linguistic terms for criterion C_3

Fig. 2 Fuzzy linguistic terms for each criterion

Decision maker completes the decision matrix based on himself/herself idea and fuzzy linguistic terms (See Figure 2). But, same scaling of decision matrix's elements, achieved decision maker ideas are transformed into normalized matrix. The decision matrix and its normalization are shown in Table 3. Note that, **step 1** is performed by this process.

TABLE III DECISION MATRIX AND ITS NORMALIZATION

	Alternative	C1	C2	C3
Decision matrix	A_1	(7,9,10)	(1,3,5)	(200,250,350)
	A_2	(1,3,5)	(9,9,10)	(200,250,350)
	A_3	(3,5,7)	(5,7,9)	(50,150,200)
	A_4	(5,7,9)	(1,3,5)	(150,200,250)
Normalized decision matrix	A_1	(0.7,0.9,1)	(0.1,0.3,0.5)	(0.1,0.2,0.3)
	A_2	(0.1,0.3,0.5)	(0.9,0.9,1)	(0.1,0.2,0.3)
	A_3	(0.3,0.5,0.7)	(0.5,0.7,0.9)	(0.3,0.3,1)
	A_4	(0.5,0.7,0.9)	(0.1,0.3,0.5)	(0.2,0.3,0.3)

In multi attribute decision making (MADM), criteria have not same importance. So, next stage must assign an appropriate weight for each criterion. According to **Step 2.1**, criteria weighting by Entropy theory ignore decision maker's judgment about each criterion. This means that obtained objective weights are less often optional. Achieved entropy of each criterion and corresponding objective weights, are shown in Table IV:

TABLE IV
ENTROPY AND OBJECTIVE WEIGHTS OF EACH CRITERION BASED ON NORMALIZED MATRIX'S ELEMENTS

Criterion	C_1	C_2	C_3
Entropy value	0.95	0.91	0.93
Objective weight	0.24	0.42	0.34

It is ran **Step 2.2** for specifying the subjective judgments of decision maker. Hence, Table V shows linguistic values of the i^{th} criterion which are used simply as MFC_i . According to the approach of **Step 2.2**, reciprocal subtraction matrix and also obtained subjective weights are calculated systematically. Results are shown in Table VI.

TABLE V
LINGUISTIC TERMS AND THEIR MEMBERSHIP FUNCTION.

Linguistic terms	Abbreviation	Membership function
Extremely unimportant	EU	(1,1,3)
Very unimportant	VU	(1,3,5)
Important	I	(3,5,7)
Very important	VI	(5,7,9)
Extremely important	EI	(7,9,9)

TABLE VI
RECIPROCAL SUBTRACTIONS AND SUBJECTIVE WEIGHTS

	C_1	C_2	C_3	RS_i	w_j^s
C_1	0	3.5	5.5	9	0.84
C_2	-3.5	0	2	-1.5	0.12
C_3	-5.5	-2	0	-7.5	0.04

It is combined weight of j^{th} attribute by geometric average according to the (8). In view of **Step 3**, achieved weights affect on normalized matrix. Finally in **Step 4**, we use equation 10 for ranking the fuzzy numbers that are acquired in the previous step. As mentioned before, the ranking order depends on two parameters of W_j^T . Accordingly, we used a different value of α to identify which subjective and objective weights of criteria is mostly impact. Table VII gives the total weights of each criteria and various ranking of each alternative based on different values of α .

TABLE VII
TOTAL WEIGHTS OF EACH CRITERIA AND FINAL RANKING WITH REGARD TO DIFFERENT α

α	W_1	W_2	W_3	Rank			
				A_1	A_2	A_3	A_4
0.10	0.74	0.14	0.05	1	2	4	3
0.23	0.63	0.16	0.07	1	2	4	3
0.31	0.57	0.18	0.08	1	2	4	3
0.76	0.33	0.31	0.20	2	1	4	3
0.87	0.28	0.36	0.26	2	1	4	3

As demonstrated in table 10, rank of A_3 and A_4 is constant, but with emphasis on subjective judgment A_1 is the better alternative than A_2 . Hence, with ignorant to decision maker's idea A_2 is the better alternative than A_1 .

V.CONCLUSION

The main purpose of this paper is to develop a fuzzy based method to select information systems appropriately for an organization from available alternatives. In this regard, a novel approach is proposed for solving MADM problems in a fuzzy environment. To determine the performance of the proposed method, we apply the proposed method in a brief case study in the tourist attraction field. Most MADM approaches regard only decision maker's subjective weights but in this manuscript illustrated an integrated approach of objective and subjective weighting to lead the simplicity of decision making. A simple index is defined to determine the ranking order of alternatives by calculating the distances to Maximum and Minimum of fuzzy numbers. This ranking is impressed by different combinations of subjective and objective weights. So, Alternatives ranking is studied in different combinations with respect to random values of α .

REFERENCES

- [1] S. Abbasbandy, B. Asady, "Ranking of fuzzy numbers by sign distance," *Information Science*, 176, 2405-2416, 2006.
- [2] A. Azar, *Fuzzy management science*; IMPSC publication, Iran, 2008, pp 238-358.
- [3] C. A. Bana, P. Vincle, "Multiple Criteria Decision Aid: an overview," *Readings in Multiple Criteria Decision Aid*, Springer Verlag, Berlin – Heidelberg, 1990.
- [4] M. Barkhordary, T. Allahviranloo, T. Hajjari, (2007). "Novel ranking method of fuzzy numbers," in *First Joint Congress on Fuzzy and Intelligent Systems*, Ferdowsi University of Mashhad, Iran 29-31, 2007.
- [5] F. H. Barron, B.E. Barreh, "Decision quality using ranked attribute weights," *Management science* 42, 1515-1523, 1996.
- [6] M. Bender, S. P. Simonovic, "A fuzzy compromise approach to water resource structures planning under uncertainty," *Fuzzy Sets and Structures* 115: 35-44, 2000.
- [7] G. Bortolan, R. Degam, "A review of some methods for ranking fuzzy subsets," *Fuzzy Sets System*, 1985, pp 15-19.
- [8] J. J. Buckley, "Ranking alternatives using fuzzy numbers," *fuzzy sets and structures* 15, 1985.
- [9] D. Y. Chang, "Fuzzy analytic hierarchy process; extend analysis method," *Fuzzy sets and systems*, no. 12, 1996, pp 43-51.
- [10] L. Cheng, B. Peng, "The Fuzzy Relation Equation with Union or Intersection Preserving Operator," *Fuzzy Sets and Structures*, no. 25, 1988, pp. 191-204.
- [11] J. Drewniak, "Fuzzy Relation Calculus," *UnivSlaski, Katowice*, 1989.
- [12] E. Fernandez, R. Olmedo, "An agent model based on ideas of concordance and discordance for group ranking problems," *Decision support structures*, vol 39(3), 2005.
- [13] G. W. Fischer, "Range sensitivity of attribute weights in multi-attribute value model," *organizational behavior & human decision operations* 62, 1985.
- [14] B. Hobbs, "A comparison of weighting methods in power plant citing," *decision sciences*, vol. 11, 1980.
- [15] F. Kong, H. Liu, "Fuzzy RBF neural network model for multiple attribute decision making," *Lecture notes in computer science*, 2006.
- [16] P. J. M. Laarhoven, W. Pedrycz, "A fuzzy extension of saaty's priority theory," *Fuzzy sets and structures*, vol. 11, 1983.
- [17] T. S. Liou, M. J. Wang, "Ranking fuzzy numbers with integral value," *Fuzzy Sets and Structures*, vol. 50, 1992, pp. 247-255.
- [18] B. Mareschal, "Weight stability intervals in multicriteria decision Aid," *European Journal of operational Research*, vol. 33, 1988.
- [19] M. R. Mehregan, H. Safari, "Combination of fuzzy TOPSIS and fuzzy ranking for multi attribute decision making," *Lecture notes in computer science*, 2006.
- [20] P. Prodanovic, "Fuzzy set ranking methods and multiple expert decision making," *project report*, 2002.
- [21] T. L. Saaty, "The analytic hierarchy process," *McGraw– Hill*, New York, 1980.
- [22] M. Sakawa, "Fuzzy sets and interactive multiobjective optimization," *applied, information Technology*, plenum Pres, New York, 1993.
- [23] E. Takeda, K. O. Cogger, P.L. Yu, "Appraising criterion weights using eigenvectors: A comparative study," *European Journal of operational Research*, vol. 29, 1998.
- [24] Tien-Chin Wang, Hsien-Da Lee, "Developing a fuzzy TOPSIS approach based on subjective weights and objective weights," *Expert Systems with Applications*, vol. 36, 2009, pp. 8980–8985.
- [25] A. Valls-Mateu, "ClusDM: A multiple criteria decision making method for heterogeneous data sets," *Polytechnic University of Catalonia*, 2002.
- [26] W. D. Von, W. Edwards, "Decision Analysis and Behavioral research," *Cambridge university Press*, London, 1986.
- [27] Y. J. Wang, H. S. Lee, "Generalizing TOPSIS for fuzzy multiple-criteria group decision making," *Computers and Mathematics with Applications*, vol. 53, 2007, pp. 1762–1772.
- [28] J. B. Yang, Y. M. Wang, D. L. XU, et al, "The evidential reasoning approach for MADA under both probabilistic and fuzzy uncertainties" *European journal of operational research*, vol. 171(1), 2006.
- [29] C. H. Yeh, H. P. Deng, "A practical approach to fuzzy utilities comparison in fuzzy multicriteria analysis," *International journal of approximate reasoning*, vol. 35(2), 2004.
- [30] L. A. Zadeh, "Fuzzy sets, information and control" vol. 8, 1965, pp. 338-353.
- [31] H. J. Zimmerman, "Fuzzy set theory and its applications," *Third Edition*, Kluwer Academic, 1996.