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Forecasting the Volatility of Geophysical Time Series with Stochastic Volatility Models

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***Abstract*—This work is devoted to the study of modeling geophysical time series. A stochastic technique with time-varying parameters is used to forecast the volatility of data arising in geophysics. In this study, the volatility is defined as a logarithmic first-order autoregressive process. We observe that the inclusion of log-volatility into the time-varying parameter estimation significantly improves forecasting which is facilitated via maximum likelihood estimation. This allows us to conclude that the estimation algorithm for the corresponding one-step-ahead suggested volatility (with ± 2 standard prediction errors) is very feasible since it possesses good convergence properties.

***Keywords*—Augmented Dickey Fuller Test, geophysical time series, maximum likelihood estimation, stochastic volatility model.

I. Introduction

TIME series forecasting and its parameter estimation constitute an important area of research in which many scholars have shown an increasing interest in recent times. The development of forecasting methodologies in geophysics yields a good estimation of the type of source that generates a recorded seismic signal, and this methodology is important in many other fields, for example, the science of meteorology, and the safety of power system [1]. So, a reliable technique of volatility forecasting, including the time-varying parameters based on the seismic signals generated, is imperative to mitigate some seismic hazards of a region.

In the present study, we develop a forecasting method for inference and prediction in a volatility model in which the logarithm of the conditional volatility follows an autoregressive time series model. One of the common volatility models is the autoregressive conditional heteroskedasticity (ARCH) model by Engle [2], which was later modified into generalized autoregressive conditional heteroskedasticity (GARCH) by Bollerslev [3]. According to the GARCH, the volatility system is driven by the observed values in a pre-deterministic fashion. The GARCH model differs from the stochastic volatility (SV) model in the sense that, unlike the SV model, it does not have any stochastic noise. The SV model is identified by the fact that it invariably contains its probability density function. Moreover, the maximum likelihood method is applied to estimate the parameters of latent volatility in order to obtain a good estimation.

It is now widely believed that the time and measurements of a sequence of geophysical time series may be stochastically dependent. In other words, there is a correlation among the numbers of data points in successive time intervals. In [4], the authors used stochastic models to describe a unique type of dependence in geophysical time series. It has been observed that the geophysical data may follow different behaviors over time, for instance, the mean reversion and fluctuation of power spectrum. Such observations would justify a rather fundamental difference from the classical modeling foundations. But the concept of time-dependent seismicity suggests that the current seismicity needs to be evaluated on the basis of its past behavior [5]. This behavior of seismogram makes it possible to do good volatility forecasting and to obtain some stylized facts of the geophysical data, namely, time-varying volatility, persistence, and clustering.

The deterministic models are extensively used due to its ability to represent the stylized facts of time series and ease of identification based on maximum likelihood estimation (MLE). However, this deterministic nature does not allow for a full statistical description of volatility [6]. In order to get the statistical description of geophysical time series, we propose to apply the stochastic volatility model and filtering technique as a way to estimate parameters. We therefore study a sequence of mining explosions and a large number of aftershocks of the magnitude M=5.2 earthquake to forecast the volatility by using estimated parameters. The adequacy of the data is determined by computing the estimated standard error.

The main difficulty of SV model is to fit it into the data (with higher accuracy in a stochastic process), since their likelihood estimations involve numerical integration over higher dimensional intractable integrals [7], whose maximization seems to be complicated. Our paper provides promising results regarding the application of the SV model in geophysical time series. It also provides a continuous time-dependent process that exhibits the long-memory feature of volatilities. The presence of long-memory behavior suggests that there is a close correlation between current information and past information at different time intervals, and this enables us to make prediction. The methodology used in this work can be applied to other disciplines such as statistics, mathematics, and finance.

The paper is organized as follows: in Section II, we present a brief overview of the stochastic volatility model with time-varying parameters. Section III includes the estimation

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procedure that has been followed to forecast the parameters with standard prediction errors. Section IV is devoted to a brief description of our data and a motivation to use the SV model in geophysics. We then analyze the stationarity and application of volatility technique to the seismograms containing the seismic waves generated by the earthquakes and the explosions. Finally, in Section V, we provide a conclusion that recapitulates the main points.

II. Stochastic Volatility Model

In this section, we briefly describe the stochastic volatility (SV) model used in this study. The stochastic volatility technique incorporated into our model implies that the volatility is driven by an innovation sequence, that is, independent of observations [8]. It causes the volatility through an unobservable process that allows it (volatility) to vary stochastically. The observations y_t of the time series used in this paper may be represented as:

$$y_t = \sigma_t \eta_t, \tag{1}$$

where σ_t is the volatility of the observations, and $\{\eta_t\}_{t\in\mathbb{N}}$ (which is independent of $\{\sigma_t\}_{t\in\mathbb{N}}$ and $\{y_t\}_{t\in\mathbb{N}}$) is a Gaussian white noise sequence.

To develop the SV model, we use the log-squared observations of the time series in (1):

$$log y_t^2 = log\sigma_t^2 + log\eta_t^2$$

which can be rewritten as:

$$m_t = s_t + log\eta_t^2, \tag{2}$$

where $m_t = log y_t^2$ and $s_t = log\sigma_t^2$. Thus the observations m_t are generated by two components namely, the unobserved volatility s_t and the unobserved noise $log\eta_t^2$. Considering the autoregression, the first term on the right hand side of (2) i.e. s_t can be expressed as:

$$s_t = \upsilon_0 + \upsilon_1 s_{t-1} + \omega_t, \tag{3}$$

where ω_t is a white Gaussian noise with the variance σ_ω^2. Equations (2) and (3) constitute the stochastic volatility model by Taylor [9]. To compute the observation noise, we take into account the mixtures of two Normal distributions with one centered at zero. Thus, we have:

$$y_t = \beta + s_t + \gamma_t, \tag{4}$$

where β is the mean of log-squared observations and $\gamma_t = B_t z_{t0} - (B_t - 1) z_{t1}$, which fulfills the following conditions:

$$\begin{aligned} z_{t0} &\sim \text{i.i.d N}(0, \sigma_0^2), \\ z_{t1} &\sim \text{i.i.d N}(\mu_1, \sigma_1^2), \\ \text{and } B_t &\sim \text{i.i.d Bernoulli } (p), \end{aligned}$$

where p is an unknown mixing probability and i.i.d means independently and identically distributed. We therefore define the time-varying probabilities $\Pr\{B_t = 0\} = p_0$ and $\Pr\{B_t = 1\} = p_1$, where $p_0 + p_1 = 1$. In this study, our approach is to estimate the parameters $\upsilon_0, \upsilon_1, \sigma_\omega, \sigma_0$, and σ_1 from the given data sets and to analyze their forecasting behavior.

III. Estimation Procedure

This section describes the estimation of time-varying parameters of the time series arising in geophysics. First, we briefly discuss some techniques that will be used to estimate the parameters of the proposed model.

A. Filtering Approach

The state space model is defined by a relation between the m-dimensional observed time series, $\mathbf{y}_t$, and the n-dimensional state vector (possibly unobserved), $\mathbf{x}_t$ [10]. An observed equation is driven by the stochastic process as follows:

$$\mathbf{y}_t = \mathbf{A}_t \mathbf{x}_t + \mathbf{w}_t, \tag{5}$$

where $\mathbf{A}_t$ is a $m \times n$ observation matrix, $\mathbf{x}_t$ is a vector of $n \times 1$, and $\mathbf{w}_t$ is a Gaussian error term ($\mathbf{w}_t \sim N(0, \delta_t)$).
The unobservable vector $\mathbf{x}_t$ is generated from the transition equation which is defined as:

$$\mathbf{x}_t = \mathbf{\Phi} \mathbf{x}_{t-1} + \mathbf{v}_t, \tag{6}$$

where $\mathbf{\Phi}$ is a $n \times n$ transition matrix and $\mathbf{v}_t \sim$ i.i.d $N(0, \psi_t)$. We assume that the process starts with a Normal vector $\mathbf{x_0}$. From Eqs. (5) and (6), we make estimation for the underlying unobserved data $\mathbf{x}_t$ from the given data $\mathbf{Y}_m = \{y_1, \ldots, y_m\}$. When $m = t$, the process is called filtering.

B. Likelihood Approximation

Let α denote the parameters of the state space model, which are embedded in the system matrices $\mathbf{A}_t, \mathbf{\Phi}, \delta_\mathbf{t}$, and ψ_t. These parameters are typically unknown, but estimated from the data $Y = y_1, \ldots, y_m$.

The likelihood $L(\alpha|Y)$ is a function that assigns a value to each point in the parameter space Δ which suggests the likelihood of each value in generating the data. However, the likelihood is proportional to the joint probability distribution of the data as a function of the unknown parameters. The maximum likelihood estimation means the estimation of the value of $\alpha \in \Delta$ that is most likely to generate the vector of the observed data y_t [11]. We may represent this as:

$$\begin{aligned} \hat{\alpha}_{MLE} &= \max_{\alpha\in\Delta} L(\alpha|Y) = \max_{\alpha\in\Delta} L_Y(\alpha) \\ &= \max_{\alpha\in\Delta} \prod_{t=1}^{m} f(y_t|y_{t-1}; \alpha), \end{aligned} \tag{7}$$

where $\hat{\alpha}$ is the maximum likelihood estimator of α. Since the natural logarithm function increases on $(0, \infty)$, the maximum value of the likelihood function, if it exists, occurs at the same points as the maximum value of the logarithm of the likelihood function. In this paper, we propose to work with the log-likelihood function which is defined as:

$$\begin{aligned} \hat{\alpha}_{MLE} &= \max_{\alpha\in\Delta} lnL(\alpha|Y) = \max_{\alpha\in\Delta} lnL_Y(\alpha) \\ &= \max_{\alpha\in\Delta} \sum_{t=1}^{m} lnf(y_t|y_{t-1}; \alpha). \end{aligned} \tag{8}$$

Since this is a highly non-linear and complicated function of the unknown parameters, we first consider the initial state vector $\mathbf{x}_0$ and develop a set of recursions for the log-likelihood function with its first two derivatives [12]. We then use Newton-Raphson algorithm [13] successively until the negative of the log-likelihood is minimized to obtain the MLE.

C. Parameter Estimation

In order to estimate the time-varying parameters, we use the filtering technique that is followed by three steps namely, forecasting, updating, and parameter estimation. In the first step, we forecast the unobserved state vector s_t on time series observations as follows:

$$s_{t+1}^t = \upsilon_0 + \upsilon_1 s_t^{t-1} + \sum_{j=0}^{1} p_{tj} K_{tj} \eta_{tj}, \tag{9}$$

where the predicted state estimators $s_t^{t-1} = E(s_t|y_1, \ldots, y_{t-1})$. The corresponding error covariance matrix is defined as:

$$M_{t+1}^t = \upsilon_1^2 M_t^{t-1} + \sigma_\omega^2 - \sum_{j=0}^{1} p_{tj} K_{tj}^2 \sum\nolimits_{tj}. \tag{10}$$

At this point, the innovation covariances are given as:

$$\sum_{t0} = M_t^{t-1} + \sigma_0^2$$
$$\text{and } \sum_{t1} = M_t^{t-1} + \sigma_1^2,$$

where $M_t^{t-1} = \Phi M_{t-1}^{t-1} \Phi^T + V$, $M_0^0 = \sum_0$, $\sum_t = \text{var}(\eta_t)$, and $V = \text{var}(w_t)$. Furthermore, we use Kalman filter [14] to measure the estimates precision, which may be shown as:

$$K_{t0} = \upsilon_1 M_t^{t-1} / (M_t^{t-1} + \sigma_0^2)$$
$$\text{and } K_{t1} = \upsilon_1 M_t^{t-1} / (M_t^{t-1} + \sigma_1^2). \tag{11}$$

The second step deals with updating results while we have a new observation of y_t at time t. The prediction errors of the likelihood function are computed using the following relations:

$$\eta_{t0} = y_t - \beta - s_t^{t-1}$$
$$\text{and } \eta_{t1} = y_t - \beta - s_t^{t-1} - \mu_1. \tag{12}$$

For estimating the parameters, we complete the updating step by assessing the time-varying probabilities (for $t = 1, \ldots, m$):

$$p_{t1} = \frac{p_1 d_1(t|t-1)}{p_0 d_0(t|t-1) + p_1 d_1(t|t-1)}$$
$$\text{and } p_{t0} = 1 - p_{t1},$$

where $d_j(t|t-1)$ is considered to be the conditional density of y_t, given the previous observations $y_1, \ldots, y_{t-1}$.

Since the observation noise of this model is not fully Gaussian, it is computationally difficult to obtain the exact values of $d_j(t|t-1)$. Hence, we use a good approximation of $d_j(t|t-1)$ that provides Normal density which is: $N(s_t^{t-1} + \mu_j, \sum_{tj})$, for $j = 0, 1$ and $\mu_0 = 0$.

Finally, we estimate the parameters ($\Theta = (\upsilon_0, \upsilon_1, \sigma_w, \beta, \sigma_0, \mu_1, \sigma_1)'$) by maximizing the expected likelihood, where the MLE is represented as:

$$lnL_Y(\Theta) = \sum_{t=1}^{m} ln\Big(\sum_{j=0}^{1} p_j d_j(t|t-1)\Big). \tag{13}$$

IV. Application of the Stochastic Volatility to Geophysical Data

A. Background of Data

The earthquakes used in this study correspond to a set of M=3.0-3.3 aftershocks of a recent M=5.2 intraplate earthquake which occurred on June 26, 2014. These earthquakes were located near the town of Clifton, Arizona, where a large surface copper mine previously triggered off several explosions due to quarry blasts activities. We selected some explosions cataloged with similar magnitude as the earthquakes (M=3.0-3.3) and located in the same region within a radius of 10 km [15]. We collected the seismograms containing the seismic waves from two nearby seismic stations, IU.TUC and IU.ANMO (see Fig. 1), located at an average distance from the epicenters of 161 km and 357 km respectively. The data contains information about the date, time, longitude, latitude, the average distance to seismic events, average azimuth, and the magnitude of each seismic event in the region (see Tables I and II).

We downloaded the broadband vertical components (Z-component) seismograms from the Incorporated Research Institutions for Seismology Data Management Centers (IRIS DMC). We used the Seismic Analysis Code (SAC) software to remove the instrument's response of the seismometer. We cut the seismograms to contain $0 - 200$ s with respect to the event time origin in order to capture the main seismic waves train [15]. The sampling rate for both recording stations is 20 samples per second. Thus, the resulting seismograms represent the time series of the vertical displacement of the ground (in nm) caused by the passing of the seismic waves generated by the explosions and earthquakes.

B. Motivation

In this subsection, we study the nature of time series regarding geophysics. Indeed, it is the dynamic behavior of the data that encourages us to apply our methodology in this paper.

In Figs. 2 and 3, we notice that the frequency components change from one interval to another in earthquake or explosion as long as it lasts. The mean of the series appears to be stable with an average magnitude of approximately zero, which reflects both the time-varying nature and the mean reversion characteristics of the data. We observe that the volatility changes at a short interval and that the periods of high volatility tend to be correlated. This shows that the volatility itself is very volatile. The fluctuations of magnitudes typically exhibit the volatility clustering i.e. small changes in the seismogram tend to be followed by small changes, and large changes by large ones. This volatile nature of the data justifies the use of volatility model to fit the data for studying their physical dynamic behavior.

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TABLE I
STATIONS INFORMATION

Station	Network	Latitude	Longitude	Avg. distance (km)	Avg. Azimuth (deg)
TUC	IU	32.3°	−110.8°	161	76
ANMO	IU	34.9°	−106.5°	357	224

TABLE II
EVENTS INFORMATION

Events	Magnitude	Date	Time (UTC)	Latitude	Longitude
Earthquake	3.0	7/12/14	7:12:53	32.58°	−109.08°
Explosion	3.2	12/23/99	21:15:48	32.65°	−109.08°

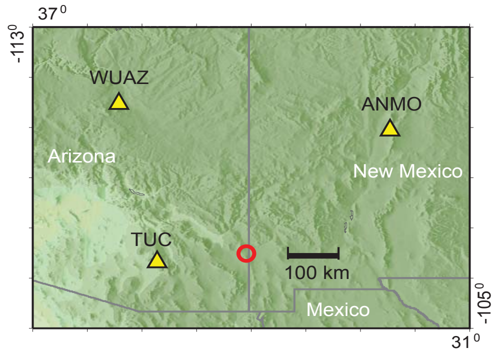
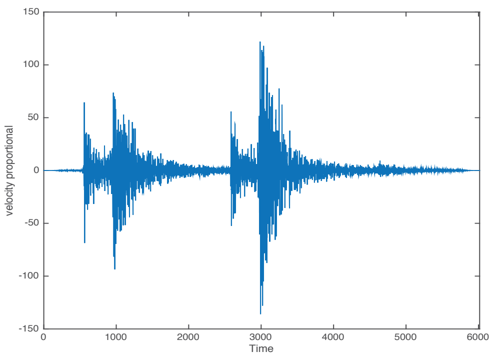
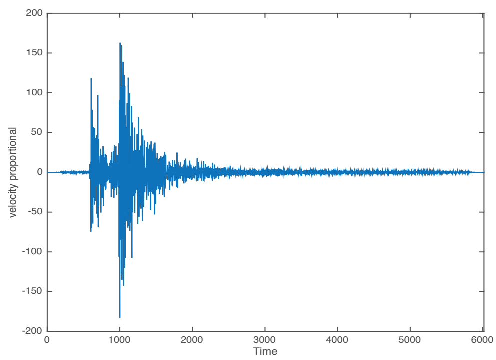


Fig. 1 The map shows the location of the seismic stations IU.TUC and IU.ANMO ([15]) used in this study (yellow color triangles). Red open circle represents the area within which the earthquakes and explosions used in this study are located

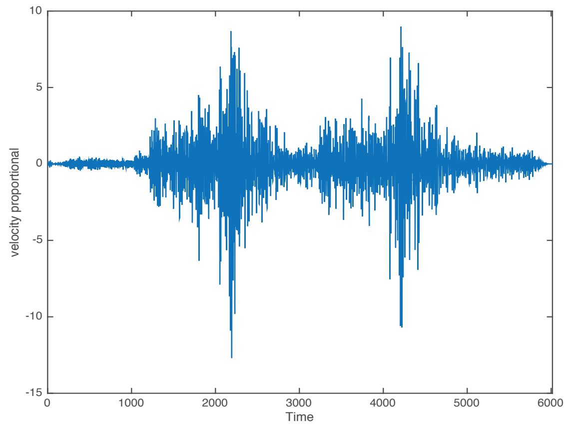


(a) Earthquake (IU.TUC)

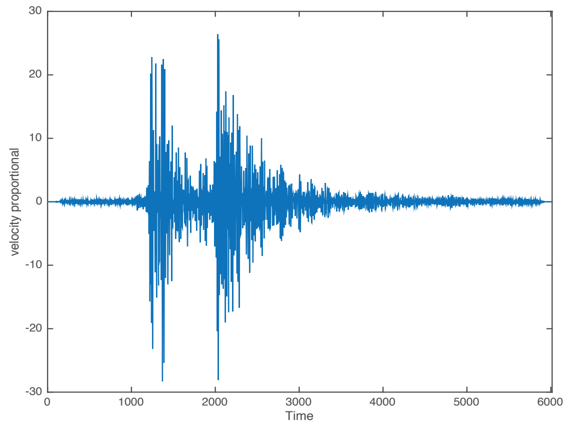


(b) Explosion (IU.TUC)

Fig. 2 The vertical ground displacement in nanometers of earthquake (a) and explosion (b) in Table II as recorded by station TUC



(a) Earthquake (IU.ANMO)



(b) Explosion (IU.ANMO)

Fig. 3 The vertical ground displacement in nanometers of earthquake (a) and explosion (b) in Table II as recorded by station ANMO

C. Results and Discussion

We begin this subsection by testing for a unit root in the seismic waves generated by the earthquake and explosion time series using the Augmented Dickey Fuller Test (ADF) tests. The ADF is a very powerful test that can handle more complex models. It tests the null hypothesis that a time series y_t is a unit root against the alternative that it is stationary, assuming that the dynamics in the data have an ARMA structure [16].

First, we test the stationarity for all the earthquake and explosion time series used in this paper by using the ADF test. The summary statistics for the results of this test for the TUC and ANMO stations are displayed in Tables III and IV, respectively.

TABLE III
AUGMENTED DICKEY FULLER T-STATISTIC TEST FOR TUC STATION

Events	Test statistic	p-value
Earthquake	-39.088	0.01
Explosion	-40.298	0.01

TABLE IV
AUGMENTED DICKEY FULLER T-STATISTIC TEST FOR ANMO STATION

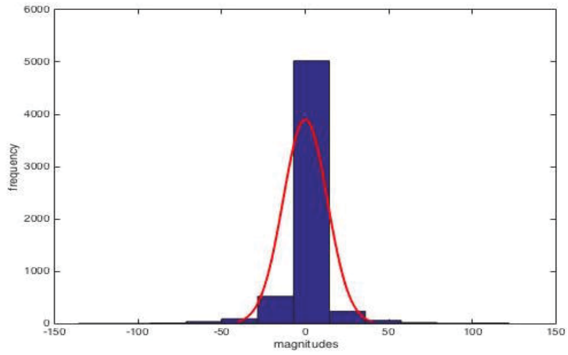
Events	Test statistic	p-value
Earthquake	-32.313	0.01
Explosion	-37.264	0.01

Test interpretation:
H_0 : There is a unit root for the time series.
H_a : There is no unit root for the time series. This series is stationary.

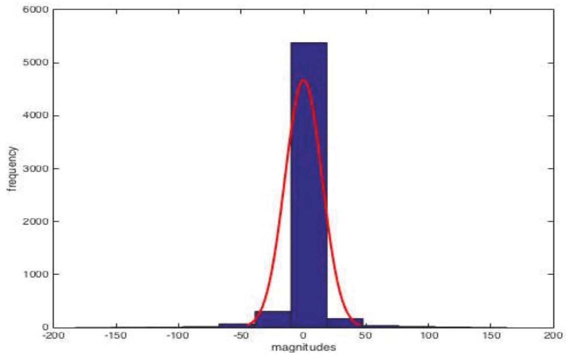
As the computed p-value is lower than the significance level $\alpha = 0.05$, we reject the null hypothesis H_0 in all four events, and accept the alternative hypothesis H_a. Thus, the events under study are all stationary time series. We also observed that the explosions are more stationary than earthquake. This is because for the ADF test, the more negative value of test statistic, the stronger the rejection of hypothesis that there is a unit root for the time series at some level of confidence.

The dynamics of the series changes with time and we forecast the time series by using volatility technique. As we see in Fig. 4, the histograms of 6016 observations of geophysical time series are well represented. The thin red line in the diagram shows the theoretical probability density function of Normal distribution with the same mean and standard deviation as geophysical data. We therefore consider the ARCH Normality assumption on the basis of volatility η_t. The time-varying parameters and the fixed parameter (β) were initialized in order to observe the performance of the SV algorithms during a set of magnitudes for each seismic event. We assumed the initial parameters to be $\upsilon_0 = 0, \upsilon_1 = 0.96, \sigma_\omega = 0.3, \sigma_0 = 1, \mu_1 = -4, \sigma_1 = 3$, and $\beta =$ the mean of the observations. In order to maximize Eq. (13), the innovation processes for (3) and (4) were fitted to the data by considering this time-varying probability ($p_1 = 0.5$). This analysis was performed by a module obtained through R statistical software.

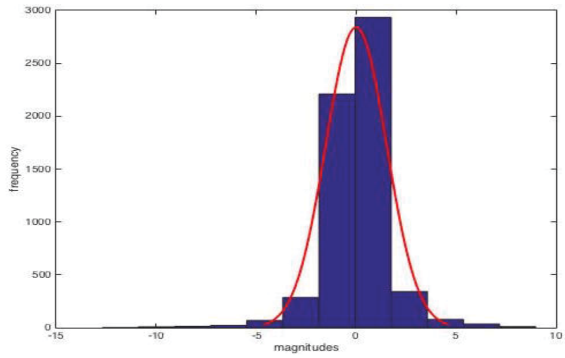
Tables V-VIII summarize the estimation of parameters (υ_0, υ_1, σ_w, β, σ_0, μ_1, and σ_1). The estimated error in these tables makes two things evident: firstly, the estimates are close to the true parameters; secondly, the algorithm of the SV model is consistent with the results obtained by using the geophysical data. The variance σ_w^2 of the log-volatility process measures the uncertainty about the future volatility of data. If the value of σ_w^2 is zero, it is not possible to identify the SV model. The parameter υ_1 is considered as a measure of the persistence of shocks to the volatility. Tables V-VIII indicate that υ_1 is less than 1, which suggests that the latent volatility process and y_t are stationary. In these tables, we notice that υ_1 is near to unity and σ_w^2 is different from 0,



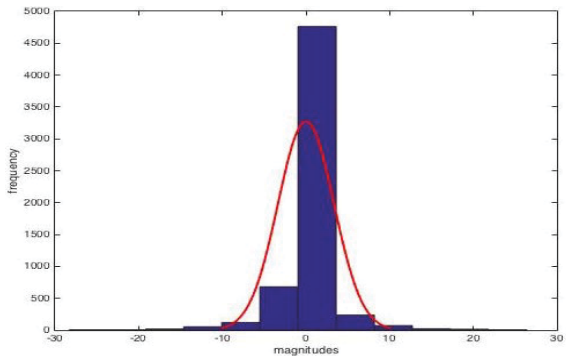
(a) Earthquake (TUC station)



(b) Explosion (TUC station)



(c) Earthquake (ANMO station)



(d) Explosion (ANMO station)

Fig. 4 The histograms of geophysical time series and the fitted Normal density which means that the evolution of volatility is not smooth over time. This also suggests that the geophysical time series could be heteroscedastic by nature, that is, there is a non-constant volatility over time. So, it is very useful to control the risk or to mitigate the effect of hazards. For example, if there are two seismic time series having the same mean but with different variances, we would then consider the series with lower variance, because it is less risky.

TABLE V
SUMMARY STATISTICS FOR EARTHQUAKE DATA FROM TUC STATION

Parameter	Estimate	Standard Error
υ_0	-0.0005	0.0093
υ_1	0.9994	0.14E-04
σ_ω	0.6127	0.0749
β	-7.1967	0.5468
σ_0	0.4723	0.0913
μ_1	-2.3616	0.0898
σ_1	2.4418	0.0566

TABLE VI
SUMMARY STATISTICS FOR EXPLOSION DATA FROM TUC STATION

Parameter	Estimate	Standard Error
υ_0	0.0176	0.0514
υ_1	0.9942	0.0019
σ_ω	0.5253	0.0466
β	-0.5354	8.5901
σ_0	0.5555	0.0544
μ_1	-2.4134	0.0884
σ_1	2.4086	0.0531

TABLE VII
SUMMARY STATISTICS FOR EARTHQUAKE DATA FROM ANMO STATION

Parameter	Estimate	Standard Error
υ_0	0.1653	0.1192
υ_1	0.9814	0.0032
σ_ω	0.7283	0.0180
β	-8.8430	5.7360
σ_0	0.0001	0.0643
μ_1	-2.3918	0.0761
σ_1	2.1528	0.0475

TABLE VIII
SUMMARY STATISTICS FOR EXPLOSION DATA FROM ANMO STATION

Parameter	Estimate	Standard Error
υ_0	0.1261	0.1000
υ_1	0.9848	0.0029
σ_ω	0.7154	0.0168
β	-7.9347	5.9560
σ_0	0.99E-05	0.0790
μ_1	-2.3382	0.0769
σ_1	2.2432	0.0494

V. CONCLUSION

In this study, we have implemented a technique that incorporates time-varying parameters, which are used to forecast the volatility of a geophysical time series.

We estimated these parameters based on recent and large datasets of magnitudes from earthquakes and mining explosions. Since the data reflects stochastic behavior of most measurements over time, we therefore use the SV model to fit the data, which is strictly recursive. The filtering technique of this model is based on three continuous steps i.e. forecasting, updating, and parameter estimation. Thus, the fitted model allows us to capture the evolution of volatility that is the physical and long-memory behavior of the data. With the use of squared logarithm of observations, we succeeded in making a good prediction despite the variation of the observational noise from a Normal mixture distribution, because the data regarding geophysics studied is not fully Gaussian (see the histograms in Fig. 4).

The adequate choice of maximum likelihood computation suggests that our proposed model aligns with the geophysical time series since the one-step-ahead predictions were made on the basis of the MLE algorithm indicated. It is evident that the estimates obtained are stable around the true value (see Tables V-VIII). In order to facilitate the understanding of the forecasting concepts, we superimposed the plot of one-step-ahead predicted volatility and ± 2 standard prediction errors in Figs. 5-8. The predicted log-volatility with $\pm 2\widehat{\sigma_t}$ is displayed as a dashed line surrounding the original output. It visually shows how the values of predicted volatility differ over time.

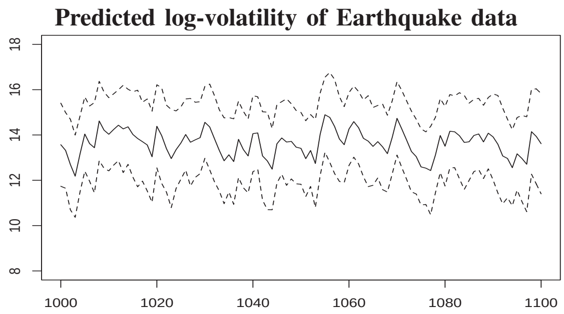


Fig. 5 One-step-ahead predicted log-volatility, with ± 2 standard prediction errors for one hundred observations of earthquake data from TUC station

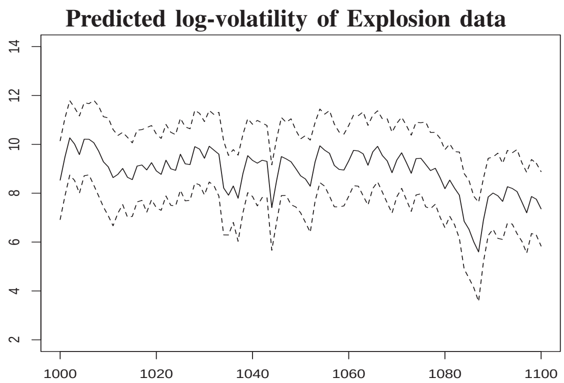


Fig. 6 One-step-ahead predicted log-volatility, with ± 2 standard prediction errors for one hundred observations of explosion data from TUC station

In these figures, we notice that the predicted volatility of explosions changes very smoothly in comparison to the earthquakes. This suggests that the persistence in the explosives volatility is higher than that of the earthquakes. Furthermore, Tables III and IV indicate that the explosion time

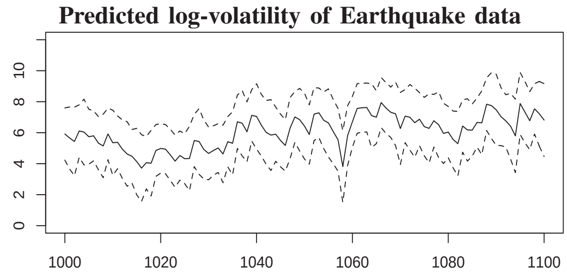


Fig. 7 One-step-ahead predicted log-volatility, with ±2 standard prediction errors for one hundred observations of earthquake data from ANMO station

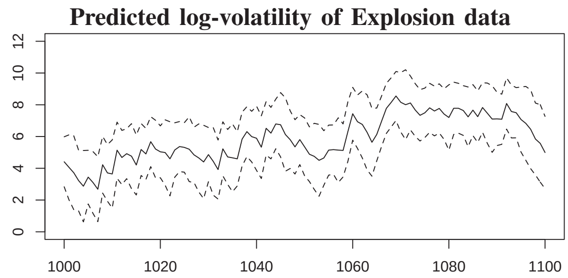


Fig. 8 One-step-ahead predicted log-volatility, with ±2 standard prediction errors for one hundred observations of explosion data from ANMO station

series is more stationary than the earthquake time series. Thus we conclude that the more stationary time series data have higher volatility persistence with time in comparison to the less stationary data.

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