International Journal of Architectural, Civil and Construction Sciences
ISSN: 2415-1734
Vol:12, No:1, 2018

Formex Algebra Adaptation into Parametric Design Tools: Dome Structures

Réka Sárközi, Péter Iványi, Attila B. Széll

***Abstract*—**The aim of this paper is to present the adaptation of the dome construction tool for formex algebra to the parametric design software Grasshopper. Formex algebra is a mathematical system, primarily used for planning structural systems such like truss-grid domes and vaults, together with the programming language Formian. The goal of the research is to allow architects to plan truss-grid structures easily with parametric design tools based on the versatile formex algebra mathematical system. To produce regular structures, coordinate system transformations are used and the dome structures are defined in spherical coordinate system. Owing to the abilities of the parametric design software, it is possible to apply further modifications on the structures and gain special forms. The paper covers the basic dome types, and also additional dome-based structures using special coordinate-system solutions based on spherical coordinate systems. It also contains additional structural possibilities like making double layer grids in all geometry forms. The adaptation of formex algebra and the parametric workflow of Grasshopper together give the possibility of quick and easy design and optimization of special truss-grid domes.

***Keywords*—**Parametric design, structural morphology, space structures, spherical coordinate system.

I. Introduction

In this paper, the adaptation of dome structures of the paper Formex Configuration Processing I. [1] to the computer program Grasshopper 3D [2] is introduced. The content of this paper covers the 'simple' dome structures of [1], including Ribbed domes, Schwedler domes and Lamella domes. It is a part of a bigger work which covers the dome and vault structures of [1] and Formex Configuration Processing II. [3]. This will contain the dome structures based on spherical and ellipsoidal coordinates, the vault and planar structures based on cylindrical coordinates, and the triangular version of these structures based on the Diamatic domes introduced in [3].

Using computer-based technologies to plan truss grid structures provides the possibility for architects and engineers to use more advanced or user-friendlier methods in the design process [4], [5]. The aim of this adaptation is to develop a tool for architects to design truss grid structures easily. For the sake of the cause the generating methods of formex algebra are reinterpreted and different calculation methods are used.

This paper shows the mathematical basics of generating dome structures in the developed Grasshopper 3D tool, the aim and logical structure of the modified dome structures and the functionality of this tool.

R. Sárközi, P. Iványi, and A. B. Széll are with the University of Pécs, Faculty of Engineering and Information Technology, Pécs, Hungary (e-mail: sarkozi.reka@mik.pte.hu, ivanyi.peter@mik.pte.hu, szellattila@mik.pte.hu).

II. About Formex Algebra and Grasshopper 3D

Formex algebra is a mathematical system created by Nooshin and Disney. They started to develop this system in 1975 and it is developed since then by Professor Nooshin and his colleagues. It is most useful for planning structural systems such as various truss-grid domes and vaults, but other geometrical forms are also covered by it [6]. The engine of formex algebra is the computer program Formian (Formian 2 and Formian K [7]), and the programming language Formian.

The main advantage of Formex algebra is that it uses coordinate system transformations to modify structures, and it makes very easy to construct a vault or a dome using a planar grid. The disadvantage of formex algebra is the necessity of programming skills for the users, which is not very common among architects.

Grasshopper is a modern parametric software, it is a plug-in for the 3D modeling program Rhino. It has a graphical algorithm editor, which makes the coding process much easier. The user has to follow the logical steps of the geometrical conversions but no programming knowledge is necessary. It already contains a lot of useful tools to create special forms or fluid geometries, and lots of additional plug-ins are available. This serves as an ideal environment for the coordinate system transformations used with Formex algebra, because it is easier to use without prior programming skills, and also allows the development of any dome structure in a multitude of ways.

III. Mathematic Calculation of Dome Structures

The calculation of dome structures is based on spherical coordinates. In formex algebra the user has to define three valuables, '*b1* (factor for scaling in the first direction (linear scale factor)), *b2* (factor for scaling in the second direction (angular scale factor), and *b3* (factor for scaling in the third direction (angular scale factor))' [1]. Every other attributes of the domes are given by the attributes of the initial grid, such as the thickness of the double layer grids and the gap in the middle of the structure. The advantage of this solution is that it uses just a few variables, nevertheless the disadvantage of it is that it requires relatively lot calculation from the user.

To make a versatile and user-friendly tool in the calculation method and by choosing the input parameters some modifications were introduced. To make it possible to design domes with different height and radius an ellipsoidal coordinate system was used. The ellipsoidal coordinate system is based on the spherical coordinate system, see Fig. 1.

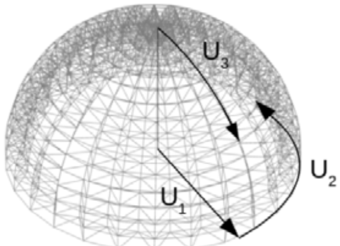

Fig. 1 Spherical coordinates

The coordinates of the spherical coordinate system are calculated from the original coordinates of the initial grid, see Fig. 2, the radius of the dome, and the angular scaling factors, as in (1):

$$U_1 = R \quad (1)$$
$$U_2 = \alpha * x$$
$$U_3 = \beta * y$$

U_1, U_2, U_3 are the coordinates of the spherical coordinate system; x, y, z are the Cartesian coordinates of the initial grid; R is the radius of the spherical dome; and α, β are the factors of angular scaling in the second and third directions.

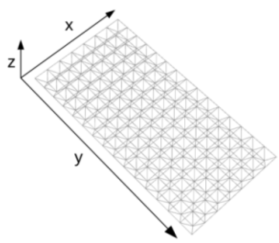

Fig. 2 Initial grid

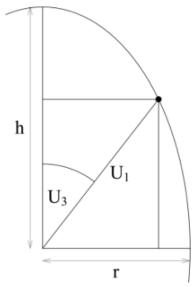

Fig. 3 Variables of the ellipsoidal coordinates

The calculation of the ellipsoidal coordinates is similar to the spherical coordinates, only the first coordinate (U_1) of the points vary according to the spherical coordinates of a point, as in (2) and Fig. 3, the calculation of U_2 and U_3 are the same.

$$U_1 = \sqrt{\frac{1}{\frac{[\sin(U_3)]^2}{r^2}+\frac{[\cos(U_3)]^2}{h^2}}} \quad (2)$$

r is the radius of the base of the ellipsoidal dome, and h is the height of the ellipsoidal dome.

Instead of using a scaling factor to define the first coordinate (U_1); the actual radius and height are the variables. This means that the user does not have to calculate the scaling factor, which would be especially difficult, when the radius and the height are different. Because of this calculation method the thickness of the double layer grid is also a required variable.

To ensure the uniform thickness through the ellipsoidal structure, the inner shell of the grid is calculated by an ellipsoid, which height and radius are reduced by the value of the thickness, as seen in Fig. 4 and (3).

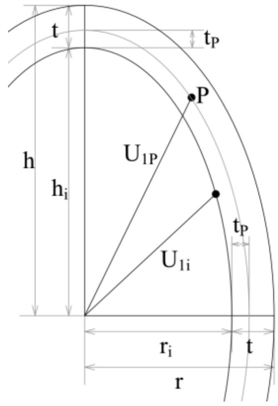

Fig. 4 Variables of ellipsoidal coordinates with thickness

$$U_{1i} = \sqrt{\frac{1}{\frac{[\sin(U_3)]^2}{(r-t)^2}+\frac{[\cos(U_3)]^2}{(h-t)^2}}} \quad (3)$$

U_{1i} is the first coordinate of the inner shell, and t is the thickness of the structure. According to this, the calculation of the first coordinate of a point is visible in (4):

$$t_P = t\frac{|z_P - z_{min}|}{|z_{max} - z_{min}|} \quad (4)$$

$$U_{1P} = \sqrt{\frac{1}{\frac{[\sin(U_{3P})]^2}{(r-t_P)^2}+\frac{[\cos(U_{3P})]^2}{(h-t_P)^2}}}$$

U_{1P}, U_{2P}, U_{3P} are the ellipsoidal coordinates of a point in the grid; t_P $_i$s the distance of a point from the inner shell; z_P is a points z coordinate in the initial grid; and z_{max}, z_{min} are the biggest and smallest of the points z coordinates in the initial grid.

In formex algebra it is possible to produce a hole in the center of the structure which is given by the position of the initial grid according to the origo. In this adaptation a more user-friendly solution is used, there is a variable to define the angle of the gap. If a gap is used in a dome structure, the calculation of the U_3 varies as in Fig 5 and (5).

$$U_{3P} = \lambda + \beta * y_P \quad (5)$$

λ is the angle of the gap, and y_P is the y coordinate of a point in the initial grid.

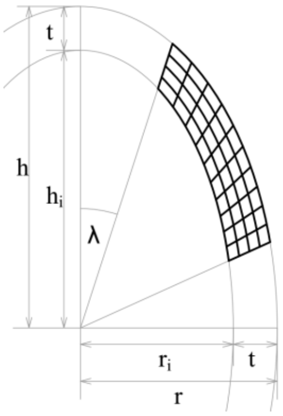

Fig. 5 Variables of an ellipsoidal dome with a gap

IV. Modified Dome Structures

A. Basics of Modifications

In order to create a diverse design tool, some modifications of the ellipsoidal coordinate system are also included. By using these modifications it is possible to create additional dome types.

Every modification is based on the transformation of one or both of the angular coordinates (U_2, U_3) to linear coordinates. The corners of the initial ellipsoidal dome structure stay unchanged during the modification, and the second direction (cupola type), the third direction (cone type) or both of them (pyramid type) are modified from an angular to a linear direction by connecting the matching corners with a straight line instead of an elliptic curve.

The value of the linear second (U_2) or third (U_3) coordinate of the modified structure is based on the ratio of the initial angular coordinates of the direction. The linear coordinate divides the linear edge of the modified grid in the same ratio as the interval of the angular coordinates of the initial grid was divided by the value of the initial angular coordinate of the vertex, as seen in Fig. 6.

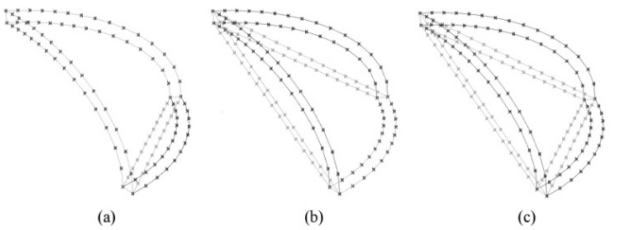

Fig. 6 Edges of the modified domes (a) Type 1 - Cupola, (b) Type 2 - Cone, (3) Type 3 - Pyramid

This modification method results that it is easy to combine and joint the different type of domes. If the value of the biggest third coordinate (U_3) of a grid is equal to the gap angle of another grid, and the first (U_1) and second (U_2) coordinates are synchronized, the two structures can be jointed precisely. At the same time, this modification method means that the thickness of the modified structures differs from the variable 'thickness'. Moreover, the thickness of the cone and pyramid type domes is different on the top and on the bottom of the structure if the value of the height and the radius is different. This difference is based on the ratio of the height and the radius. If the third coordinates (U_3) of the vertices of a structure vary on the interval [0°-90°] the ratio of the top thickness and the bottom thickness is the same as the ratio of the radius and the height, as seen in Fig. 7 and (6).

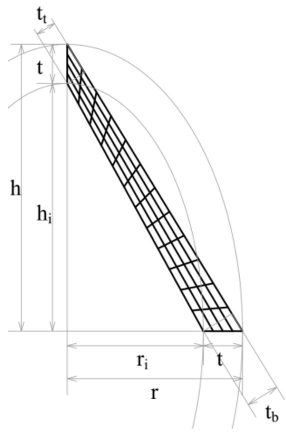

Fig. 7 Thickness of the cone and pyramid type domes

$$\frac{t_t}{t_b} = \frac{r}{h} \tag{6}$$

t_t is the top thickness of the structure, and t_b is the bottom thickness of the structure.

During the development of this dome tool other modification methods were also tested, where the thickness of the structure were based on the variable *'thickness'*. In this case the joining of two dome sections was much more difficult, such like the mathematical calculation of the grid structures.

The modification method works correctly if both the second and third coordinates of the vertices are less than 180°.

B. Type 1- Cupola

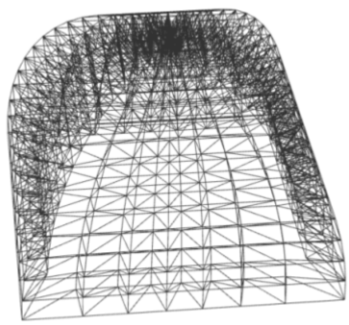

Fig. 8 Cupola type dome

The second coordinates (U_2) of the cupola type domes are modified from angular to linear coordinates. The first and second coordinates (U_1, U_2) of this structure are linear, the third (U_3) is angular.

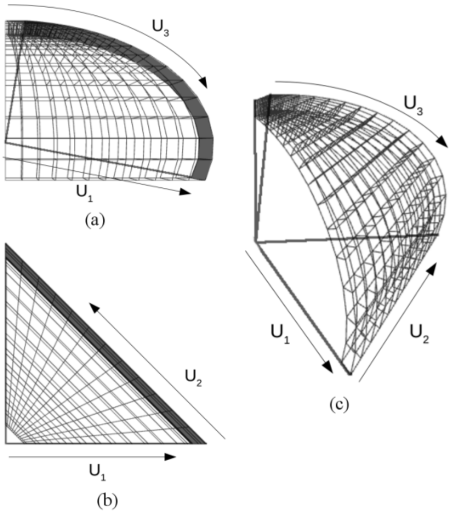

Fig. 9 Coordinates of cupola type domes (a) front view, (b) top view, (c) perspective view

C. Type 2 - Cone

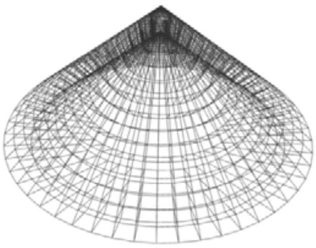

Fig. 10 Cone type dome

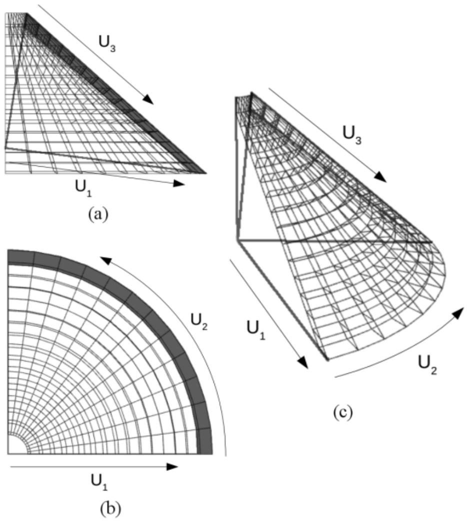

Fig. 11 Coordinates of cone type domes (a) front view, (b) top view, (c) perspective view

The third coordinates (U_3) of the cone type domes are modified from angular to linear coordinates. The first and third coordinates (U_1,U_3) of this structure are linear, the second (U_2) is angular.

D. Type 3 - Pyramid

The second and third coordinates (U_2, U_3) of the pyramid type domes are modified from angular to linear coordinates. Both three coordinates (U_1, U_2, U_3) of this structure are linear.

E. Jointed Structures

If the coordinates of the domes are synchronized it is possible to combine dome sections along the second or third direction. The coordinates of the grid structures by the jointed edges have to be the same type.

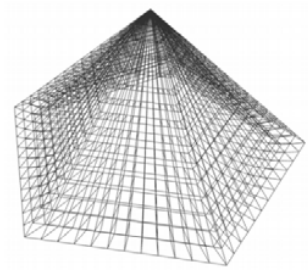

Fig. 12 Pyramid type dome

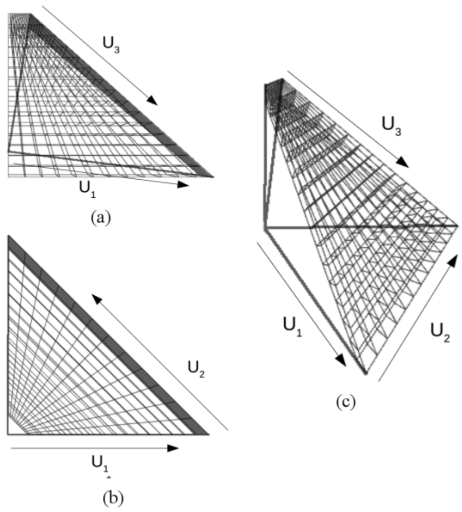

Fig. 13 Coordinates of pyramid type domes (a) front view, (b) top view, (c) perspective view

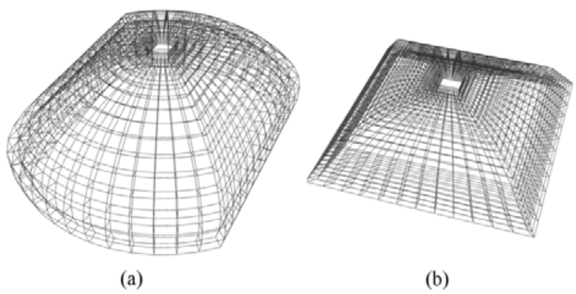

Fig. 14 Jointed domes (a) ellipsoid and cupola segments, jointed vertically (b) cupola and pyramid segments, jointed horizontally

V. User Interface

The dome tool in Grasshopper is made up from two components. The first one is called the 'Dome' component; this contains the calculation of the dome structure. The input parameters are the initial grid and the basic variables of the dome: the height (h) and radius (r), the angular scaling factor of U_1 and U_2 (α, β), and the type of the dome (ellipsoid/ cupola/cone/pyramid). These are the necessary settings for this tool. Additional settings are placed in the tool 'dome settings', like thickness for double layer grids (t), the gap angle (λ), the axis (a) of the structure and the origo (O), and there is a possibility to use curve segments instead of lines.

The users can use the already existing components of Grasshopper to create the initial grid in Cartesian coordinate system. The 'dome' component transforms it to the dome structure given by the input parameters, together with the 'dome settings' tool.

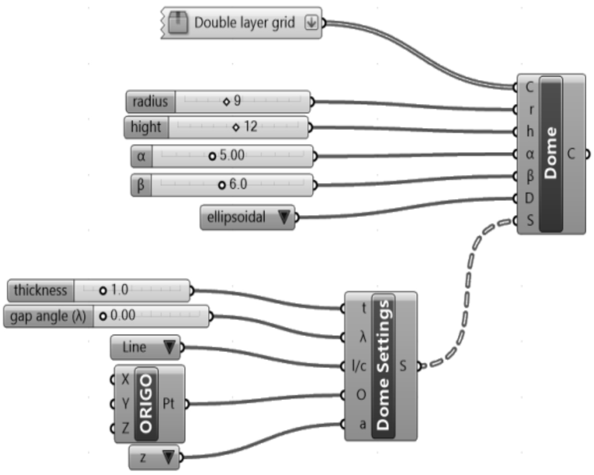

Fig. 15 Grasshopper user interface

VI. Conclusion and Further Research

The adaptation and development of the dome tool from formex algebra to Grasshopper resulted in an easy-to-use grid design tool for architects. This is the first phase of adaptation of the dome and vault structures of formex algebra to Grasshopper. The aim is to construct a plug-in for Grasshopper which contains these two structures together with the triangular grid version of them, and with an additional freeform modification tool.

Acknowledgment

Emberi Erőforrások Minisztériuma

This research is supported by the ÚNKP-17-3-IV New National Excellence Program of the Ministry of Human Capacities.

References

[1] H. Nooshin, P. L. Disney, "Formex Configuration Processing I," in *International Journal of Space Structures*, Vol 15, No 1, 2000, pp 1-52, ISSN: 0266-3511.
[2] A. Tadeshi, AAD: Algorithms Aided Design, *Le Penseur Publisher*, Brienza, 2014.
[3] H. Nooshin, P. L. Disney, "Formex Configuration Processing II," in *International Journal of Space Structures*, Vol 16, No 1, 2001, pp 1-56, ISSN: 0266-3511.
[4] S. Suresh, J. Poornaselvan, C. Divyapreya, "Optimal path planning of approach to Grid environment," in *Pollack Periodica*, Vol. 6, No. 1, 2011, pp 131-140.
[5] A. Cârstea, G. Macariu, "Towards a grid enabled symbolic computation architecture," in *Pollack Periodica*, Vol. 3, No. 2, 2008, pp 15-26.
[6] H. Nooshin, O. A. Samavati, "Some Morphological Aspects of Configuration," in Proceedings of the IASS Symposium, Tokyo, Japan, September 2016.
[7] H. Nooshin, O. A. Samavati, A. Sabzali, "Basics of Formian-K," 2016 (77 pages), ISBN 978-1-5272-0161-3.
[8]