# Frequency and Amplitude Measurement of a Vibrating Object in Water using Ultrasonic Speckle Technique

Hongmao Zhu, Jun Chu, Lei Shen, and Zhihua Luo

*Abstract*—The principle of frequency and amplitude measurement of a vibrating object using ultrasonic speckle technique is presented in this paper. Compared with other traditional techniques, the ultrasonic speckle technique can be applied to vibration measurement of a nonmetal object with rough surface in water in a noncontact way. A ultrasonic speckle measurement system is set up, with which the frequency and amplitude of a vibrating cantilever beam was detected. The result show that the experimental data is in good agreement with the calibrating data.

*Keywords*—Amplitude, Frequency, Ultrasonic speckle, Vibration measurement.

## I. INTRODUCTION

IN industry and scientific research, vibration measurements are commonly needed. Traditional testing methods include the use of strain gages, capacitance sensors, piezoelectric sensors, fiber optic sensors, eddy current sensors and laser displacement sensors etc. Doubtlessly these methods are effective in vibration measurement under regular conditions. But, strain gages[1] and piezoelectric sensors[2] were only capable of contact measurements at single points. Fiber optic sensors[3] and laser displacement sensors[4] are hard to be applied to rough surfaces, and eddy current sensors[5] is limited its applicability to metallic objects, besides, they are not applicable for underwater applications because they are easily influenced by the disturbance of water. From the above, a new method is needed to deal with both the rough surfaces and the underwater noncontact measurements. Thus, ultrasonic speckle technique is suggested to meet the needs of solving these problems.

When ultrasonic waves are incident upon a rough interface, ultrasonic speckles are formed in the back-scattering space[6]. The ultrasonic speckles exist not only in the air, but in liquid and inside of solid. They carry the information of the object surface structure and their statistical properties has been evaluated[7][8]. Moreover, the movement of the back-scattered ultrasonic speckles is dependent on the movement of the object surface only. Based on this fact, the motion of a rough surface of an object can be evaluated according to the motion of the

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ultrasonic speckles in the back-scattering space if the relationship of the movement between the object surface and the ultrasonic

## II. THEORY

### A. Ultrasonic Speckle Movement

In order to find out the relationship between the motion of the ultrasonic speckles and the motion of an object surface, an ordinate $O-XYZ$ in the space and another ordinate $O'-X'Y'Z'$ on the object surface are set up respectively as shown in Fig.1. Before the object moves, these two ordinates coincide with each other. Ultrasound source E is located in the ($Y$-$Z$)-plane and it transmits ultrasound on to the object surface with an incident angle $i = \angle EOZ$. Ordinate $O_1 - \xi \eta \varsigma$ is also set up on the focus plane of probe R, which is used for receiving ultrasonic speckles. F and $\Phi$ are its focus length and aperture respectively. $O_1\xi$ is parallel to $OX$. $OO_1$ and $O_1\varsigma$ are in the same direction.
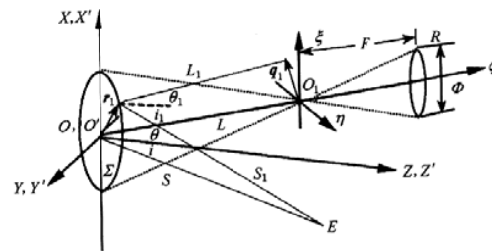


Fig. 1 Diagrams of the ordinates set up on an object surface and in the space

$\theta = \angle O_1OZ$ is the observing angle According to Kirchhoff diffraction theory , the complex amplitude of the speckle at $q_1$ is

$$A(q_1) = \int_{-\infty}^{\infty} \int P(r_1)A(r_1)\frac{\exp(jkL_1)}{L_1}dxdy, \qquad (1)$$

where $k = 2\pi/\lambda$ , $\lambda$ is the ultrasonic wavelength, $P(r_1)$, the aperture function of the area $\sum$ , and $A(r_1)$, the sound complex amplitude on the object surface is

$$A(r_1) = \frac{D(r_1)}{j\lambda}\frac{\exp(jkS_1)}{S_1}\frac{\cos i_1 + \cos\theta_1}{2}, \qquad (2)$$

In Eq.2, $D(r_1)$ is a circular symmetric complex Gaussian random variable. Let the position of a point on the object

surface be $r = (x, y, 0)^T$ in the space ordinate $O - XYZ$ and be $r' = (x', y', 0)^T$ in the surface ordinate $O' - X'Y'Z'$, where $(...)^T$ means the transposed matrix. As the object surface has small translations $u, v, w$ in the directions of $X$, $Y$, $Z$ and small rotation angles $\alpha, \beta, \gamma$ about the axes of $X$, $Y$, $Z$ respectively, Using the method proposed by Wu [9], we find the rules of the ultrasonic speckle movement in the space as the following,

$$\begin{cases} U = L(\cos i + \cos\theta)\beta + L(\sin i - \sin\theta)\gamma + u(1+\dfrac{L}{S}) \\ V = -L(1+\dfrac{\cos i}{\cos\theta})\alpha + v(\cos\theta + \dfrac{L\cos^2 i}{S\cos\theta}) + \\ \qquad w(\dfrac{L\sin i\cos i}{S\cos\theta} - \sin\theta) \\ W = v(\sin\theta - \dfrac{L^2}{S^2}\sin i) + w(\cos\theta + \dfrac{L^2}{S^2}\cos i) \end{cases} \quad (3)$$

where $U, V, W$ are the translations of the speckles in the $\xi, \eta, \varsigma$ directions respectively. It must be pointed out that the translations $u, v, w$ and the rotation angles $\alpha, \beta, \gamma$ of the object can not be directly figured out from the displacement $U, V, W$ of the speckles in the back-scattering space according to Eq. 6. However, if $\alpha = \beta = 0$, $L = 0$, $\theta = 0$, we have $U=u$, $V=v$, $W=w$. Thus, the movement of an object surface can be directly figured out according to the movement of the speckles on the object surface.

### B. Digital signal Correlation Operation

The principle of the digital correlation operation[10] used for detecting the speckle displacement is as follows. Before the object motion, the speckle signal on the object surface received by the probe can be expressed as

$$f(t) = \sum_{n=1}^{m} f(nT_s), n = 1 \cdots m$$

where $T$s is the sampling period. After the object motion, the speckle signal at the same point on the object surface becomes to be

$$g(t') = \sum_{n=1}^{m} g(nT_s - \Delta t), n = 1 \cdots m$$

where $t' = t - \Delta t$, $\Delta t$ is the time delay caused by object motion. The correlation coefficient of $f(t)$ and $g(t')$ is defined as

$$C(\tau) = \dfrac{\left[f(t) - \bar{f}\right] \cdot \left[g(t'+\tau) - \bar{g}\right]}{\sqrt{\left[f(t) - \bar{f}\right]^2}\sqrt{\left[g(t'+\tau) - \bar{g}\right]^2}}, \quad (4)$$

where $\bar{f} = \sum_{n=1}^{m} f(nT_s) \Big/ m$, $\bar{g} = \sum_{n=1}^{m} g(nT_s - \Delta t + \tau) \Big/ m$ and $0 \le C(\tau) \le 1$. $C(\tau) = 1$ if $f(t)$ and $g(t')$ is totally correlated. By using the exhausting search method to find out the maximum of $C(\tau)$, the speckle displacement can be calculated according to the value of $\tau$.

### III. EXPERIMENT

#### A. Experimental Arrangement

The measurement system is shown in Fig.2. A cantilever beam, the surface of which was coated with fine aluminium powder, was immersed in water and stimulated by a vibration generator (HEV-02). Focus probe R ( φ20mm, f = 5MHz, F = 40mm )was perpendicularly placed and focused on a measuring point on the surface of vibrating cantilever
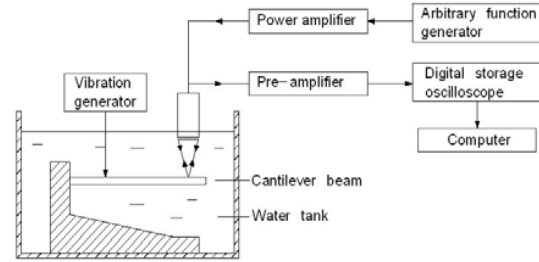


Fig. 2 Experimental setup of frequency and amplitude measurement using ultrasonic speckle technique

beam. R was generated by an arbitrary function generator (Agilent 33250A) via a power amplifier. The sweeping frequency and triggering time delay of the ultrasound transmitted by probe R were adjusted by the arbitrary function generator. The speckle signal was received by probe R too when it worked in a self transmitting receiving mode. Afterwards, the signal was amplified by the pre-amplifier and then was sent to a digital oscilloscope (Tektronix TDS 2014). After the signal was digitized and displayed, it was sent to a PC. In this case, according to Eq.12 the dynamic displacement of the speckle located on point O due to vibration was equal to the dynamic displacement of measuring point O on the cantilever beam surface.

#### B. Experiment and Results

The arbitrary function generator was adjusted to change the sweeping frequency $f_s$ of the transmitting ultrasound until the displayed speckle signal had been standing on the oscilloscope screen. In this case we had $f_s = f/n$ (n=1,2,3…), where $f$ was the vibrating frequency of the cantilever beam, otherwise the signal displayed would keep moving on the oscilloscope scope. This situation was judged by using a MATLAB program. After that, the sweeping frequency was lowered to the lowest $f_s^{min}$, with which the speckle signal was still kept standing on the oscilloscope screen. Thus, $f_s^{min}$ was equal to the vibrating frequency $f$ of the beam. The comparison of the frequency $f$ measured by using ultrasonic speckle technique with the frequency indicated on the vibration generator is shown on Table I. Next, the triggering time delay of the arbitrary function generator was adjusted to make the displayed speckle signal being located on the maximum time position and the minimum time position on the oscilloscope screen respectively. It meant that following the beam vibration the speckle arrived at the positive maximum position and the negative maximum position respectively, which was also judged by a MATLAB program. Based on these two triggering time data, the vibration amplitude of the speckle could be calculated with digital

correlation operation. According to Eq. 6 the vibration amplitude of the speckle was equal to that of the cantilever beam. The comparison of the vibration amplitude measured by using ultrasonic speckle technique with that measured by eddy current sensor is shown in Table II.

TABLE I
COMPARISON OF THE MEASURED FREQUENCY WITH
THE VIBRATION GENERATOR FREQUENCY

| NUMBER | GENERATOR FREQUENCY [HZ] | MEASURED FREQUENCY [HZ] |
|---|---|---|
| 1 | 20.00 | 20.05±0.01 |
| 2 | 22.50 | 22.48±0.01 |
| 3 | 25.00 | 24.97±0.01 |
| 4 | 27.50 | 27.64±0.01 |
| 5 | 30.00 | 30.05±0.01 |
| 6 | 32.50 | 32.52±0.01 |

TABLE II
COMPARISON OF THE VIBRATING AMPLITUDE MEASURED BY ULTRASONIC
SPECKLE TECHNIQUE AND EDDY CURRENT SENSOR

| TESTING POINT | EDDY CURRENT SENSOR MEASUREMENTS [MM] | ULTRASONIC SPECKLE MEASUREMENTS [MM] | ERROR [%] |
|---|---|---|---|
| $P_1$ | 0.3795 | 0.3655±0.0123 | 3.69 |
| $P_2$ | 0.3887 | 0.4042±0.0155 | 3.99 |
| $P_3$ | 0.6033 | 0.6300±0.0181 | 4.43 |
| $P_4$ | 0.7357 | 0.7097±0.0204 | 3.53 |
| $P_5$ | 1.0606 | 1.0389±0.0239 | 2.05 |
| $P_6$ | 1.1587 | 1.1556±0.0263 | 0.27 |

## IV. CONCLUSION

The relationship between the motion of an object surface and the motion of the speckles back-scattered from the object surface was deduced based on the Kirchhoff diffraction theory and the correlation principles of random signals. Based on this relationship, the movement of an object in water can be figured out according to the dynamic displacement of the ultrasonic speckles on the object surface. To perform the vibrating frequency and amplitude measurement using ultrasonic speckle technique, a special experimental arrangement was set up. In the experiment, frequency is measured by adjusting the function generator to have to the same frequency with the vibration frequency of the cantilever beam, which is indicated by the standing still speckle signal on the oscilloscope screen. The amplitude is measured by the correlation calculations using speckle signals from the highest position and the lowest position of the testing point. The measured results were compared with the frequency shown on vibration generator and the amplitude detected by eddy current sensor. Good agreement was obtained.

## ACKNOWLEDGMENT

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