

# Free Vibration Analysis of Carbon Nanotube Reinforced Laminated Composite Panels

B. Ramgopal Reddy, K. Ramji and B. Satyanarayana

**Abstract**—In this paper, free vibration analysis of carbon nanotube (CNT) reinforced laminated composite panels is presented. Three types of panels such as flat, concave and convex are considered for study. Numerical simulation is carried out using commercially available finite element analysis software ANSYS. Numerical homogenization is employed to calculate the effective elastic properties of randomly distributed carbon nanotube reinforced composites. To verify the accuracy of the finite element method, comparisons are made with existing results available in the literature for conventional laminated composite panels and good agreements are obtained. The results of the CNT reinforced composite materials are compared with conventional composite materials under different boundary conditions.

**Keywords**—CNT Reinforced Composite Panels; Effective Elastic Properties; Finite Element Method; Natural Frequency;

## I. INTRODUCTION

CARBON nanotubes, discovered first by Iijima[1], have attracted researcher's great attention due to their outstanding mechanical, electrical, thermal, chemical and even biological properties. Computational approach can play a significant role in the development of the CNT reinforced composites. At the nanoscale, it becomes a tedious task to solve the analytical models and also the experimental tests are very expensive to be conducted. However, modeling and simulations of nanocomposites can be cost effectively achieved using a desktop computer.

Although CNTs embedded in a matrix have modeled and analyzed extensively and successfully using molecular dynamics (MD) [2], [3] and continuum mechanics (CM) models [4]-[8], there have been very few reported studies in modeling fiber/CNTs/matrix.

Vibration, bending and buckling behavior of CNT has been a subject of interest in the past five years. Molecular dynamics or atomistic model has been used in order to look into the mechanics of nanotubes. Moreover, many authors have employed a continuum or structural mechanics approach for more practical and efficient modeling. For this purpose, rod, beam and shell theories have been used by researchers [9-14].

In this study, free vibration analysis for three types of CNT reinforced composite panels such as flat, concave and convex

is carried out using finite element method. The elastic properties required for frequency analysis are calculated using numerical homogenization.

## II. NUMERICAL HOMOGENIZATION

A composite is usually composed of fibers and matrix with quite different properties, and will demonstrate non-uniform response when even subjected to a uniform loading in a micromechanical view. However, in classical composite theory the composite is modeled as a homogeneous orthotropic medium with certain effective moduli that describe the average material properties of the composite. One of the most powerful tools to speed up the modeling process, both the composite discretization and the computer simulation of composites in real conditions, is the homogenization method. In this method, a representative volume element (RVE) shown in Fig.1 is chosen by assuming that the reinforcing material is in a periodic arrangement, and it is assumed that the average mechanical properties of a RVE are equal to the average properties of the particular composite. The average stresses and strains in a RVE are defined

$$\overline{\sigma}_{ij} = \frac{1}{V} \int_V \sigma_{ij} dV \quad (1)$$

$$\overline{\epsilon}_{ij} = \frac{1}{V} \int_V \epsilon_{ij} dV \quad (2)$$

The indices  $i$  and  $j$  denote the global three dimensional coordinate directions of a composite

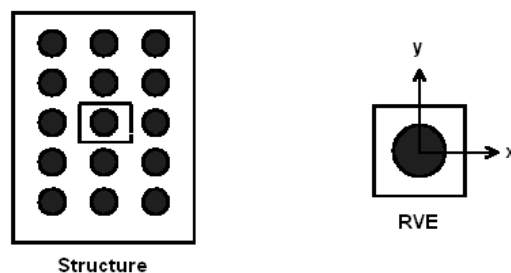


Fig.1 Periodic microstructures and an RVE

The total strain energy  $U$  stored in the volume  $V$  of the effective medium is given by

$$U = \frac{1}{2} \overline{\sigma}_{ij} \overline{\epsilon}_{ij} V \quad (3)$$

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With the appropriate boundary conditions applied to the RVE model, a finite element analysis can be performed to provide the required numerical results by which the effective mechanical properties can be calculated from Hooke's law and strain energy equation, i.e.

$$E = \frac{2U}{V\varepsilon^2} \quad \text{and} \quad G = \frac{2U}{V\gamma^2} \quad (4)$$

### III. EFFECTIVE ELASTIC PROPERTIES OF CNT REINFORCED COMPOSTIES

To evaluate the effective material constants of CNT-based nanocomposites, a Square RVE for a single-walled carbon nanotube reinforced in a matrix material as well as in glass fiber composites, is studied using finite element method. The material properties and dimensions taken for the analysis are

#### A. Short CNT in an Epoxy resin matrix

##### Carbon Nanotube (CNT)

The CNT type used in all the cases throughout this paper is the single-walled armchair (9, 9) type taken from literature [15]

Outer radius  $r_{\text{cnt}} = 0.61 \text{ nm}$ ; Length  $L_{\text{cnt}} = 50 \text{ nm}$

Effective thickness of the CNT  $t_{\text{cnt}} = 0.34 \text{ nm}$

Young's modulus  $E_{\text{cnt}} = 1000 \text{ nN/nm}^2$

Poisson's ratio  $\nu_{\text{cnt}} = 0.3$

#### Epoxy Resin Matrix

Side length of RVE = variable

Length  $L_m = 100 \text{ nm}$

Young's modulus  $E_m = 3.5 \text{ nN/nm}^2$

Poisson's ratio  $\nu_m = 0.3$

From the result of the finite element analysis and the strain energy formulas described in equation (4), the effective elastic properties are obtained as shown in Table I. For verification, these results are compared with analytical results obtained by modified Halpin-Tsai equations.

#### B. Glass/CNT/Epoxy composite

The material properties and dimensions taken for the analysis are as follows

##### Equivalent matrix:

Young's modulus  $E_{\text{em}} = 5.04$ ;  $\nu_{\text{em}} = 0.36$ ;

Length  $L_{\text{em}} = 1000\mu\text{m}$ ; Side length = variable;  $V_{\text{em}} = \text{variable}$ ;

##### Glass Fiber [16]:

Young's modulus  $E_f = 70 \text{ GPa}$ ;  $\nu_f = 0.2$ ;

Length of fiber  $L_f = 1000\mu\text{m}$ ;

Diameter  $d_f = 20\mu\text{m}$ ; Volume fraction  $V_f = \text{variable}$ ;

The finite element analysis of equivalent homogeneous matrix/Glass fiber composite (Glass/CNT/Epoxy) can be performed in way similar to that described in the previous section. The results of the effective elastic moduli of the glass/CNT/matrix composite are listed in Table II.

TABLE I  
EFFECTIVE ELASTIC PROPERTIES OF THE CNT/EPOXY COMPOSITE AT DIFFERENT VOLUME FRACTIONS OF CARBON NANOTUBES

$V_{\text{cnt}} (\%)$	$E_L/E_m$ FEM	$E_L/E_m$ Analytical	$E_T/E_m$ FEM	$E_T/E_m$ Analytical	$E_R/E_m$	$G_R/G_m$	$\nu_R/\nu_m$
1.0	1.583	1.632	1.029	1.064	1.237	1.183	1.196
2.0	1.740	1.783	1.060	1.104	1.315	1.255	1.206
3.0	1.817	1.835	1.091	1.135	1.363	1.300	1.210
4.0	1.868	1.901	1.123	1.176	1.402	1.336	1.213
5.0	1.902	1.937	1.157	1.209	1.436	1.370	1.206

TABLE II  
EFFECTIVE ELASTIC PROPERTIES OF CNT/GLASS/EPOXY COMPOSITE AT DIFFERENT VOLUME FRACTIONS OF GLASS FIBER

Volume fraction of Glass fiber ( % )	$E_L/E_m$ FEM	$E_L/E_m$ Analytical	$E_T/E_m$ FEM	$E_T/E_m$ Analytical	$G_{LT}/G_m$ FEM	$G_{LT}/G_m$ Analytical	$\nu_{LT}$ FEM	$\nu_{LT}$ Analytical
0	1.003	1.000	1.128	1.000	0.966	1.000	0.364	0.360
10	3.302	3.293	1.925	1.818	1.405	1.636	0.352	0.344
20	5.162	5.150	2.166	2.272	1.791	1.954	0.339	0.328
30	7.019	7.006	2.536	2.825	2.219	2.347	0.323	0.312
40	8.876	8.862	3.067	3.509	2.510	2.844	0.302	0.296
50	10.735	10.719	3.885	4.181	3.491	3.496	0.294	0.280
60	12.498	12.575	5.177	5.529	4.550	4.385	0.272	0.264
70	14.215	14.431	7.555	7.107	6.512	5.672	0.257	0.248

## IV. FINITE ELEMENT SIMULATION

Finite element method is an approximate numerical method which has been successfully used for solutions of problems in various fields. In the present work free vibration analysis of CNT reinforced composite panels is performed using ANSYS 11.0.

## V. VERIFICATION OF FINITE ELEMENT MODEL

To verify the applicability of software for free vibration analysis, simulation results are compared with the reference solution taken from literature [17]. The properties of the material are listed in Table III.

TABLE III  
MATERIAL PROPERTIES OF AS1/3501-6 (CARBON/EPOXY COMPOSITE)

$E_1$ (Gpa)	$E_2$ (Gpa)	$G_{12}=G_{13}$ (GPa)	$G_{23}$ (GPa)	$\nu_{12}$	Density (Kg/m <sup>3</sup> )
128	11	4.48	1.53	0.25	1500

The plate parameters are as follows:

$a = 250$  mm;  $b = 500$  mm;  $t = 1.04$  mm;

The layer stacking sequence of the plate is  $[0/\pm 45/90]_s$ . The plate thickness considered as 0.13 mm. The plate is clamped at all edges. Numerical results which are obtained by finite element simulation for the first five natural frequencies are presented in the table IV

TABLE IV  
NATURAL FREQUENCY (HZ) OF CONVENTIONAL COMPOSITE PLATE

Mode	literature	ANSYS
1	85.1	85.084
2	134.0	134.02
3	207.4	206.58
4	216.1	215.90
5	252.5	251.88

From the results it may be noted that the predictions show very close correlations with that of literature. This implies that ANSYS can be used for solving dynamic response of composite panels with good engineering accuracy.

## VI. FREE VIBRATION OF CARBON NANOTUBE REINFORCED LAMINATED COMPOSITE PANELS

The three types of CNT laminated composite panels (flat, concave and convex) are considered for the vibration analysis. To understand the dynamic properties of structures under vibrational excitation, a typical hull panel of size 1.5 m x 0.6 m x 0.02 m (thickness) is used. The panel is assumed to have 40 layers of 0.005 m thick. Lamination sequence of  $[0/90]_s$  is assumed for the panel. In case of concave and convex hull panels, shell rise ratio 0.05 with projected dimensions 1.5m x 0.6m x 0.02m is used. The properties of the material are listed in Table V.

TABLE V  
MATERIAL PROPERTIES OF GLASS/EPOXY AND GLASS/CNT/EPOXY

Material	$E_L$ (Gpa)	$E_T$ (Gpa)	$G_{LT}$ (GPa)	$\nu_{LT}$	Density (Kg/m <sup>3</sup> )
Glass/Epoxy	43.4	43.4	8.74	0.30	1980
Glass/CNT/Epoxy	44.0	44.0	15.92	0.27	1984

Numerical results are presented for three types of orthotropic panels (i.e. flat, concave and convex). Plates with three different boundary conditions, such as plate (a), (b) and (c) as shown in Fig. 2-4 are analyzed. In plate (a), all edges are fixed (CC), in plate (b), all edges are simply supported (SS) and in plate (c), one edge is fixed (C).

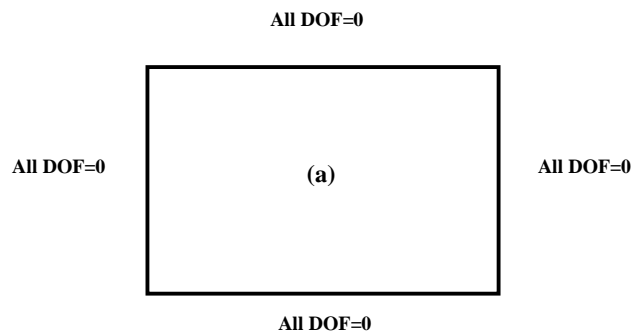


Fig. 2 Plate with clamped boundary conditions

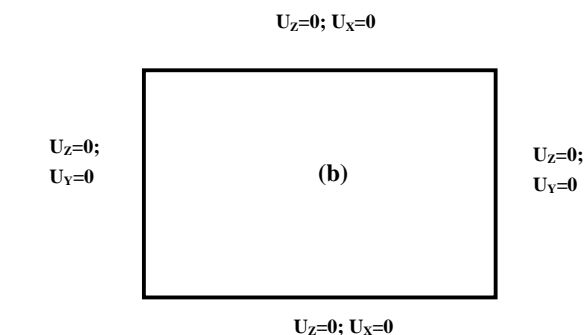


Fig. 3 Plate with simply supported boundary conditions

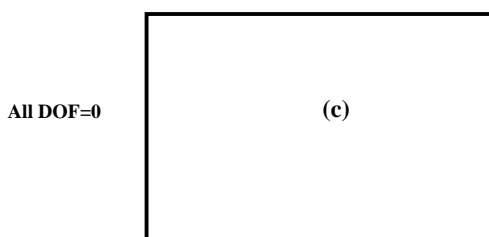


Fig. 4 Plate with cantilever boundary conditions

## VII. RESULTS AND DISCUSSION

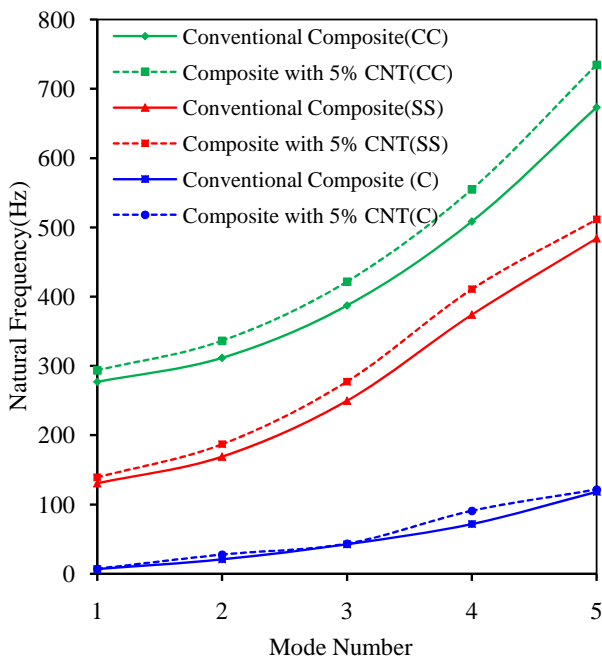


Fig. 5 Variation of natural frequency with mode number in case of flat plate

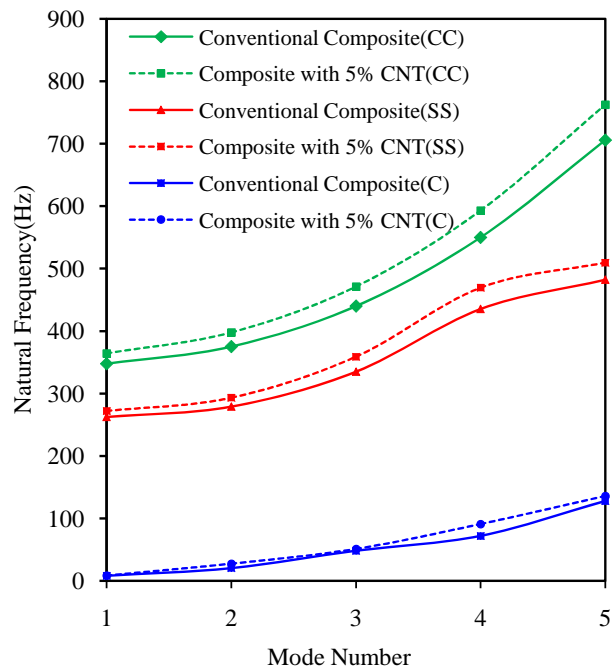


Fig. 7 Variation of natural frequency with mode number in case of convex plate

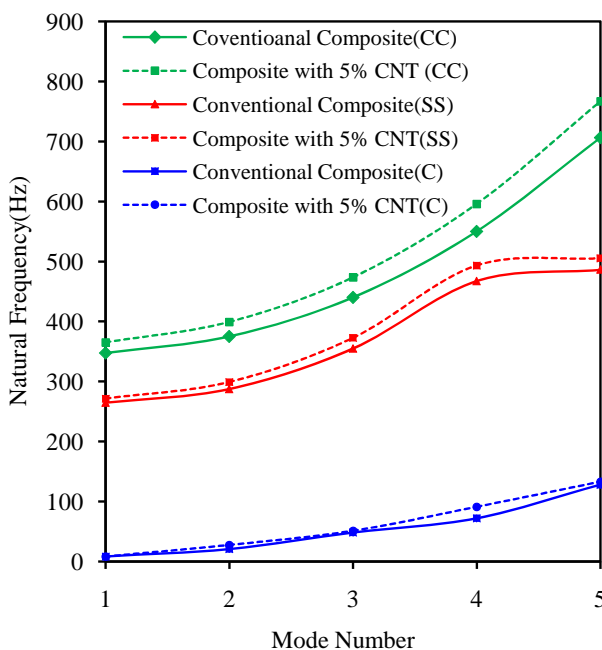


Fig. 6 Variation of natural frequency with mode number in case of concave plate

Fig.5-7 shows the effect of boundary conditions on the natural frequency for both conventional and CNT reinforced composite panels. From the results it may be noted that the reinforcing effect for adding CNTs into glass fiber composite have significant effect on the natural frequency of composite panels. This effect varies depending on the boundary conditions. In case of clamped boundary conditions the natural frequency increases monotonically with the mode number for all types of composite panels. As the mode number increases the natural frequency increases upto mode four and beyond this it is more or less constant for simply supported boundary conditions. However, the increase in natural frequency is very small in case of cantilever boundary conditions. From the results, it is also observed that the increase in natural frequency is more significant in case of concave panels.

## VIII. CONCLUSION

Free vibration analysis of CNT reinforced composite panels is carried out using finite element method. The accuracy of this method is verified by comparing the results with the literature for conventional composite materials. Three types of panels and the influence of boundary conditions on the natural frequency of CNT reinforced composite panels are analyzed in great detail. From the analysis it is found that the increase in natural frequency is more significant for adding CNTs into the glass fiber composite. From these results, we can have a better insight into the reinforcing effect of CNTs, and this will help the further development of new nanocomposites.

## NOMENCLATURE

a	Length of the plate
b	Width of the plate
C	Cantilever boundary conditions
CC	Clamped boundary conditions
E	Axial Young's modulus
G	Shear modulus
SS	Simply supported boundary conditions
t	Thickness of the plate
U	Total strain energy of effective medium
V	Volume of effective medium

## Greek symbol

$\bar{\varepsilon}$	Average strain
$\bar{\sigma}$	Average stress
$\gamma$	Shear Strain
$\varepsilon$	Normal strain
$\nu$	Poisson's ratio

## Subscript

cnt	Carbon nanotube
em	Equivalent matrix
f	Glass fiber
L	Longitudinal
LT	In plane
m	matrix
R	Randomly oriented CNTs
T	Transverse

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