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Frequency-Variation Based Method for Parameter Estimation of Transistor Amplifier

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Abstract—In this paper, a frequency-variation based method has been proposed for transistor parameter estimation in a common-emitter transistor amplifier circuit. We design an algorithm to estimate the transistor parameters, based on noisy measurements of the output voltage when the input voltage is a sine wave of variable frequency and constant amplitude. The common emitter amplifier circuit has been modelled using the transistor Ebers-Moll equations and the perturbation technique has been used for separating the linear and nonlinear parts of the Ebers-Moll equations. This model of the amplifier has been used to determine the amplitude of the output sinusoid as a function of the frequency and the parameter vector. Then, applying the proposed method to the frequency components, the transistor parameters have been estimated. As compared to the conventional time-domain least squares method, the proposed method requires much less data storage and it results in more accurate parameter estimation, as it exploits the information in the time and frequency domain, simultaneously. The proposed method can be utilized for parameter estimation of an analog device in its operating range of frequencies, as it uses data collected from different frequencies output signals for parameter estimation.

Keywords—Perturbation Technique, Parameter estimation, frequency-variation based method.

I. INTRODUCTION

ANALOG circuit designing involves equivalent modelling of various analog devices and simulations. The equivalent models require to perform as close as possible to the actual physical models of analog circuits, for various applied input signals of varying voltages, currents and frequencies, under changing environmental conditions. Variation of device parameters in view of applied signal variations and environmental conditions (such as variation of temperature) cannot be ruled out. If, the variation in devices parameters is ignored during the circuit design process, then the circuit model and simulation results would not match with the performance of actual physical circuit. Therefore analog devices parameter estimation is of great importance for analog circuit designing and simulations. Various techniques for analog devices parameters estimation have been presented in existing literatures such as parameter estimation based on vectorial large-signal measurements [4], concurrent global optimization technique [2], waveform based parameter estimation [5], analytical approach [6] , using wavelet transform [1] and so on. In [8] combinatorial algorithms for BJT (Bipolar Junction Transistor) parameter extraction have been presented. V.B. vats et al. in [7] have presented BJT parameter estimation from state analysis and simulations. Sudipta Majumdar et al. in [3] have presented wavelet based transistor parameter estimation. In this paper we present a parameter estimation method which utilizes noisy measured variable frequency output voltages to estimate transistor parameters. As compared to least squares method of parameter estimation, our proposed method results in improved accuracy of parameter estimation as it takes into consideration the information in time and frequency domain simultaneously. Although, the transistor is represented by Ebers-Moll current equations in the model of common emitter amplifier circuit yet only linear part of each equation is considered in order to obtain linearized transistor model. As the thermal voltage is more affected by thermal fluctuations than the other parameters such as $I_{C0}, I_{E0}, \alpha_r, \alpha_f$, we have estimated the thermal voltage V_T. In the same way other parameters can also be estimated. The proposed method for parameter estimation involves minimization of the squared error energy function calculated by varying the input frequencies.

The proposed method applies K.A.M.(Kolmogorov Arnold Moser) technique, during the process of parameter estimation. A brief introduction of the K.A.M. technique is as follows: K.A.M. is the abbreviation of the famous theorem on averaging for quasi-periodic motion due to Kolmogorov Arnold and Moser. They used the technique to analyze the trajectories of 'n' body gravitational problem. Averaging can be used to determine qualitative characteristics of the motion, like, the average time spent by the trajectories inside a set. In our paper, we use K.A.M. method to calculate the Fourier components averaging of the output waveform at any given frequency. If the output has the waveform: $y(t) = \sum_{k=1}^{m} A_k e^{j\omega_k t}$, where, $\omega_1, \omega_2, ..., \omega_m$ are incommensurate, then y(t) is quasi-periodic signal and application of K.A.M. gives, $A_k = \lim_{T \longrightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} y(t) e^{-j\omega_k t} dt$. This paper has been organized in five sections. In section (2) we have developed frequency-variation based method for transistor parameters estimation. In section (3) this technique has been applied to the common emitter transistor amplifier circuit, in order to estimate the transistor parameters. Section (4) presents simulations and discussions. In section (5) conclusions are presented.

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II. PARAMETER ESTIMATION USING FREQUENCY-VARIATION BASED METHOD

An analog circuit can be represented by a system of multivariable state equations. The system of equations consists of r

first order differential equations and $n-r$ algebraic equations for the n state variables $x_i, i = 1, 2, ..., n$. These can be expressed as

$$x_i' = \sum_{j=1}^{n} a_{ij}x_j + \epsilon f_i(\mathbf{x}) + u_i(t), 1 \le i \le r$$

$$\sum_{j=1}^{n} a_{ij}x_j + \epsilon f_i(\mathbf{x}) = 0, r+1 \le i \le n$$

Equivalently, writing

$$\mathbf{x}_1 = (x_i)_{1\le i\le r}, \mathbf{x}_2 = (x_i)_{r+1\le i\le n}$$

The above equations can be expressed as

$$\mathbf{x}_1' = \mathbf{A}_{11}\mathbf{x}_1 + \mathbf{A}_{12}\mathbf{x}_2 + \epsilon\mathbf{f}_1(\mathbf{x}) + \mathbf{u}_1$$

$$\mathbf{A}_{21}\mathbf{x}_1 + \mathbf{A}_{22}\mathbf{x}_2 + \epsilon\mathbf{f}_2(\mathbf{x}) = \mathbf{0}$$

These can be expressed in the following form:

$$\mathbf{T}\mathbf{x}' + \mathbf{C}\mathbf{x} = \epsilon\mathbf{f}(\mathbf{x}) + \mathbf{u}(t)$$

where

$$\mathbf{T} = \begin{pmatrix} \mathbf{I}_r \\ \mathbf{0}_{n-r\times r} \end{pmatrix}$$

$$\mathbf{C} = -\mathbf{A} = -\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix}$$

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} \mathbf{f}_1(\mathbf{x}) \\ \mathbf{f}_2(\mathbf{x}) \end{pmatrix}$$

After ignoring the nonlinear part and having put $\mathbf{u}(t) = \mathbf{b}_0 + \mathbf{b}_1 w(t)$, the differential equations for an analog circuit (transistor amplifier circuit in our application example) can be put in the following state variable form.

$$\mathbf{T}(\theta)\mathbf{x}'(t) + \mathbf{C}(\theta)\mathbf{x}(t) = \mathbf{b}_0 + \mathbf{b}_1 w(t) \quad (1)$$

where the parameter vector θ is given by

$$\theta = (V_T, I_{C0}, I_{E0}, \alpha_r, \alpha_f) \quad (2)$$

$w(t)$ is the input voltage signal to the amplifier and we shall assume it to be a sine wave, i.e.,

$$w(t) = A.cos(\omega t) \quad (3)$$

The output of the amplifier (which we take as the voltage across the load) has the form

$$y(t) = \mathbf{c}^T\mathbf{x}(t) \quad (4)$$

We assume that $\mathbf{T}$ and $\mathbf{C}$ are $d \times d$ matrices while $\mathbf{x}(t), \mathbf{b}_0, \mathbf{b}_1$ and $\mathbf{c}$ are $d \times 1$ vectors. The steady state state-variables have the form

$$\mathbf{x}(t) = \mathbf{k} + Re(\mathbf{z}.exp(j\omega t) \quad (5)$$

where $\mathbf{k}$ is a real vector and $\mathbf{z}$ is a complex vector, both depending on the parameter vector θ. Substituting this solution into the state equations give us

$$\mathbf{T}.Re(j\omega\mathbf{z}.exp(j\omega t)) + \mathbf{C}\mathbf{k} + \mathbf{C}.Re(\mathbf{z}.exp(j\omega t))$$

$$= \mathbf{b}_0 + \mathbf{b}_1 Re(A.exp(j\omega t)) \quad (6)$$

Equating the dc terms and the ac terms separately on both sides, we get

$$\mathbf{C}\mathbf{k} = \mathbf{b}_0, j\omega\mathbf{T}\mathbf{z} + \mathbf{C}\mathbf{z} = A\mathbf{b}_1 \quad (7)$$

so that

$$\mathbf{k} = \mathbf{C}^{-1}\mathbf{b}_0, \mathbf{z} = A(j\omega\mathbf{T} + \mathbf{C})^{-1}\mathbf{b}_1 \quad (8)$$

The output is thus given by

$$y(t) = \mathbf{c}^T(\mathbf{k} + Re(\mathbf{z}.exp(j\omega t)))$$

$$= \mathbf{c}^T\mathbf{C}^{-1}\mathbf{b}_0 + A.Re(\mathbf{c}^T(j\omega\mathbf{T} + \mathbf{C})^{-1}\mathbf{b}_1 exp(j\omega t)) \quad (9)$$

We now apply K.A.M. technique, in which both sides of the above equation are multiplied by $e^{-j\omega t}$and a time average is taken. We find that with $< . >$ denoting time average,

$$< y(t) >= \mathbf{c}^T\mathbf{C}^{-1}\mathbf{b_0} \quad (10)$$

$$< y(t).exp(-j\omega t) >= \frac{1}{2}\mathbf{A}\mathbf{c}^T(j\omega\mathbf{T} + \mathbf{C})^{-1}\mathbf{b_1} \quad (11)$$

Thus we have one real equation and one complex equation which amount in all to three real equations for four parameters. We can vary the frequency and get more equations from which, using a least squares approach combined with the gradient algorithm for function minimization, we can estimate the parameter vector θ as follows:

$$\hat{\theta} = Argmin_\theta(| < \hat{v}(t) > -\mathbf{c}^T\mathbf{C}(\theta)^{-1}\mathbf{b_0}|^2$$

$$+| < \hat{v}(t).exp(-j\omega t > -\mathbf{c}^T(j\omega\mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1}\mathbf{b_1}|^2)$$

Here, energy function $E(\omega, \theta)$ is written as follows:

$$E(\omega, \theta) = (| < \hat{v}(t) > -\mathbf{c}^T\mathbf{C}(\theta)^{-1}\mathbf{b_0}|^2$$

$$+| < \hat{v}(t).exp(-j\omega t > -\mathbf{c}^T(j\omega\mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1}\mathbf{b_1}|^2)$$

Which is the error corresponding to the measurement at frequency ω. The minimization of $E(\omega, \theta)$ is carried out using the gradient search algorithm as follows:

$$\hat{\theta}_{n+1} = \hat{\theta}_n - \mu\sum_{k=1}^{N}\frac{\partial E(\omega_k, \theta_n)}{\partial\theta}$$

In other words, we select a sequence of frequencies $(\omega)_{k=1}^{N}$ and minimize $\sum_{k=1}^{N} E(\omega_k, \theta)$, the total sum of errors squared over every measured frequency using gradient search algorithm.

In our simulations, we have computed the time averages by sampling the signal and summing over a finite number of samples. Thus, if the sampling time interval is Δ, we replace $< y(t) >$ by $\frac{1}{N}\sum_{n=0}^{N-1} y(n\Delta)$ and $< y(t).exp(-j\omega t) >$ by $\frac{1}{N}\sum_{n=0}^{N-1} y(n\Delta)exp(-j\omega n\Delta)$. The sampling interval Δ and the number of samples N must be chosen appropriately. for example, if the 3 dB bandwidth of $y(t)$ is $[-\sigma, +\sigma]$ radians per second, then Δ may be chosen as one hundredth of $min(\sigma^{-1}, \omega^{-1})$ and N may be chosen so that $N\Delta$ equals the duration over which the signals are simulated.

A. *Gradient algorithm for minimizing a function of several variables*

Suppose $E(\theta_1, ..., \theta_n)$ is the function of the n variables $\theta_1, ..., \theta_n$ that is to be minimized. Ideally, the local minima can be found as solutions to the system

$$E_{,i}(\theta) = 0, ((E_{,ij}(\theta)))_{1 \le i,j \le n} \ge \mathbf{0} \quad (12)$$

In vector notation, these can be expressed as

$$\nabla E(\theta) = 0, \nabla \nabla^T E(\theta) \ge 0 \quad (13)$$

where

$$\nabla = (\frac{\partial}{\partial \theta_1}, ..., \frac{\partial}{\partial \theta_n})^T$$

The gradient algorithm can be expressed as

$$\theta[n+1] = \theta[n] - \mu[n] \nabla E(\theta[n]), n = 0, 1, ... \quad (14)$$

with $\mu[n]$ a positive constant or a slowly varying parameter called the adaptation constant. If $\parallel \theta[n+1] - \theta[n] \parallel$ is large, then $\mu[n+1]$ can be made large while if $\parallel \theta[n+1] - \theta[n] \parallel$ is small, then $\mu[n+1]$ is made small.

B. *Parameter estimation using time-domain least squares method*

In the time-domain least squares method, in the process of the estimating the parameters, we collect many time-sample values from an output waveform and then the total squared error between the output time samples and the predicted values is minimized over all possible parameters.

Let the amplifier output voltage be denoted by $v_o(t)$. This can be represented as follows:

$$v_o(t) = \int h(\tau, \theta) v_i(t) d\tau$$

Where $v_i(t)$ is input voltage. Now ignoring the d.c. component of the output voltage(d.c. is filtered by the coupling capacitor),from (9),

$$H(j\omega, \theta) = \mathbf{c}^T (j\omega \mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1} \mathbf{b_1}$$

or,

$$H(s, \theta) = \mathbf{c}^T (j\omega \mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1} \mathbf{b_1},$$

$$h(t, \theta) = L^{-1}(H(s, \theta))$$

Where, $s = j\omega$, L^{-1} denotes inverse laplace transform. Implementation of time-domain least squares method requires discretization of the output voltage equation as follows:

$$v_o(n\Delta) \approx \sum_{k=0}^{n-1} h(k\Delta, \theta) v_i(n\Delta - k\Delta)$$

Let $\hat{v}_o(t)$ represents the noisy measured voltage at the output of amplifier and in discretized form let it be denoted by $v_o(n\Delta)$. Now minimization of $E(\theta)$(where, θ is the parameters vector, to be estimated) is carried out as follows:

$$\hat{\theta} = argmin_\theta \sum_{k=0}^{n-1} |v_o(n\Delta) - \sum_{k=0}^{n-1} h(k\Delta, \theta) v_i(n\Delta - k\Delta)|^2$$

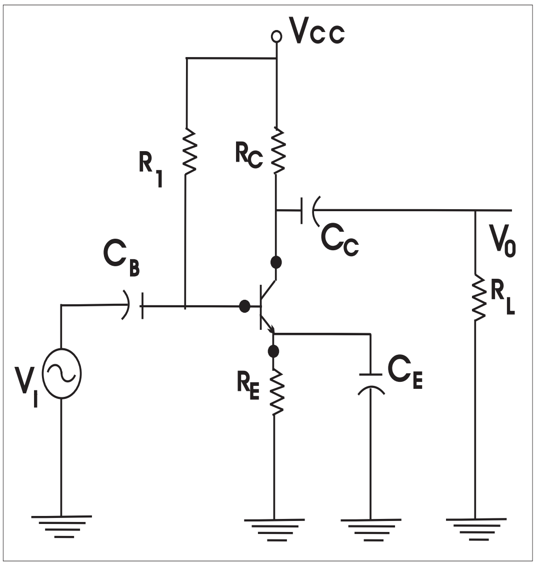

Fig. 1. Common emitter amplifier circuit

III. Application of frequency-variation based method to common emitter amplifier circuit

We consider a common emitter transistor amplifier built out of an npn transistor, resistors and capacitors (fig(1)). The input voltage to the amplifier is a periodic sinusoidal signal. We define appropriate state variables for the amplifier and write down a complete set of the KCL and KVL equations. The two port model used for the transistor in order to write these equations is the standard Ebers-Moll model. The two port model of the transistor is decomposed into a linear and a nonlinear part and a perturbation parameter ϵ is incorporated into the nonlinear part.Only linear part of Ebers-Moll current equations are utilized to obtain linearized model of the transistor.

Common emitter amplifier shown in fig(1)can be described completely by following three differential and one algebraic equations. The common emitter amplifier circuit, shown in Fig. 1, can be described completely by the following three differential and one algebraic equations:

$$i_C = \frac{V_{CC} - v_C}{R_C} - \frac{v_o}{R_L} \quad (15)$$

$$i_E = C_E \frac{dv_E}{dt} + \frac{v_E}{R_E} \quad (16)$$

$$i_E - i_C = C_B \frac{d(v_i - v_B)}{dt} + \frac{V_{CC} - v_B}{R1} \quad (17)$$

$$C_C \frac{d(v_C - v_o)}{dt} = \frac{v_o}{R_L} \quad (18)$$

From the transistor Ebers-Moll model i_C and i_E currents are given as follows:

$$i_C = \alpha_F I_{EO}(e^{v_{BE}/V_T} - 1) - I_{CO}(e^{v_{BC}/V_T} - 1) \quad (19)$$

$$i_E = I_{EO}(e^{v_{BE}/V_T} - 1) - \alpha_R I_{CO}(e^{v_{BC}/V_T} - 1) \quad (20)$$

Separating the linear and nonlinear parts of (19) and (20), and incorporating a small perturbation parameter ε into the nonlinear part we have

$$i_C = xc_1 - zc_2 + \varepsilon[c_1 f(x) - c_2 f(z)] \tag{21}$$

$$i_E = c_4 x - c_3 z + \varepsilon[c_4 f(x) - c_3 f(z)] \tag{22}$$

where $x = \frac{v_{BE}}{V_T}$
$z = \frac{v_{BC}}{V_T}$
$c_1 = \alpha_F I_{EO}$
$c_2 = I_{CO}$
$c_3 = \alpha_R I_{CO}$
$c_4 = I_{EO}$
$f(x) = e^x - 1 - x$
$f(z) = e^z - 1 - z$

Considering only linear parts of i_C and $i_E$from (21) and (22) we have

$$i_C = xc_1 - zc_2 \tag{23}$$

$$i_E = c_3 z - c_4 x \tag{24}$$

Putting $x = \frac{V_{BE}}{V_T}$, $z = \frac{V_{BC}}{V_T}$, also, $v_{BE} = v_B - v_E$ and $v_{BC} = v_B - v_C$ in the equations (23) and (24), we get

$$i_C = c_1\frac{v_B}{V_T} - c_1\frac{v_E}{V_T} - c_2\frac{v_B}{V_T} + c_2\frac{v_C}{V_T} \tag{25}$$

$$i_E = c_3\frac{v_B}{V_T} - c_3\frac{v_C}{v_T} - c_4\frac{v_B}{V_T} + c_1\frac{v_E}{V_T} \tag{26}$$

Putting i_C and i_E from equations (25) and (26) in (15),(16),(17) and (18) we get the following state variable equations for the state variables v_B, v_E, v_C and v_O

$$\frac{c_1 - c_2}{V_T}v_B - \frac{c_1}{V_T}v_E + (\frac{c_2}{v_T} + \frac{1}{R_C})v_C + \frac{v_O}{R_L} = \frac{V_{CC}}{R_C} \tag{27}$$

$$(\frac{c_4}{V_T} - \frac{c_3}{V_T})v_B - (\frac{c_4}{V_T} + \frac{1}{R_E})v_E - C_E\frac{dv_E}{dt} + \frac{c_3}{V_T}v_C = 0 \tag{28}$$

$$(\frac{c_4}{V_T} - \frac{c_1}{V_T} - \frac{c_3}{V_T} + \frac{c_2}{V_T} + \frac{1}{R_1})v_B + C_B\frac{dv_B}{dt}$$
$$+(\frac{c_1 - c_4}{V_T})v_E + (\frac{c_3}{V_T} - \frac{c_2}{V_T})v_C = C_B\frac{dv_i}{dt} + \frac{V_{CC}}{R_1} \tag{29}$$

$$C_C\frac{dv_C}{dt} - C_C\frac{dv_O}{dt} - \frac{v_O}{R_L} = 0 \tag{30}$$

Now considering equations (1) through (9) , output voltage v_O is given as follows:

$$y(t) = v_O(t) = \mathbf{c}^T\mathbf{C}^{-1}\mathbf{b}_0 + A.Re(\mathbf{c}^T(j\omega\mathbf{T}+\mathbf{C})^{-1}\mathbf{b}_1 exp(j\omega t))$$

Where following matrices and vectors, obtained from the state variable equations are employed to calculate output voltages $v_O(t)$ at many frequencies.

The matrix $(jn\omega\mathbf{T} + \mathbf{C})$ is given by

$$\begin{pmatrix} \frac{c_2-c_1}{V_T} & \frac{-C_1}{V_T} & \frac{C_2}{V_T} + \frac{1}{R_C} & \frac{1}{R_L} \\ \frac{c_4}{V_T} - \frac{c_3}{V_T} & m6 & \frac{c_3}{V_T} & 0 \\ m9 & j\omega C_B & \frac{c_1-c_4}{V_T} & \frac{c_3}{V_T} - \frac{c_2}{V_T} \\ 0 & 0 & i\omega C_C & -\frac{1}{R_L} - j\omega C_C \end{pmatrix}.$$

where,

$$m6 = -(\frac{c_4}{V_T} + \frac{1}{R_E}) - j\omega C_E$$

$$m9 = \frac{c_4}{V_T} - \frac{c_1}{V_T} - \frac{c_3}{V_T} + \frac{c_2}{V_T} + \frac{1}{R_1}$$

Matrix $(\mathbf{C})$is given by

$$\begin{pmatrix} \frac{c_2-c_1}{V_T} & \frac{-C_1}{V_T} & \frac{C_2}{V_T} + \frac{1}{R_C} & \frac{1}{R_L} \\ (\frac{c_4}{V_T} - \frac{c_3}{V_T}) & -(\frac{c_4}{V_T} + \frac{1}{R_E}) & \frac{c_3}{V_T} & 0 \\ m19 & 0 & \frac{c_1-c_4}{V_T} & (\frac{c_3}{V_T} - \frac{c_2}{V_T}) \\ 0 & 0 & 0 & -\frac{1}{R_L} \end{pmatrix},$$

where,

$$m19 = \frac{c_4}{V_T} - \frac{c_1}{V_T} - \frac{c_3}{V_T} + \frac{c_2}{V_T} + \frac{1}{R_1}.$$

$$\mathbf{b_0} = \begin{pmatrix} \frac{V_{CC}}{R_C} \\ 0 \\ \frac{V_{CC}}{R_1} \\ 0 \end{pmatrix}$$

$$\mathbf{b_1} = \begin{pmatrix} 0 \\ 0 \\ C_B \\ 0 \end{pmatrix}$$

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

It is evident from matrices elements that T and C matrices are functions of transistor parameters, θ. In this case let θ be V_T. In order to estimate V_T, we take many output voltage measurements at different frequencies and deliberately add random noise to the measured voltages, let these voltages be represented by $\hat{v}(t)$ (are called as noisy measured voltages). Making use of equations (10) and (11),

$$< \hat{v}(t) >= \mathbf{c}^T\mathbf{C}(\theta)^{-1}\mathbf{b_0} \tag{31}$$

$$< \hat{v}(t).exp(-j\omega t >= \mathbf{c}^T(j\omega\mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1}\mathbf{b_1} \tag{32}$$

Now, the frequency-domain least-squares method (the proposed method) involves estimating θ (here V_T) in the following way:

$$\hat{\theta} = Argmin_\theta(| < \hat{v}(t) > -\mathbf{c}^T\mathbf{C}(\theta)^{-1}\mathbf{b_0}|^2$$
$$+| < \hat{v}(t).exp(-j\omega t > -\mathbf{c}^T(j\omega\mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1}\mathbf{b_1}|^2) \tag{33}$$

Here energy function $E(\omega, \theta)$ is written as follows:

$$E(\omega, \theta) = (| < \hat{v}(t) > -\mathbf{c}^T\mathbf{C}(\theta)^{-1}\mathbf{b_0}|^2$$
$$+| < \hat{v}(t).exp(-j\omega t > -\mathbf{c}^T(j\omega\mathbf{T}(\theta) + \mathbf{C}(\theta))^{-1}\mathbf{b_1}|^2)$$

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Which is the error corresponding to the measurement at frequency ω. The minimization of $E(\omega, \theta)$ in (33) is carried out using the gradient search algorithm as follows:

$$\hat{\theta}_{n+1} = \hat{\theta}_n - \mu \sum_{k=1}^{N} \frac{\partial E(\omega_k, \theta_n)}{\partial \theta} \tag{34}$$

In other words, we select a sequence of frequencies $\omega_{k=1}^{N}$ and minimize $\sum_{k=1}^{N} E(\omega_k, \theta)$, the total sum of errors squared over every measured frequency using gradient search algorithm.

IV. Simulations and Discussions

Simulations have been done in MATLAB. In our case θ is V_T. For estimation of V_T, we take ten noisy measurements of the output voltage at variable frequency from 100 Hz to 160 Hz. The amplitude of the input voltage is kept constant. A noisy measured output voltage is represented by $\hat{v}(t)$. Now making use of equations 31, 32 and 33, we estimate V_T, by minimizing $E(\omega, \theta)$ through gradient search algorithm (using (34)). Therefore, our method of estimating a parameter can be regarded as frequency-domain-least-squares method, as it involves minimizing a squared error energy function from many data values, each obtained from different frequency output signals. It is to be noted that a data point at a frequency is obtained by applying K.A.M on the measured voltage at that frequency (equations 31, 32). Figs.(2) and (3) present plot of estimated value of V_T, using the proposed method and the conventional time domain least squares method. The time domain least squares method makes use of 5500 time-sample points and the sample time interval is .000001 second. The proposed method makes use of ten data points obtained from ten different frequencies (as d.c. value of the output voltage is zero). Initial value of V_T is .0180 volts in case of Fig.(1) and 0.016 in Fig.(2). Fig.(4) shows estimation of the V_T making use of fifteen data values collected from fifteen output measured voltages at fifteen different frequencies from 100 Hz to 180 Hz. Tables (1) and(2) present SNRs ,calculated for conventional time domain least squares method at different nos. of time-samples taken at a time, for two different values of initial values of V_T taken during application of gradient search algorithm. The calculations of SNRs have been done using following relation:

$$SNR = \frac{\sum_n \hat{V}_T^2(n)}{\sum_n e^2(n)}$$

It is evident from tables (1) and (2) that as the no. of time-sample points decrease in time domain least squares method, SNR also decreases. Tables (3) and (4) present SNRs obtained using the proposed method. It is to be noted that the SNR using the proposed method is more than that of the conventional time domain least squares method. Also the data storage requirement in the time domain least squares method is 1000-5500 points (in our simulation system), while the proposed method employs 10-15 data points.

V. Conclusions

We have presented a frequency-variation based method for transistor parameter estimation in a common-emitter amplifier circuit. This method involves parameter estimation from

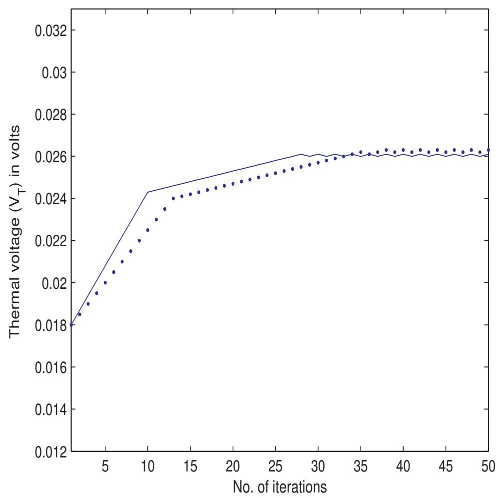

Fig. 2. Plot for the estimated value of V_T. Initial value of V_T=0.018V. The solid line represents the proposed method(freq. variation), and the dotted line shows the time-domain least-squares method.

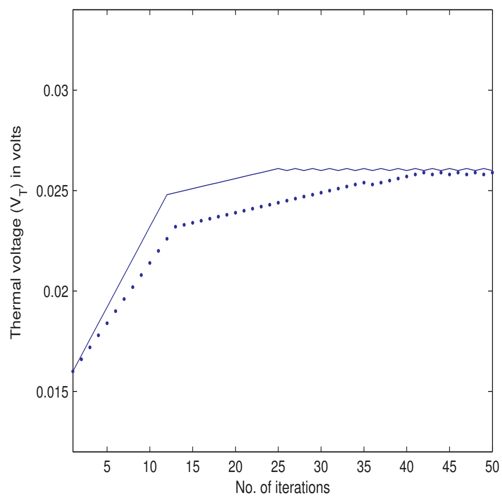

Fig. 3. Plot for the estimated value of V_T. Initial value of V_T=0.016 V. The solid line represents the proposed method (freq. variation), and the dotted line shows time-domain least-squares method.

TABLE I
SNR by time domain least squares method, for different nos. of time samples taken at a time. Initial value of $V_T = 0.0018$.

Sr. No.	Time Samples	SNR(dB)
1	1000	21.8926
2	2000	22.4669
3	3000	22.7995
4	5500	22.8213

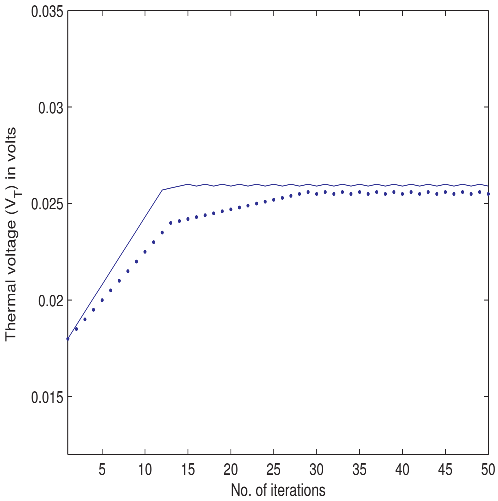

Fig. 4. Plot for the estimated value of V_T. Initial value of V_T=0.018 V. The solid line represents the proposed method (freq. variation based), fifteen different frequencies taken at a time , and the dotted line shows time domain least squares method.

TABLE II
SNR BY TIME DOMAIN LEAST SQUARES METHOD, FOR DIFFERENT NOS. OF TIME SAMPLES TAKEN AT A TIME. INITIAL VALUE OF $V_T = 0.0016$.

Sr. No.	Time Samples	SNR(dB)
1	1000	20.7048
2	2000	20.9532
3	3000	22.9538
4	5500	20.9721

noisy measurements of the output voltages of the transistor amplifier at variable input frequency, but fixed amplitude input signal. The algorithm of parameter estimation is based on the application of K.A.M. technique and the gradient search algorithm. For parameter estimation of the transistor, we applied K.A.M. to each measured output voltage. This way data values were obtained from the measured output voltages at variable frequency, which were employed for parameter estimation. Hence, there is only one data value (if d.c. value of the output is zero) per signal, which is employed during the process of parameter estimation. Therefore, if we employ ten different frequency output signals, then only ten data values have to be stored. Which is much less as compared to the conventional time-domain least-squares method of parameter estimation. Hence, the data storage requirement in the proposed method is very low. From the simulations and calculations of the SNRs, it is evident that the proposed method is more accurate as compared to the time domain least squares method, as the proposed method exploits the information in the time and frequency domains simultaneously. Also, this method can be a handy tool for parameter estimation of analog devices in their operating range of frequencies.

TABLE III
SNR BY THE PROPOSED METHOD (FREQ. DOMAIN LEAST-SQUARES METHOD), TEN DIFFERENT FREQUENCIES INPUT TAKEN AT A TIME.

Sr. No.	Initial value of V_T	SNR(dB)
1	.018	24.1742
2	.016	23.3931

TABLE IV
SNR BY THE PROPOSED METHOD (FREQ.-DOMAIN LEAST-SQUARES METHOD), FIFTEEN DIFFERENT FREQUENCIES INPUT TAKEN AT A TIME.

Sr. No.	Initial value of V_T	SNR(dB)
1	.018	25.0547

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