Free Convection in a MHD Porous Cavity with using Lattice Boltzmann Method

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Abstract—We report the results of an lattice Boltzmann simulation of magnetohydrodynamic damping of sidewall convection in a rectangular enclosure filled with a porous medium. In particular we investigate the suppression of convection when a steady magnetic field is applied in the vertical direction. The left and right vertical walls of the cavity are kept at constant but different temperatures while both the top and bottom horizontal walls are insulated. The effects of the controlling parameters involved in the heat transfer and hydrodynamic characteristics are studied in detail. The heat and mass transfer mechanisms and the flow characteristics inside the enclosure depended strongly on the strength of the magnetic field and Darcy number. The average Nusselt number decreases with rising values of the Hartmann number while this increases with increasing values of the Darcy number.

I. INTRODUCTION

THE problem of natural convection in a cavity has been a major topic for research studies due to its frequent occurrence in industrial and technological applications. This includes crystal growth, electronic cooling, oil extraction, solar collectors, etc. Some of recent studies considered hydro magnetic flows and heat transfer in much different porous and non porous geometry, for example, Oreper and Szekely 1983; Vajravelu and Hadjinicolaou 1998; Al-Nimr and Hader 1999; Chamkha 2002 and Borjini *et al.* 2006 [1]-[5].

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To the best of our awareness, the first study of this problem is due to Alchar *et al.* [6] who considered the stability of a conducting fluid saturating a porous medium with the attendance of a uniform magnetic field using the Brinkman model. However, some comments on the MHD convection in a porous medium have been done very recently by Nield [7]. Also a very recent paper by Barletta *et al.* [8] has studied the mixed convection with heated effect in a vertical porous annulus with the radially varying magnetic field. Natural convection of an electrically conducting fluid in a rectangular enclosure in the presence of a magnetic field is studied numerically by Rudraiah *et al* [9]. They pointed out that the average Nusselt number decreases with an increase in the Hartmann number and the Nusselt number approaches unity for a strong magnetic field. Recently Robillard *et al* [10] investigated numerically as well as analytically the effect of an electromagnetic field on the free convection in a vertical rectangular porous cavity saturated with an electrically conducting binary mixture. They conclude that under the condition of constant fluxes of heat and mass imposed at the long side walls of the layer, the flow is parallel in the core of the cavity and turns through 180° in regions close to the end boundaries. This flow structure is not affected by the imposition of a magnetic field. Pangrle *et al* [11] performed an experimental research of magnetic resonance imaging an incompressible, laminar fluid flow in porous tube and shell systems flow. They used porous tube module in closed end mode for Reynolds number of 100 to 200 based on the tube radius to study the flow behavior and heat transfer. Other experimental studies dealing with MHD flows in porous media were reported by McWhirter *et al* [12] and Kuzhir *et al* [13]. Khanafer and Chamkha [14] studied numerically hydromagnetic natural convection heat transfer in an inclined square enclosure filled with a fluid-saturated porous medium with heat generation. Their results specify that the effects of magnetic field and the porous medium are found to reduce the heat transfer and fluid circulation within the cavity. However, there are few studies on the natural convection of a conducting fluid saturating a porous medium in the presence of a magnetic field in an enclosure.

The LBM is a new method for simulating fluid flow and modeling physics in fluids [15]-[17].This method has also been successfully applied to flow in porous media and MHD flow. The works done by LBM in porous media and MHD flow are separated from each other and no combination of them such as MHD flow in a cavity filled by porous medium is solved by LBM approach. Therefore in this section a briefly of past studies of these two fields (solved by LBM), are reviewed.

A most commonly approach to apply LBM to porous flow (with magnetic field or without it) is to model the fluid at the Reprehensive Elementary Volume (REV) scale [18]. This is accomplished by including an additional term to the standard Lattice Boltzmann Equation (LBE) to account for the presence of a porous medium. For instance, Dardis and McCloskey [19] proposed a lattice Boltzmann model by introducing a term describing the no-slip boundary condition. Spaid and Phelan proposed a model based on the Brinkman equation for single-component flow in porous media [20]. However, although the Brinkman model has been widely used to describe flows in porous media, some limitations still exist in this model. As pointed by Vafai and Kim [21], without a convective term, there is no mechanism for the development of the flow field, and this will lead to a physically flawed and unrealistic condition. The nonlinear inertial term is not included in the Brinkman model either, and thus, is suitable for low-speed flows only. In this paper, we consider linear and nonlinear matrix drag components as well as the inertial and viscous forces by using Brinkman- Forchheimer model [18]-[22]. In this model, the inertial force is included based on a recently developed technique [22], and the equilibrium distribution function is modified to account for the porosity of the medium. The model is applicable for a medium with both a constant and a variable porosity, and can be used to transient flows. Through the Chapmann-Enskog expansion, the generalized Navier-Stokes equations for flow in porous media can be derived from the model in the incompressible limit. The first magneto-hydrodynamic Lattice Gas Automata (LGA) was developed by Montgomery and Doolen [23] shortly after the Frisch, Hasslacher, and Pomeau (FHP) gas. Their model is an extension of the original FHP gas. It includes additional degrees of freedom of the particles for the vector potential, which has only one component in two dimensions and satisfies a passive scalar equation similar to the temperature. Therefore, the model is confined to two dimensions. Additionally, the model does not include the Lorentz force, which must be appended factitiously as an external force that needs some space averages when simulating. In general, these lattice Boltzmann MHD models fall into two categories: the multi-speed (MS) approach and multi-distribution-function (MDF) approach. The MS approach is a uncomplicated extension of the lattice Boltzmann models [24]-[26] presented by Martinez, Chen and Matthaeus in which a tensor (i.e., two-indexed) particle representation and a bi-directional streaming mechanism are used. For each one of these particles, there are two vectors attached, representing the momentum and magnetic field.

These MS models introduce some additional discrete velocities and the equilibrium distributions usually include higher order velocity terms. However, this model is confined to low-Reynolds because the values of the transport coefficients at the stability threshold are finite and its extension to three dimensions would require a large amount of computational memory. Such limitations severely restrict the MDF model's applications. However, the limitations of the MS approach can be partly overcome by the MDF approach [27]. In the MDF model presented by Dellar [28], the Lorentz force can be introduced as a point-wise force, the induction equation is also solved using an LBGK equation by introducing an independent distribution function. MDF models can improve the numerical stability. The accuracy of the MDF models has been verified by several benchmark studies [29] – [30]. Despite the benefit of the MDF models, there are still some limitations. For instance, in order to get the correct macroscopic equation from the MDF models, it must be assumed that the Mach number of the flow is small and the density varies slowly.

The main and particular objective of the present numerical investigation is to solve the hydro magnetic natural convection in a rectangular porous cavity by lattice Boltzmann method. The mathematical formulations for porous media are based on the Brinkman- Forchheimer equation model [22] and for consideration the magnetic effect use the MDF model [28]. Detailed results are presented in the form of the streamlines and isotherms and investigation the Nusslet number in the wide range of effective parameters.

II. MATHEMATICAL MODEL

Consider the steady two dimensional natural convection flow in a rectangular cavity filled with an electrically conducting fluid-saturated porous medium of height H and width L as shown in Fig. 1. A uniform magnetic field is applied in the vertical direction. It is assumed that the left and right walls are maintained at a constant temperature T_h and T_c ($T_h > T_c$). The both top and bottom walls are considered to be adiabatic. The physical properties are considered to be constant except the density variation in the body force term of the momentum equation which is satisfied by the Boussinesq's approximation. The magnetic Reynolds number is assumed to be very small so that the induced magnetic field and Hall effect are negligible [31].Consequence of small magnetic Reynolds number is the uncoupling of the Navier-Stokes equations from Maxwell's equation. In the present investigation the porous medium is assumed to be hydro dynamically and saturated with a fluid that is in local thermal equilibrium (LTE) with the solid matrix.
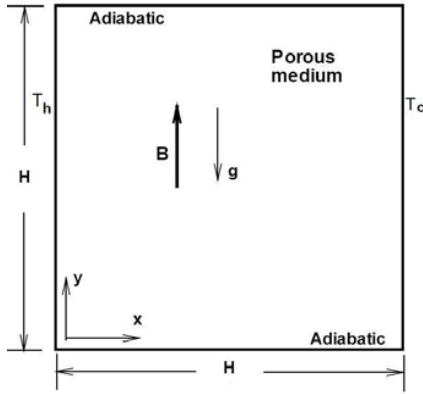
Fig 1. Geometry of the problem and coordinate system

A. LBM in MHD Porous Media

The Lattice Boltzmann model for incompressible fluid flow in porous media includes external force was proposed by several groups. In this work we take the form proposed by Guao et al [22] and Seta et al [32] which is applicable for a medium with both a constant and a variable porosity. The main particularity difference between [22] or [32] with the model that presented in this paper, is considering the magnetic effect by modifying the density distribution function, f_i^{eq}. The LBM originates from the lattice-gas automata method, and can also be viewed as a special discrete scheme for the Boltzmann equation with discrete velocities. In LBM, the fluid is modeled by a single-particle distribution function (DF). The evolution of the DF is governed by a lattice Boltzmann equation: [22]

$$f_i\left(\vec{x} + \vec{e}_i \delta_t, t + \delta_t\right) = f_i\left(\vec{x},t\right) - \frac{f_i\left(\vec{x},t\right) - f_i^{eq}\left(\vec{x},t\right)}{\tau_v} + \delta_t F_i \quad (1)$$

$$g_i\left(\vec{x} + \vec{e}_i \delta_t, t + \delta_t\right) = g_i\left(\vec{x},t\right) - \frac{g_i\left(\vec{x},t\right) - g_i^{eq}\left(\vec{x},t\right)}{\tau_c} \quad (2)$$

For the D2Q9 model, the discrete velocities are defined by:

$$\vec{e}_i = \begin{cases} (0,0) & for & i = 0 \\ c(\cos[(i-1)\frac{\pi}{4}], \sin[(i-1)\frac{\pi}{4}] & for & i = 1...4 \\ \sqrt{2}c(\cos[(i-1)\frac{\pi}{4}], \sin[(i-1)\frac{\pi}{4}] & for & i = 5...8 \end{cases} \quad (3)$$

Here, c and δ_t are the lattice spacing and the lattice time step, respectively. The equilibrium function for the density distribution function (f_i^{eq}) for the D2Q9 model in presence of porous media and considering the MHD effect is given by:

$$f_i^{eq} = \left\{ \omega_i \rho \left[1 + \frac{3\vec{e}_i.\vec{u}}{c^2} + \frac{9(\vec{e}_i.\vec{u})^2}{2\varepsilon c^4} - \frac{3|\vec{u}|^2}{2\varepsilon c^2} \right] \right\} + \left\{ \frac{\omega_i}{2c_s^2} \left[\frac{|\vec{B}|^2 |\vec{e}|^2}{2} - (\vec{e}.\vec{B})^2 \right] \right\} \quad (4)$$

Where   is the porosity of the medium and B is the magnetic field and ω_i is weighting factor and c_s is the speed of sound and defined by $c_s = \frac{c}{\sqrt{3}}$ [22]. The equilibrium distribution function (f_i^{eq}) shown in Eq. (4) has porosity   and $\vec{B}$ to include the effects of porous medium and MHD effect. The weighting factors are:

$$\omega_i = \left\{ \frac{4}{9} for \ \ i = 0 \ \ , \frac{1}{9} \ \ for \ \ i = 1...4, \ \ \frac{1}{36} \ \ for \ \ i = 5...8 \right\} \quad (5)$$

Similarly to density equilibrium function (f_i^{eq}), for calculating the magnetic field, a magnetic equilibrium function (h_i^{eq}) has been introduced: [28]

$$h_{ix}^{eq} = \lambda_i \left[\vec{B}_x + \frac{1}{c_s^2} \vec{e}_{ix} (\vec{u}_y \vec{B}_x - \vec{u}_x \vec{B}_y) \right]$$

$$h_{iy}^{eq} = \lambda_i \left[\vec{B}_y + \frac{1}{c_s^2} \vec{e}_{iy} (\vec{u}_x \vec{B}_y - \vec{u}_y \vec{B}_x) \right] \quad (6)$$

For solving the velocity and magnetic field the bellow equation must be considered [29]:

$$h_i\left(\vec{x} + \vec{e}_i \delta_t, t + \delta_t\right) = h_i\left(\vec{x},t\right) - \frac{h_i\left(\vec{x},t\right) - h_i^{eq}\left(\vec{x},t\right)}{\tau_m} \quad (7)$$

Where λ_i is the weighting factor of magnetic field and defined in fifth direction by:

$$\lambda_i = \begin{cases} \dfrac{1}{3} & for & i = 0 \\ \dfrac{1}{6} & for & i = 1...4 \end{cases} \quad (8)$$

Likewise the equilibrium distribution functions for the thermal energy distribution g_i^{eq} can be written as: [35]

$$g_i^{eq} = \omega_i T \left(1 + \frac{3}{c^2} \vec{e}_i . \vec{u} \right) \quad (9)$$

And the Brinkman-Forchheimer equation is: [32]

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)(\frac{\vec{u}}{\varepsilon}) = -\frac{1}{\rho_f} \nabla(\varepsilon P) + \upsilon \nabla^2 u + \vec{F} \quad (10)$$

Here, $p = \frac{c^2 \rho}{3\varepsilon}$, and the viscosity $\upsilon = c^2 \left(\tau_v - 0.5 \right) \frac{\delta_t}{3}$ [22]. The body force with Ergun's relation[33] can be expressed as:

$$\vec{F} = -\frac{\varepsilon \upsilon}{K} \vec{u} - \frac{1.75}{\sqrt{150} \ \varepsilon K} |u| \vec{u} + \varepsilon \vec{G} \quad (11)$$

$$K = Da \; H^2, \qquad F_\varepsilon = \frac{1.75}{\sqrt{150 \; \varepsilon^3}} \quad (12)$$

Where υ is the viscosity of the fluid, K is the permeability; G is the acceleration due to gravity, Da is the Darcy number, and H is the characteristic length. The total body force ($\vec{F}$) encompasses the viscous diffusion and the inertia due to the presence of a porous medium, and an external force. It is proved that the most suitable choice for the forcing term F_i (see (1)) to obtain correct equations of hydrodynamics is taking: [34]

$$F_i = \omega_i \rho (1 - \frac{1}{2\tau_\upsilon}) \left[\frac{3\vec{e}_i.\vec{F}}{c^2} + \frac{9(\vec{u}.\vec{F} : \vec{e}_i \vec{e}_i)}{\varepsilon c^4} - \frac{3\vec{u}.\vec{F}}{\varepsilon c^2} \right] \quad (13)$$

The forcing term F_i shown in Eq. (14) defines the fluid velocity $\vec{u}$ as:

$$\rho \vec{u} = \sum_i \vec{e}_i f_i + \frac{\delta_t}{2} \rho .\vec{F} \quad (14)$$

As shown in (11), $\vec{F}$ contains the velocity $\vec{u}$. Equation (14) is a nonlinear equation for the velocity $\vec{u}$. By using a temporal velocity $\vec{v}$, one can solve this nonlinear problem as follows [22]:

$$\vec{u} = \frac{\vec{v}}{c_o + \sqrt{c_o^2 + c_1 |\vec{v}|}} \qquad \vec{v} = \frac{\sum_i \vec{e}_i f_i}{\rho} + \frac{\delta_t}{2} \varepsilon \vec{G} \quad (15)$$

$$c_0 = \frac{1}{2}(1 + \frac{\delta_t}{2} \varepsilon \frac{\upsilon}{K}) \qquad c_1 = \frac{\delta_t}{2} \varepsilon \frac{1.75}{\sqrt{150 \varepsilon^3 K}} \quad (16)$$

The fluid density and temperature and magnetic field are defined as:

$$\rho = \sum_i f_i \quad , \quad T = \sum_i g_i \quad , \quad \vec{B} = \sum_i h_i \quad (17)$$

Through the Chapman-Enskog procedure, in the limit of small Mach number, Eq. (1) recovers the continuity equation:

$$\nabla.u = 0 \quad (18)$$

Equation (2) describes the evolution of the thermal energy and leads to the energy equation: [35]

$$\frac{\partial T}{\partial t} + \nabla .(\vec{u} T) = \alpha \nabla^2 T \quad (19)$$

Where α is the thermal diffusivity which is defined as[35]:

$$\alpha = c_s^2 (\tau_c - \frac{1}{2}) \quad (20)$$

Moreover Equations (6) and (7) describes the progress of the magnetic field and leads to the:[29]

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B} \quad (21)$$

And η is the magnetic resistivity which is defined as:

$$\eta = c_s^2 (\tau_m - \frac{1}{2}) \quad (22)$$

III. CODE VALIDATION

The present study validated by performing simulation for the analysis of natural convection in rectangular cavity filled with porous medium in the absence of a magnetic field which is reported by Seta [33] and Nithiarasu [36]. Table I clearly shows good agreement of the average Nusselt number between present study and the works done by Seta [33] and Nithiarasu [36]. Also in Fig 2, the current results are compared with the outcomes of Rudraiah et al [9]. Rudraiah et al [9] numerically investigated the effect of a horizontal magnetic field on natural-convection flow inside a rectangular enclosure without porous medium. These effects provide credence to the accuracies of the present numerical solutions.

IV. RESULTS AND DISCUSSIONS

Numerical computations in the present study were carried out for $Pr=0.7$. The effect of magnetic field on the buoyancy-driven convection of an electrically conducting fluid in a porous medium cavity with constant porosity parameter is investigated by LBM approach.

TABLE I
COMPARISON OF THE PRESENT RESULTS WITH PREVIOUS STUDY

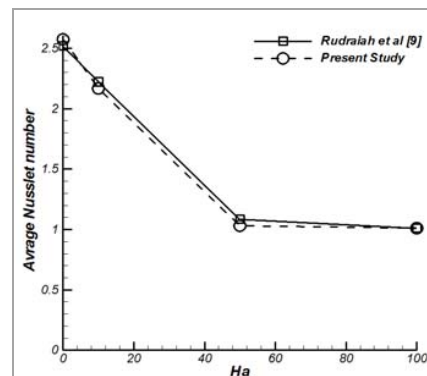| Da	Ra	ε	Present study	Seta[33]	Nithiarasu [36]
10^{-2}	10^4	0.4	1.408	1.339	1.408
		0.6	1.543	3.55	3.55
		0.9	1.685	1.659	1.64

Figure 2. Average Nusselt number versus at $Gr = 2 \times 10^4$ under various strengths of the harizontal magnetic field..
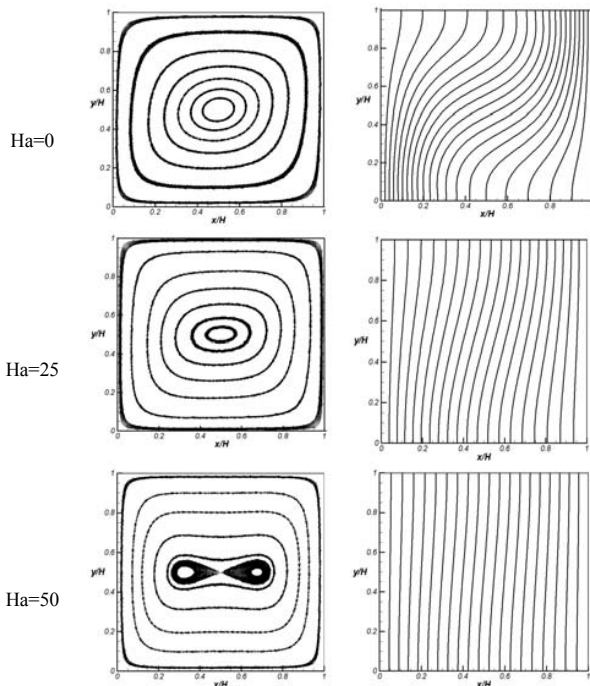
Fig. 3 Streamlines (on the left), isotherm lines (on the right) for the vertical magnetic field with $Ra = 10^4$, $Da = 10^{-2}$ and $\varepsilon = 0.6$

The present computation will be focused on the parameters having the following ranges:The Darcy number from Da= 10^{-4} to 10^{-1}, the porosity from ε=0.4 to 0.8.

Uniform grid is employed in the present study. The buoyancy force is naturally more effective for higher Rayleigh numbers. The Lorentz force reduces velocities and suppresses the convection. In general fluid circulation is strongly dependent on the Hartmann number as shown in Fig.3.

Figs.3 illustrates the effect of Hartmann number (*Ha)*, on the streamlines (on the left) and isotherms (on the right) . To highlight on the effect of *Ha*, the thermal Rayleigh number is kept constant $Ra = 10^5$, $Pr = 0.7$, $Da = 10^{-2}$ and $\varepsilon = 0.6$
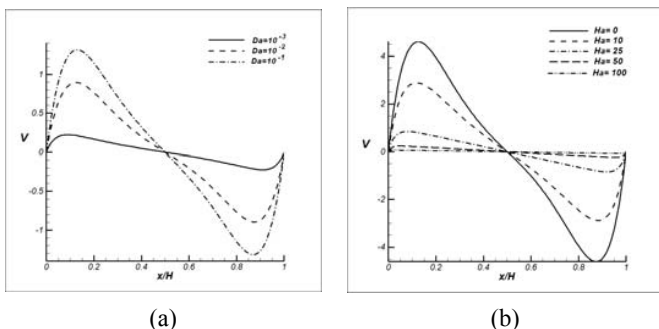
Fig.3 shows the very strong clockwise cell is observed as well as the streamlines are very crowded near the vertical walls in the absence of magnetic field. As the magnetic field is imposed $Ha = 25$, the flow strength slightly reduces and the streamlines penetrates slightly to the cavity core.As the Hartmann number increases, the flow strength is damped more and the streamlines penetrate more towards the cavity center. The isotherm lines is paralleled with side wall at the Ha=50. The effect of magnetic field and porosity on the velocity profile for different values of the Hartmann number and Darcy number at mid-section of the cavity is depicted in Fig. 4. The presence of a magnetic field within the cavity results in a force, opposite to the flow direction, which tends to resist the flow. This is clearly noticed from the vertical velocity profiles at the center of the cavity.

The results of diverse values of Da on velocity distribution have been illustrated in Fig.4 b. The results show with decreasing the Da, velocity reaches to steady distribution. As well Fig .5 illustrates that with increasing the Hartman number the pattern flow changing same. In general for a constant value of the Rayleigh number, the average Nusselt number decreases with increasing values of the Ha.

V. Conclusion

In this paper, natural convection in a porous cavity in the presence of the vertical magnetic field is studied numerically. The lattice Boltzmann method is employed for the solution of the present problem. The streamlines and the isotherms for various parametric conditions are presented and discussed. It is found that the heat transfer is strongly dependent on the strength of the Darcy number and the magnetic field. The effect of the magnetic field is found to reduce the heat transfer and fluid circulation within the cavity. In general, for fixed value of Rayleigh number, the average Nusselt number decreases with rising values of the Hartmann number.
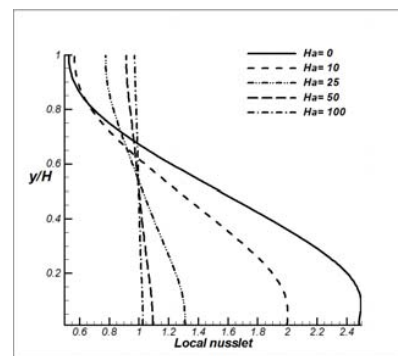

(a)   (b)

Figure 4. Vertical velocity profiles at mid-plane of the cavity for Ra=10^4 ,ε= 0.6 and a)Ha=25 b)$Da = 10^{-2}$

Fig. 5 Local Nusselt number for Ra=10 ^{4} ,ε=0.6 and Da=10^{-2}

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