Footbridge Response on Single Pedestrian Induced Vibration Analysis

J. Kala, V. Salajka and P. Hradil

Abstract—Many footbridges have natural frequencies that coincide with the dominant frequencies of the pedestrian-induced load and therefore they have a potential to suffer excessive vibrations under dynamic loads induced by pedestrians. Some of the design standards introduce load models for pedestrian loads applicable for simple structures. Load modeling for more complex structures, on the other hand, is most often left to the designer. The main focus of this paper is on the human induced forces transmitted to a footbridge and on the ways these loads can be modeled to be used in the dynamic design of footbridges. Also design criteria and load models proposed by widely used standards were introduced and a comparison was made. The dynamic analysis of the suspension bridge in Kolin in the Czech Republic was performed on detailed FEM model using the ANSYS program system. An attempt to model the load imposed by a single person and a crowd of pedestrians resulted in displacements and accelerations that are compared with serviceability criteria.

Keywords—Footbridge, Serviceability, Pedestrian action, Numerical analysis.

I. INTRODUCTION

AMONG different types of human-induced loads on footbridges, walking force caused by a single pedestrian was established in the past as the most important load type because of its most frequent occurrence. Also, almost all existing force models for this type of load (defined either in the time or frequency domain) are developed from the assumption of perfect periodicity of the force and are based on force measurements conducted on rigid (i.e. high frequency) surfaces. However, footbridges which exhibit vibration serviceability problems are low-frequency flexible structures with natural frequencies within the normal walking frequency range. In such a situation, walking at a near resonant frequency is expected to generate the highest level of response as considered in the published literature. However, the walking force is not perfectly periodic [4] and it could be attenuated due to interaction between the pedestrian and the structure [9]. These two facts deserve more attention in future force modeling. Apart from a single person walking, a group of pedestrians walking at the same speed to maintain the group consistency are a very frequent load type on footbridges in urban areas.

Dynamic load impact by crowd was not researched much in the past, especially in relation to pedestrian bridges. Wheeler [12] and Grundmann et al. [7] were among a handful of researchers who investigated this issue. They found that, under this type of load, footbridges with a natural frequency of around 2 Hz are prone to experience vibrations at a higher level than those induced by a single pedestrian because of synchronization of walking steps between people in the group. However, there is no group force model which is generally accepted.

II. DYNAMIC LOADS INDUCED BY PEDESTRIANS

Pedestrian loading, whether walking or running, was studied rather thoroughly and is translated as a point force exerted on the support, as a function of time and pedestrian position. Considering that x is the pedestrian position in relation to the footbridge centerline, the load of a pedestrian moving at constant speed v can therefore be represented as the product of a time component $F(t)$ by a space component $\delta(x - vt)$, δ being the Dirac operator, that is:

$$P(x,t) = F(t)\,\delta\,(x - vt) \tag{1}$$

In common design practice, only $F(t)$ is taken into consideration.

A. Vertical loads

Several measurements were conducted to quantify vertical loads imposed by pedestrians on structures. Most measurements indicate that the shape of the vertical force produced by one person taking one step is of the kind shown in Fig. 1.
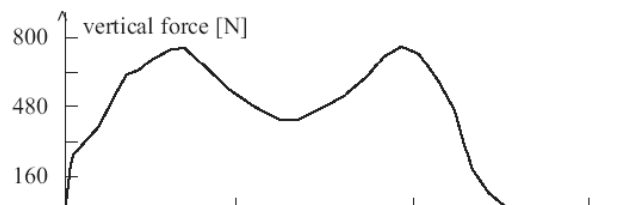

Fig. 1 Vertical force produced by one person taking one step [1].

Measurements of continuous walking were also done. The measured time histories were near periodic with an average period equal to the average step frequency. General shapes for

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continuous forces in both vertical and horizontal directions were constructed assuming a perfect periodicity of the force, see Figure 2: Periodic walking time histories in vertical directions [13]. As mentioned in the previous section, the vertical forcing frequency is generally in the region of 1.4 – 2.4 Hz [2]. This was confirmed by several experiments, for example by Matsumoto, who investigated a sample of 505 persons. He concluded that the pacing frequencies followed a normal distribution with a mean of 2.0 Hz and a standard deviation of 0.173 Hz.
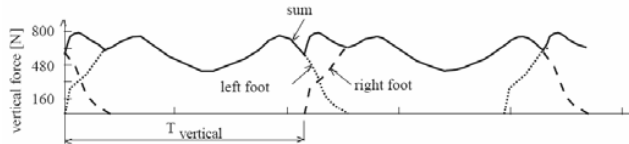
Fig. 2 General shapes for continuous vertical forces

To verify the different authors conclusions the vertical pressure measurements invoked during the walking was performed. Three sensors with average base 0.20 m placed on rigid platform were used. Distance between the gauge axes in the direction of movement was equal to 0.9 m. Configuration gauging basis can be seen on Figure 3. In Figure 4 are in different color the effects from each gauge (normalized to static load). In Figure 5 is added blue resulting curve.

In terms of the experiment was examined also the force transmission at the step from heel to the toe (one gauge for heel, second for toe, third for second legs heels) what can be seen in figure 6. Measurements results for this configuration can be seen in figure 7. This configuration confirmed measurements results for the configuration with one sensor for each step. Measurements were effected in sports also with home footwear, in addition was placed on surface sensors various mats (e.g . 15 mm of polystyrene). In terms of these variants there wasn't ascertained the measurable influence neither using footwear nor adjustment surface on resulting force record.

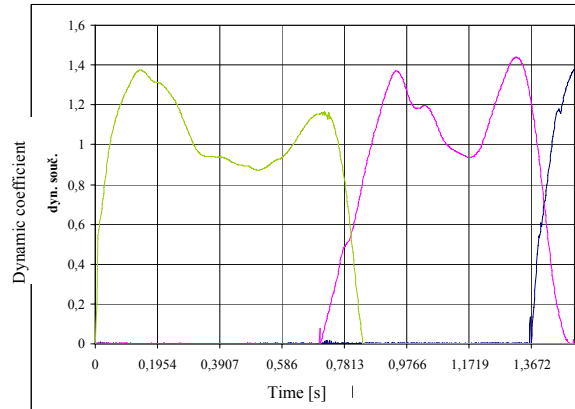Fig. 3 Gauge configuration – variant 1

Fig. 4 Force record from gauge configuration – variant 1
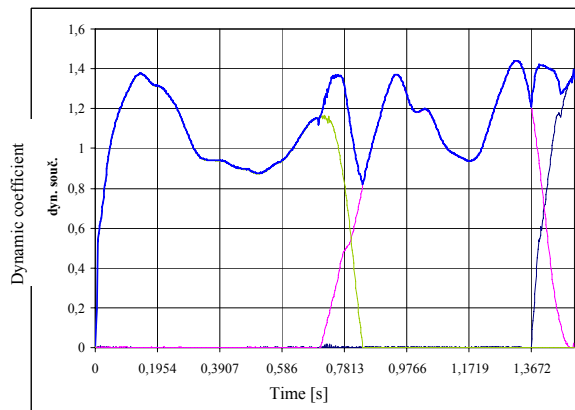

Fig. 5 Force record from gauge configuration – variant 1

This force record was represented as a Fourier series. The step frequency for this record was approximately 1.55 Hz, i. e. the speed 1.4 m.s^{-1}. Dynamic coefficient for harmonics members ($α_1 = 0.32$, $α_2 = 0.09$, $α_3 = 0.12$, $α_4 = 0.02$) were derived from the graph 8. The record is normalized to the static weight. Experimentally Obtained results are agreement with results in table I.
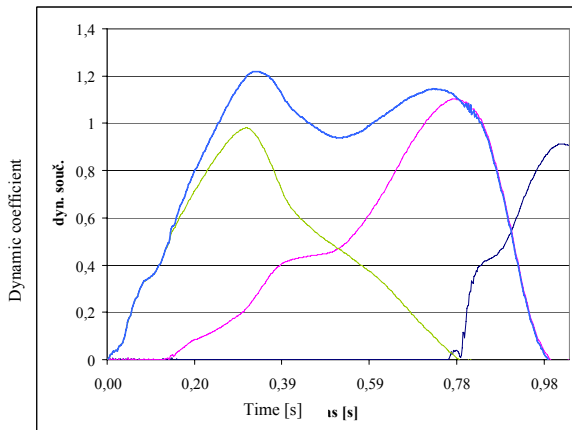

Fig. 6 Gauge configuration – variant 2

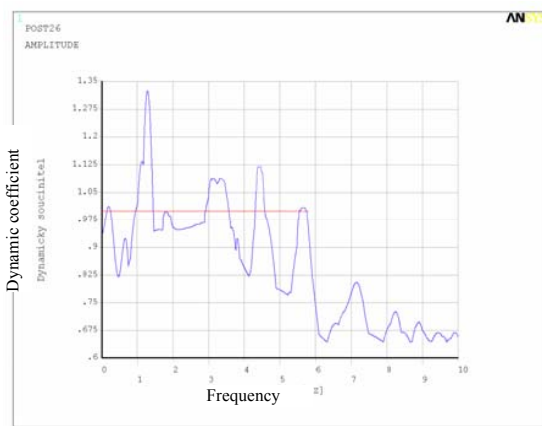Fig. 7 Force record from gauge configuration – variant 2

Fig. 8 Force-frequency record

B. Periodic load models

Periodic load models are based on an assumption that all pedestrians produce exactly the same force and that this force is periodic [38]. It is also assumed that the force produced by a single pedestrian is constant in time.

Dynamic loading caused by a moving pedestrian may be considered a periodic force. This force $F(t)$ can be represented as a Fourier series in which the fundamental harmonic has a frequency equal to the pacing rate [1]:

$$F(t) = G + \sum_k \alpha_n G \cdot \sin(2\pi ft + \varphi_n) \qquad (2)$$

where G is the pedestrian's weight, α_n is the load factor of the n-th harmonic, f is the frequency of the force, φ_n is the phase shift of the n-th harmonic, n is the number of the harmonic and k is the total number of contributing harmonics [13].

Several measurements were made in order to quantify the load factor α_n which is essential for this load model. The results of such measurements are shown in Table I.

In 1977, Blanchard et al. proposed a vertical dynamic load factor of 0.257. Ten years later, Bachmann and Ammann reported the first five harmonics of the vertical as well as the horizontal force. They found the first harmonic of the vertical dynamic load to be 37% of the vertical static load and the first

harmonic of the horizontal dynamic load to be 3,9% of the vertical static load.

TABLE I
INPUT RANDOM QUANTITIES

Author	α		Freq.
Young [11]	$\alpha_1 = 0.37(f\text{-}0.95) \leq 0.5$ $\alpha_2 = 0.054+0.0044f$ $\alpha_3 = 0.026+0.0050f$ $\alpha_4 = 0.010+0.0051f$	walk – vertical	average α
Setra [10]	$\alpha_1 = 0.4$. $\delta_2 = \delta_3 = 0.1$	walk – vertical	
	$\alpha_{1/2} = \delta_{3/2} = 0.05$, $\delta_1 = \delta_2 = 0.01$	walk – transverse	
	$\alpha_{1/2} = 0.04$, $\alpha_1 = 0.2$, $\alpha_{3/2} = 0.03$, $\alpha_2 = 0.01$	walk – longitudal	
Bachmann [1]	$\alpha_1 = 1.8/1.7$, $\alpha_2 = 1.3/1.1$, $\alpha_3 = 0.7/.5$	normal jump	2.0 - 3.0
	$\alpha_1 = 1.9/1.8$, $\alpha_2 = 1.6/1.3$, $\alpha_3 = 1.1/.8$	high jump	2.0 - 3.0
	$\alpha_1 = 0.17/0.38$, $\alpha_2 = 0.1/0.12$, $\alpha_3 = 0.04/0.02$	swaying	1.6 – 2.4
	$\alpha_1 = 0.5$	swaying - standing	0.6

In 2001, a year after the opening of the Millennium Bridge, Young presented the work of several researchers. The principles of this work are now used by Arup Consulting Engineers when modeling walking forces and the corresponding structural responses. Young proposed the first four harmonics of the vertical force as a function of the walking frequency f, see Table I [13].

All these tests, performed in order to quantify the load factors, were carried out by direct or indirect force measurements on rigid surfaces [13]. It has already been stated that horizontal movements of the surface seem to increase the horizontal pedestrian force.

III. DESIGN CODE APPROACH

A. British standard

The BD 29/04 further states that all footbridges shall satisfy the vibration serviceability requirements set out in BD 37/01 [3]: Appendix B5.5. There, it is stated that if the fundamental natural frequency of vibration exceeds 5 Hz for the unloaded bridge in the vertical direction and 1,5 Hz for the loaded bridge in the horizontal direction, the vibration serviceability requirement is deemed to be satisfied.

If the fundamental frequency of vertical vibration, on the other hand, is less than, or equal to 5 Hz, the maximum vertical acceleration of any part of the bridge shall be limited to

$$a_{lim} = 0.5\sqrt{f} \qquad (3)$$

The maximum vertical acceleration can be calculated either with a simplified method or a general method. The simplified method for deriving the maximum vertical acceleration given in BD 37/01 is only valid for single span, or two-or-three-span continuous, symmetric, simply supported superstructures of constant cross section. For more complex superstructures, the maximum vertical acceleration should be calculated assuming

that the dynamic loading applied by a pedestrian can be represented by a pulsating point load F, moving across the main span of the bridge at a constant speed v_t as follows:

$$F = 180 \sin(2\pi f t) \qquad (4)$$

$$v_t = 0.9 \cdot f$$

where f is the fundamental natural frequency of the bridge and t is the time. If the fundamental frequency of horizontal vibration is less than 1.5 Hz, special consideration shall be given to the possibility of excitation by pedestrians of lateral movements of unacceptable magnitude. Bridges having low mass and damping and expected to be used by crowds of people are particularly susceptible to such vibrations. The method for deriving maximum horizontal acceleration is, however, not given [3].

B. Eurocode

In EN1990: Basis of Structural Design [5], it is stated that pedestrian comfort criteria for serviceability should be defined in terms of maximum acceptable acceleration of any part of the deck. Also, recommended maximum values for any part of the deck are given, see Table II [5].

TABLE II
MAXIMUM ACCEPTABLE ACCELERATION, EN1990.

	Maximum acceleration
Vertical vibrations	0.7 m.s^{-2}
Horizontal vib., normal use	0.2 m.s^{-2}
Horizontal vibration, crowd	0.4 m.s^{-2}

The standard Eurocode 1: Part 2, defines models of traffic loads for the design of road bridges, footbridges and railway bridges. Chapter 5.7 deals with dynamic models of pedestrian loads. It states that, depending on the dynamic characteristics of the structure, the relevant natural frequencies of the main structure of the bridge deck should be assessed from an appropriate structural model. Further, it states that forces exerted by pedestrians with a frequency identical to one of the natural frequencies of the bridge can result into resonance and need be taken into account for limit state verifications in relation with vibrations. Finally, Eurocode 1 states that an appropriate dynamic model of the pedestrian load as well as the comfort criteria should be defined. The methods for modeling pedestrian loads are, however, left to the designer.

Eurocode 5, Part 2 [6] contains information relevant to design of timber bridges. It requires the calculation of the acceleration response of a bridge caused by small groups and streams of pedestrians in both vertical and lateral directions. The acceptable acceleration is the same as in EN1990, 0.7 and 0.2 m.s^{-2} in the vertical and the horizontal directions, respectively. A verification of these comfort criteria should be performed for bridges with natural frequencies lower than 5 Hz for the vertical modes and below 2.5 Hz for the horizontal modes [13]. A simplified method for calculating vibrations caused by pedestrians on simply supported beams is given in Eurocode 5: Annex B [6]. Load models and analysis methods for more complex structures are, on the other hand, left to the

designer. In Eurocode 5, it is also stated that the data used in the calculations, and therefore the results, are subject to very high uncertainties. Therefore, if the comfort criteria are not satisfied with a significant margin, it may be necessary to make provision in the design for the possible installation of dampers in the structure after its completion [6].

C. ISO 10137

The ISO 10137 guidelines [8] are developed by the International Organization for Standardization with the objective of presenting the principles for predicting vibrations at the design stage and also to assess the acceptability of vibrations in structures.

ISO 10137 defines the vibration source, path and receiver as three key issues which require consideration when dealing with the vibration serviceability of structures.

The vibration source produces dynamic forces or actions (pedestrians). The medium of the structure between source and receiver constitutes the transmission path (the bridge). The receiver of the vibrations is then again the pedestrians of the bridge. According to ISO 10137, the analysis of response requires a calculation model that incorporates the characteristics of the source and of the transmission path and which is then solved for the vibration response at the receiver [8].

ISO 10137 states that the designer shall decide on the serviceability criterion and its variability. Further, ISO 10137 states that pedestrian bridges shall be designed so that vibration amplitudes from applicable vibration sources do not alarm potential users. In Annex C, there are given examples of vibration criteria for pedestrian bridges. There, it is suggested to use the base curves for vibrations in both vertical and horizontal directions given in ISO 2631-2, multiplied by a factor of 60, except where one or more persons are standing still on the bridge, in which case a factor of 30 should be applicable. This is due to the fact that a standing person is more sensitive to vibrations than a walking one [13].

However, according to Zivanovic [13], these recommendations are not based on published research pertinent to footbridge vibrations. According to ISO 10137, the dynamic actions of one or more persons can be presented as force-time histories. This action varies with time and position as the persons traverse the supporting structure.

The design situation should be selected depending on the pedestrian traffic to be admitted on the footbridge during its lifetime. It is recommended to consider the following scenarios:

- One person walking across the bridge
- An average pedestrian flow (group size of 8 to 15 people)
- Streams of pedestrians (significantly more than 15 persons)
- Occasional festive of choreographic events (when relevant)

According to ISO 10137: Annex A, the dynamic force F(t) produced by a person walking over a bridge can be expressed in the frequency domain as a Fourier series, Eq. 3.3 and 3.4 [8].

D. Standard conclusion

Pedestrian induced vibrations are a criterion for serviceability. It was therefore assumed that structures respond linearly to applied pedestrian loads and that dynamic response can be found by solving the equation of motion.

The British standard BS 5400 requires a check of vibration serviceability in both vertical and horizontal directions. However, it only proposes a load model and design criteria for vertical vibrations. The load modeling and the evaluation of the design criteria for horizontal vibrations are left to the designer.

The standard ISO 10137 proposes load models for calculation of vertical and horizontal vibrations caused by one pedestrian. It also proposes design criteria for vertical and horizontal vibrations.

Eurocode proposes load models for both vertical and horizontal loads only for simplified structures. For more complex structures, the modeling of pedestrian loads are left to the designer. Eurocode proposes frequency independent maximum acceleration limits both for vertical and horizontal vibrations. The load models proposed by the above mentioned standards are all based on the assumptions that pedestrian loads can be approximated as periodic loads. They also seem to be incapable of predicting structure sensitivity to excessive horizontal vibrations caused by a crowd of pedestrians.

IV. DESCRIPTION OF THE BRIDGE STRUCTURE

The pedestrian bridge over the Labe River in Kolín is designed as a three span suspended bridge structure. The main middle span crosses the river bed width of 99 m. Due to site conditions, the ideal configuration with spans 49.5+99+49.5 has to be reduced to that of 30+99+30 m. Thus, the length of the suspended floor deck equals 159 m. The width between the railings equals 3.5 m, so that two traffic lanes for pedestrians and two lanes for bicyclists are available. Exceptionally one vehicle with mass up to 12000 kg is allowed to overpass the bridge.

The superstructure of the bridge consists of two A shaped pylons bearing two steel suspension cables with ends anchored in reinforced concrete blocks on river banks. Pylons with the height of 16.3 m are anchored through bolts in reinforced concrete foot blocks founded on pin piles. The anchoring reinforced concrete blocks of suspension cables are founded by prestressed land ties. Over their whole length, the suspension cables are protected against corrosion by steel tubes filled under pressure with special grout. Over the pylon seats the cables are guided inside properly shaped thick-walled tubes.

Fig. 9 Three span suspended pedestrian bridge with 159 m deck

The bridge deck has been designed as composed of prefabricated reinforced concrete panels, clipped together by prestressed tendons. The panels are 3.0 m long and their width equals 4.5 m. The panels with nominal thickness of 0.45 m are lightened by bottom face shaping. The panels are suspended through rod hangers of appropriate lengths on covering tubes of suspension cables. Panels are provided with electric resistance heating cells embedded under the upper surface of panels. Along both sides the bridge deck is provided with railings and lighting. Along one side the panels are provided with consoles carrying a gas pipeline.

V. RESPONSES OF THE BRIDGE STRUCTURE TO STATIC LOAD

Several complex structural analyses have been carried out in the course of the bridge design. All relevant standards have been respected. The main analysis has involved the design of the load carrying structural system. Besides the global spatial computation model, five more models of details of the system have been developed using the finite element method. The computations have been performed using the program package ANSYS, version 8.1. Finite elements mostly of the types SOLID45, SHELL43, BEAM44, CONTAC52 and COMBIN12 have been used. The structural analysis has been performed as a geometrically nonlinear one. Material deformation properties have been described as linear.

The determination of responses of the bridge structure to selected static loads has been performed using the global computation model. With regard to the nonlinearity of the model, response for every design situation (characterized by a certain combination of loads) has to be computed separately.

Every response analysis has been accomplished in several steps. In the first phase the initial configuration of the suspension system with fixed ends of cables have been determined. The contact constraints between cables and pylon cable seats have been modeled using elements transferring exclusively compressive forces. The cables have been modeled by elements transferring exclusively tensile forces. In next phases of the analysis the loads corresponding to the selected design situation have been introduced. The analysis has been always performed for both ultimate and serviceability limit states. Altogether thirteen load combinations have been considered. Results of preliminary computations have been used in the course of the design of the structure. The suspension system of pylons, cables and flooring hangers has been optimized. Final analyses have

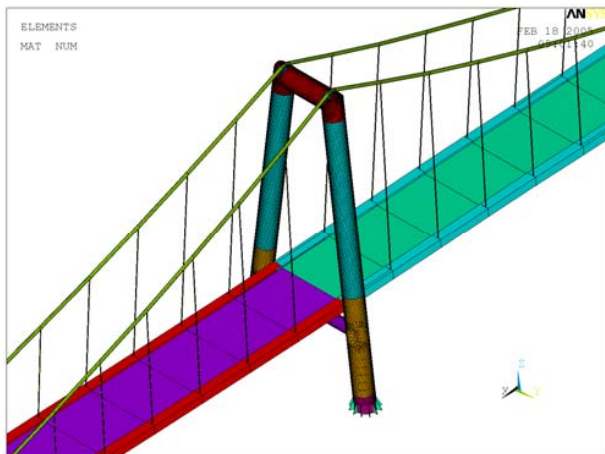proved the reliability of the construction with respect to static loads.

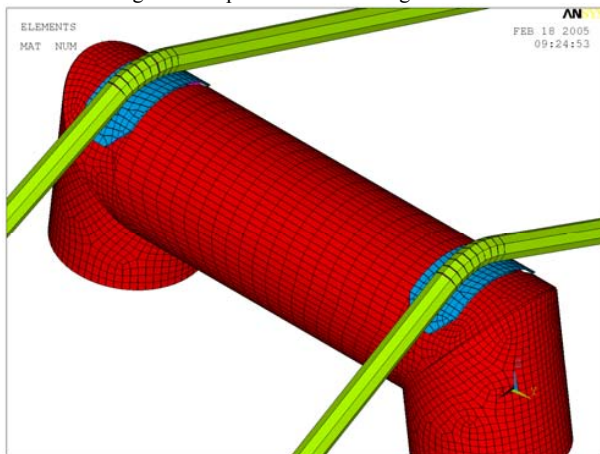Fig. 10 Computational model – general view

Fig. 11 Computational model – detail view

VI. MODAL ANALYSIS

For computational model calculation of natural frequency and eigenshape modes, the block Lanczos method was used. Fifty lower natural frequencies and eigenshape modes were extracted. The calculations were performed for a model with a nominal operational dead weight construction mass and for a model with the mass equal to the sum of nominal operational dead weight construction mass and the mass corresponding to category I. crowd load - one person per m^2, i. e. 70 kg.m^{-2}. This case presents the deck occupation by uniformly distributed 556 persons. Table III shows natural frequencies where meaningful structure response invoked by movement of persons can be expected. Shapes for empty and full construction are equal, in pictures 12 to 17 only empty construction shapes are presented.

TABLE III
MODAL STRUCTURE CHARACTERISTIC

Empty		Crowded		Shape description
No.	Frequency [Hz]	No	Frequency [Hz]	
9	1.3673	9	1.3397	Lateral
10	1.5289	10	1.5016	Torzional
11	1.6768	11	1.6427	Vertical
15	1.9646	13	1.9119	Vertical
17	2.0027	15	1.9728	Torzonal
18	2.2858	18	2.2230	Vertical
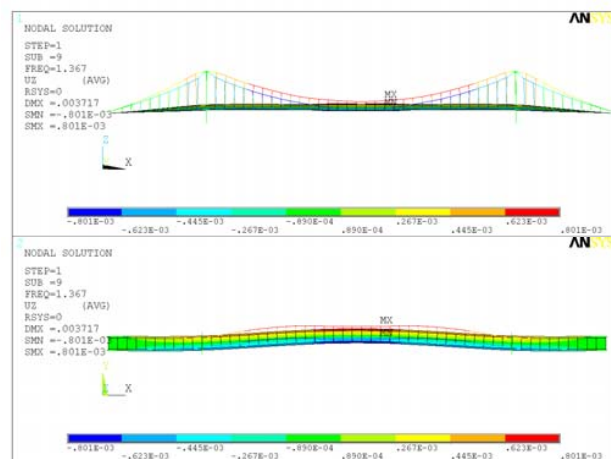
Fig. 12 Eigen shape according to f = 1.367 Hz
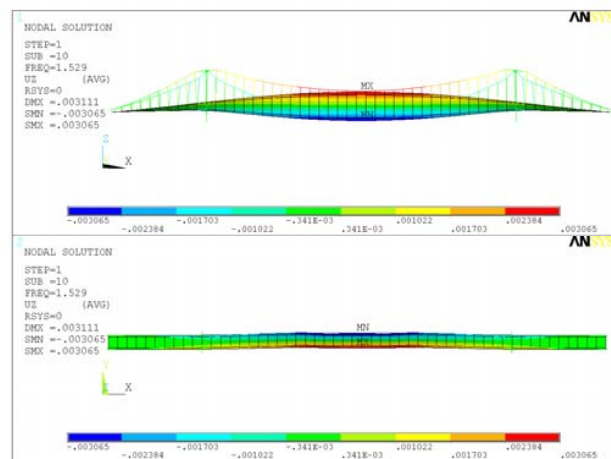
Fig. 13 Eigen shape according to f = 1.529 Hz
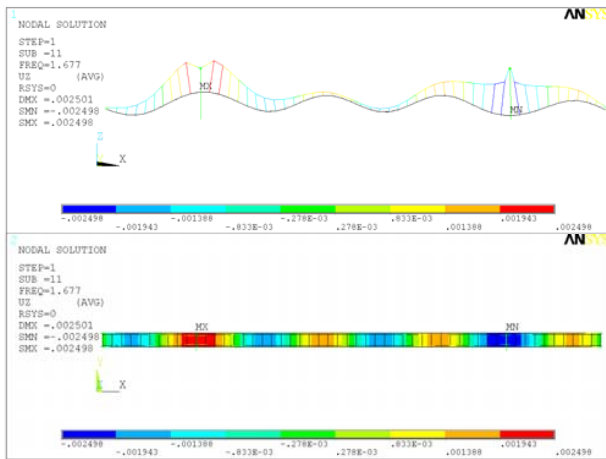
Fig. 14 Eigen shape according to f = 1.677 Hz
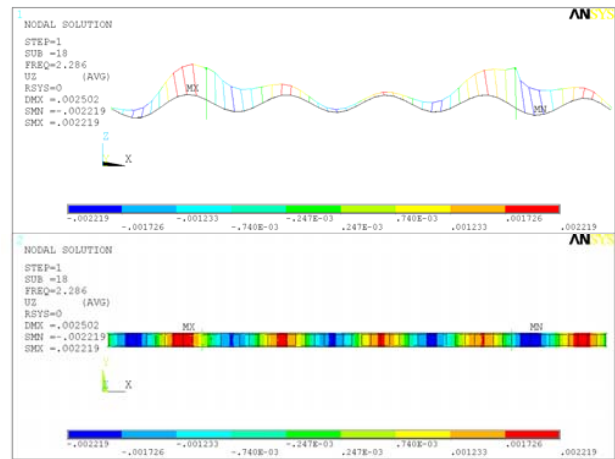
Fig. 17 Eigen shape according to f = 2.286 Hz
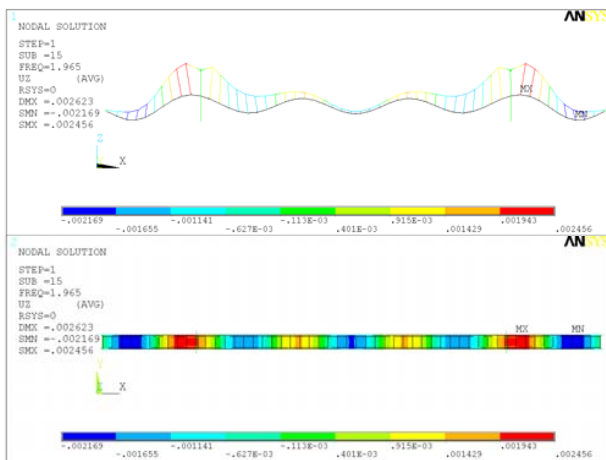
Fig. 15 Eigen shape according to f = 1.965 Hz
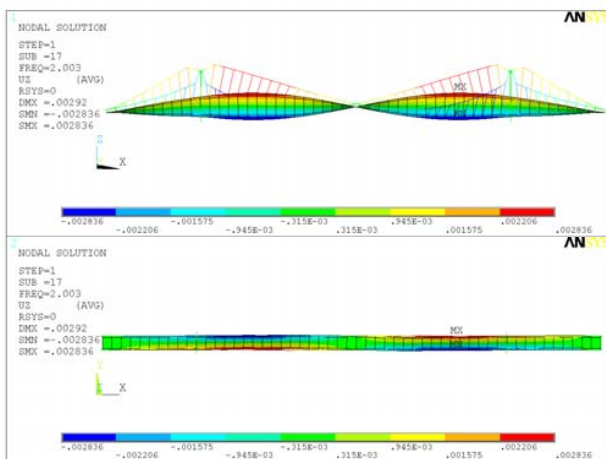
Fig. 16 Eigen shape according to f = 2.003 Hz

VII. DYNAMIC RESPONSE OF THE BRIDGE STRUCTURES ON SINGLE PEDESTRIAN LOAD

Dynamic analysis of the bridge structure has been carried out using the same global computation model as that applied for the static analysis. The dynamic loads due to walking pedestrians have been considered. Both ultimate and serviceability limit states have been assessed. Peak stresses as well as stress amplitudes in fatigue exposed parts of the structure have been checked with regard to the required service life of 100 years. In fatigue strength assessment the rule of linear fatigue damage accumulation has been applied. The vibration level has been assessed with respect to tolerance criteria for vibration exposure of humans, as stated in environmental hygienic regulations.

There have been computed responses of the structure to excitation due to walking of pedestrians intentionally at a synchronized pace corresponding to one of the natural frequencies.

According to the method introduced in [1], [10], the single person vibration response calculation was developed. The pedestrian is walking/running through the whole length of the empty bridge deck. Since prior studies showed that the influence of a single person mass on structure modal characteristics is insignificant, a persons weight in calculation (2) was considered using only a constant member $G = 700$ N. Harmonic load coefficients were considered according to [65], for vertical direction $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\alpha_3 = 0.1$ with shift phase $\varphi_2 = \varphi_3 = \pi/2$; for lateral direction $\alpha_{1/2} = 0.05$, $\alpha_1 = 0.01$, $\alpha_{3/2} = 0.05$, $\alpha_2 = 0.01$; for longitudal direction $\alpha_{1/2} = 0.04$, $\alpha_1 = 0.2$, $\alpha_{3/2} = 0.03$, $\alpha_2 = 0.1$. These coefficients are frequency independent and correspond with walking movement of a person. For 2.2858 Hz frequency, the response calculation for running movement with $\alpha_1 = 1.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.2$ in vertical direction was performed, in the phase without contact the functional was put equal to zero. In Fig. 18 to 21, the force-time functions for vertical, lateral and longitudal directions for walking and vertical force for running are shown. For all frequencies, the step length was considered equal to 0.9 m.

The speed of movement is defined by the frequency multiplied by the step length.
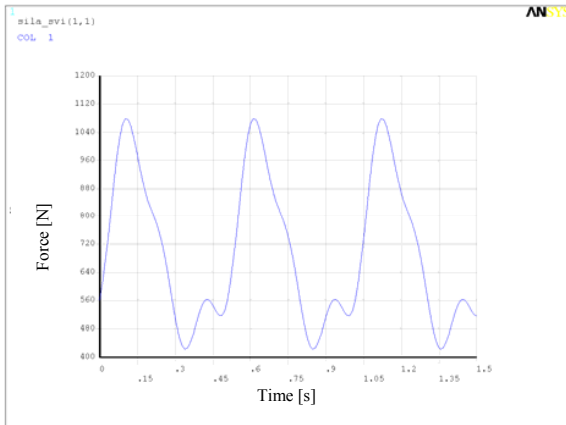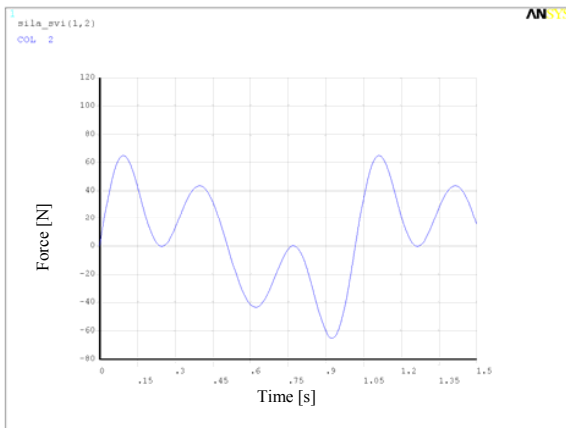

Fig. 18 Vertical force time record
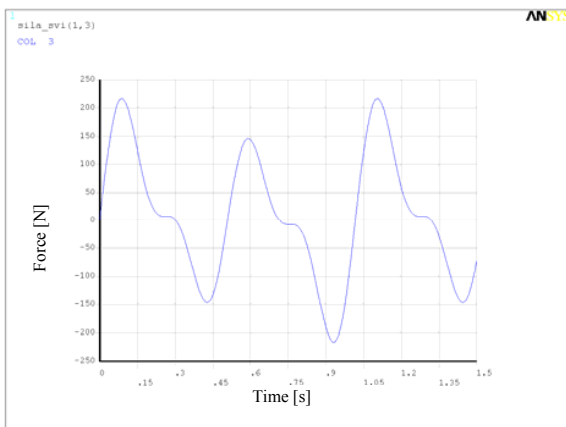

Fig. 19 Lateral force time record

Fig. 20 Longitudal force time record
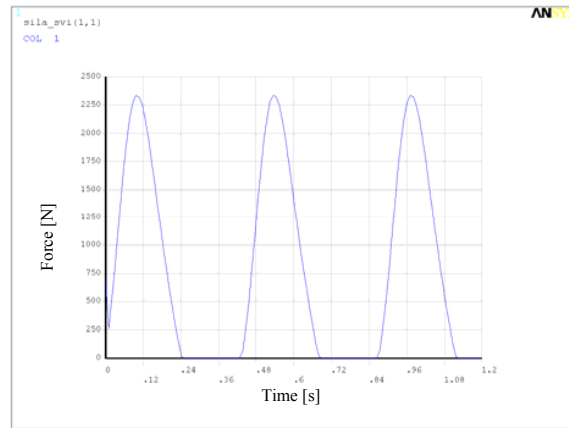
Fig. 21 Vertical force time record for running
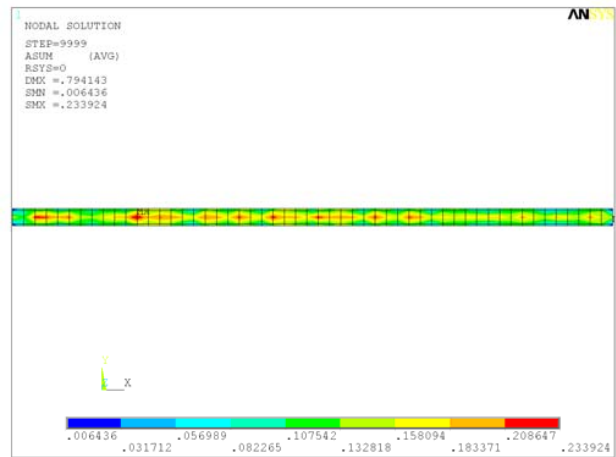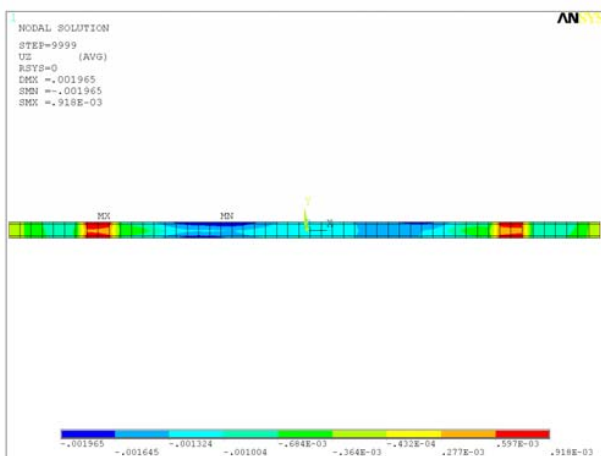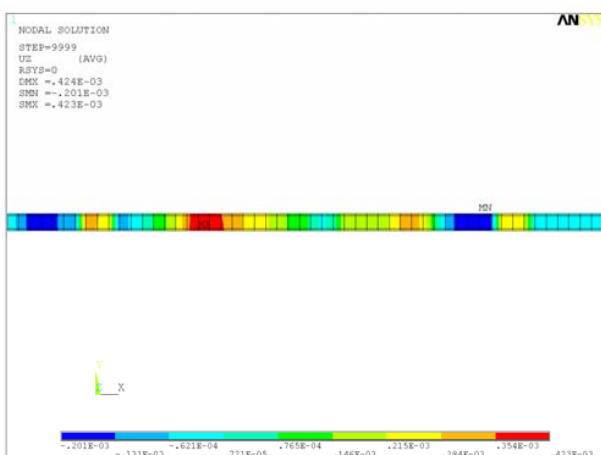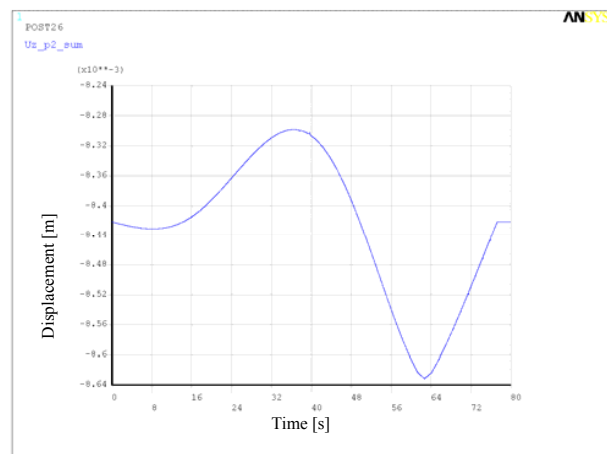
TABLE IV
PEDESTRIAN LOADING VARIANTS

Var.	Frequency	Movement	Position
1	1.367	Walk	Longitude centerline
2	1.529	Walk	Longitude centerline
3	1.529	Walk	Railway side
4	1.677	Walk	Longitude centerline
5	1.965	Walk	Longitude centerline
6	2.003	Walk	Railway side
7	2.003	Walk	Longitude centerline
8	2.286	Walk	Longitude centerline
9	2.286	Run	Longitude centerline
10	Static response on constant moving force $G = 700$ N		

In a post-processor, the absolute maxima envelopes for the whole structure through all the time steps were obtained. The maximal vertical displacement U_z, the velocity V_z in vertical direction and acceleration in vertical direction A_z and total acceleration A_{SUM} were localized. Figs 22 to 27 present the results for variant 9. Fig 26 represents the dynamic excitation without initial static displacement of the structure. Since geometrically nonlinear calculations required the construction initial stress state, the self-weight load calculation needed to be performed. For all load cases, except for variants 3 and 6, the maximal response was localized near the pylons in the deck centre. The transverse and longitudal direction response maximals were evaluated as well, but their amplitude reached insignificant values. Variant 3 and 6 constitute eccentrically acting torsional vibration shapes. Relative vertical displacements Uz for variants 3 and 6 are displayed in figures 28 and 29. Compared to graphs in figs. 30 and 31 – the response on the pedestrian movement in the longitudal deck centerline - clear torsional excitation can be observed for variants 3 and 6.
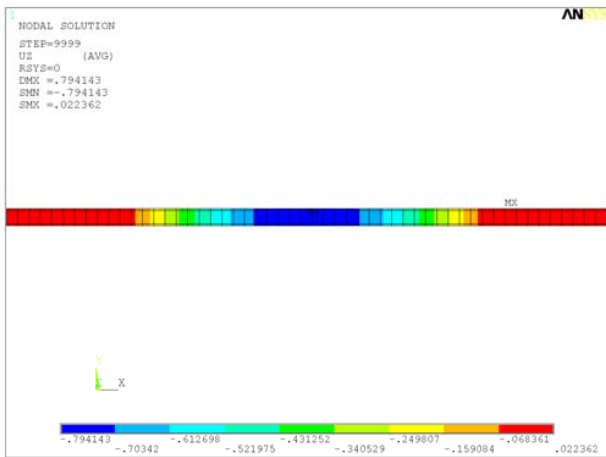
Fig. 22 Maximal envelope U_z total, Variant No. 9

Fig. 23 Maximal envelope V_z, Variant No. 9

Fig. 24 Maximal envelope A_z, Variant No. 9

Fig. 25 Maximal envelope A_{SUM}, Variant No. 9

Fig. 26 Maximal envelope U_z dynamic, Variant No. 9
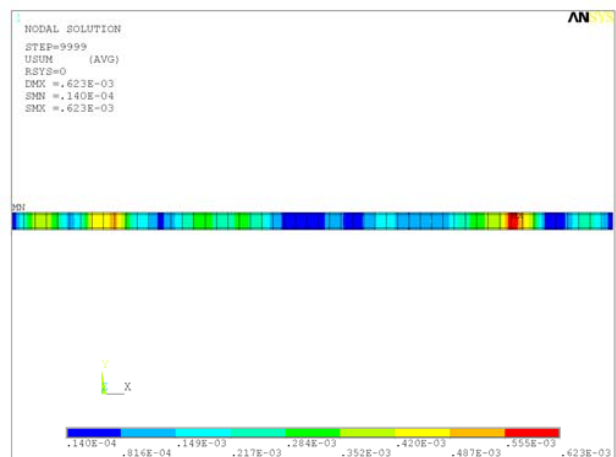
Fig. 27 Maximal envelope U_{SUM}, Variant No. 9
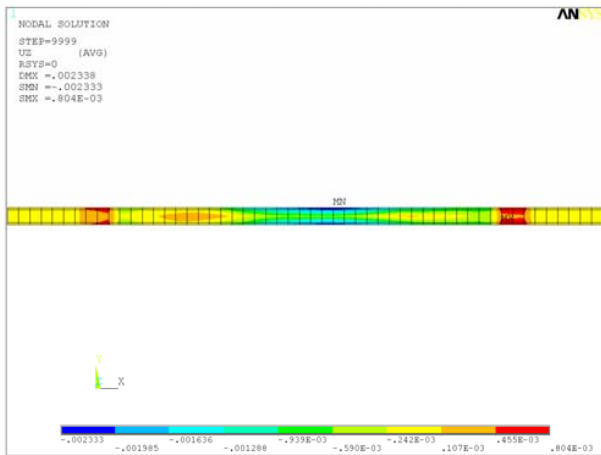
Fig. 28 Maximal envelope U_z dynamic, Variant No. 3 – torsional
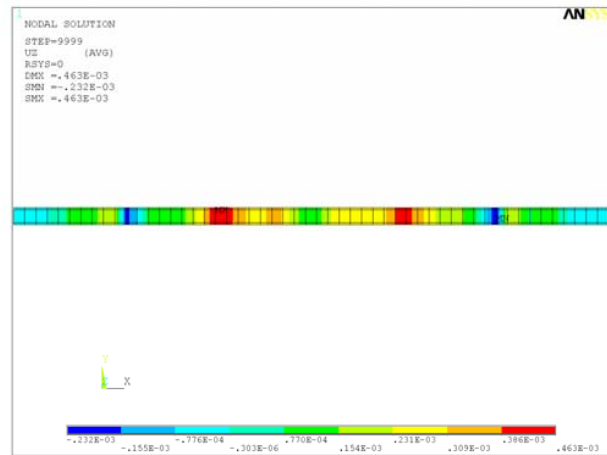
Fig. 31 Maximal envelope U_z dynamic, Variant No. 7

For variants 1 to 9, the displacement and acceleration response of the deck centre near both pylons (pylon p_1 is nearer the excitation beginning, i. e. has a lower x ordinate) and in the mid span through the whole time interval was evaluated. The reference points were chosen considering the maximal envelope occurrence. The results for variant 10 represent a static response on a moving constant force 700 N, see figures 32 and 33; the deck near pylon response is labeled $U_{z\_p1\_sta}$ and the mid span is labeled $U_{z\_st\_sta}$. By subtracting this response, the dynamic displacement for variant 1 – 9 was obtained. The single point time response analysis was performed in the ANSYS program post-processor /POST26. Figures 34 and 35 presents variant 3 results.

Fig. 29 Maximal envelope U_z dynamic, Variant No. 6 - second torsional

Fig. 32 Vertical displacement U_z, deck near pylon p_2, Variant No. 10

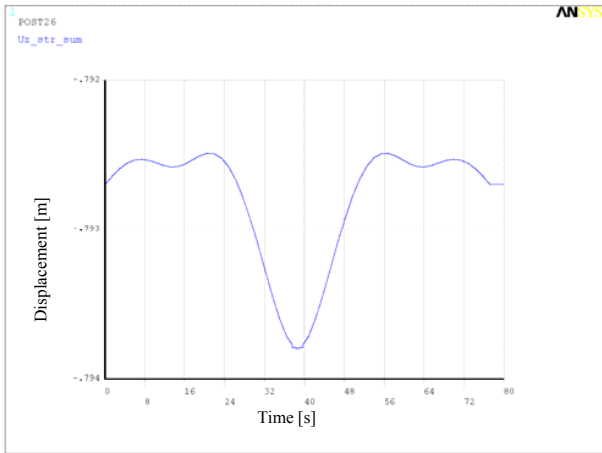Fig. 30 Maximal envelope U_z dynamic, Variant No. 2

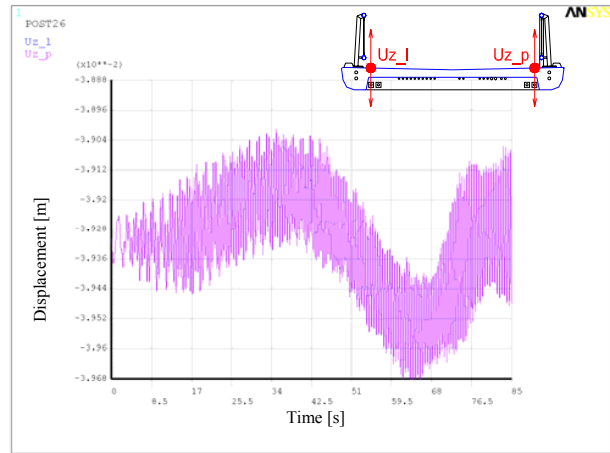Fig. 33 Vertical displacement U_z, mid span, Variant No. 10

Fig. 36 Total vertical displacements, mid span U_{z_l} , U_{z_p} , Variant No. 2
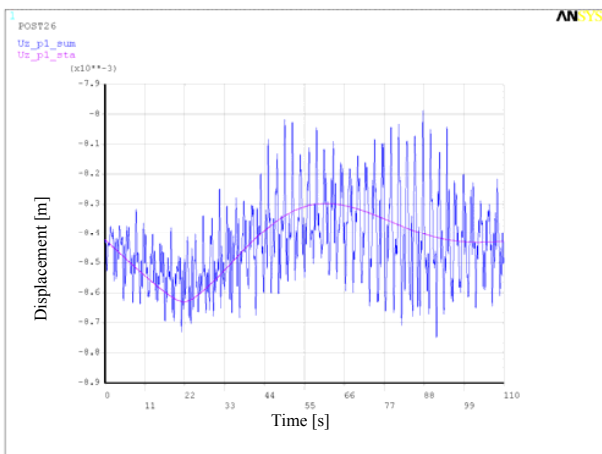

Fig. 34 Total and static vertical displacement U_z, near pylon p_1, Variant No. 3
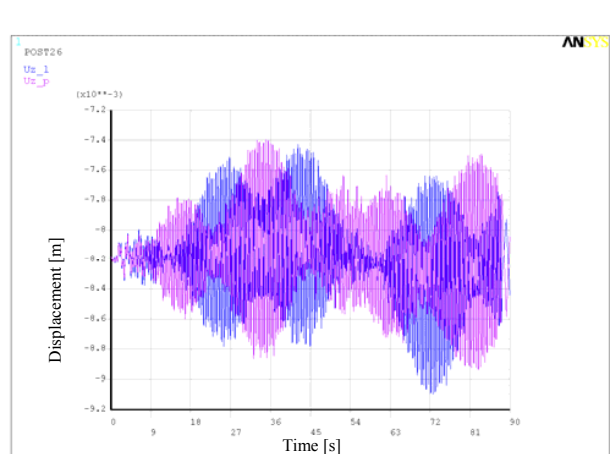

Fig. 37 Total vertical displacements, mid span U_{z_l} , U_{z_p} , Variant No. 3
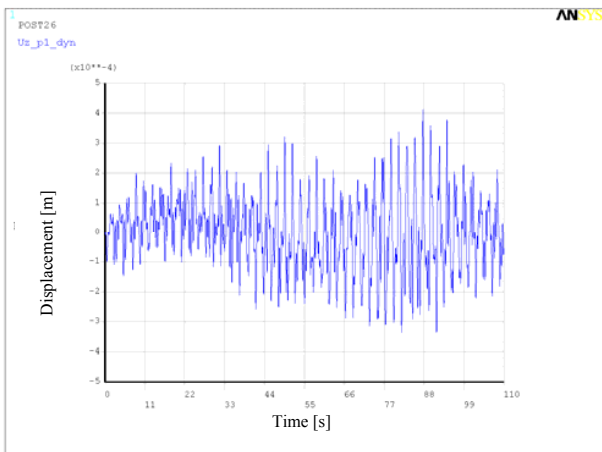

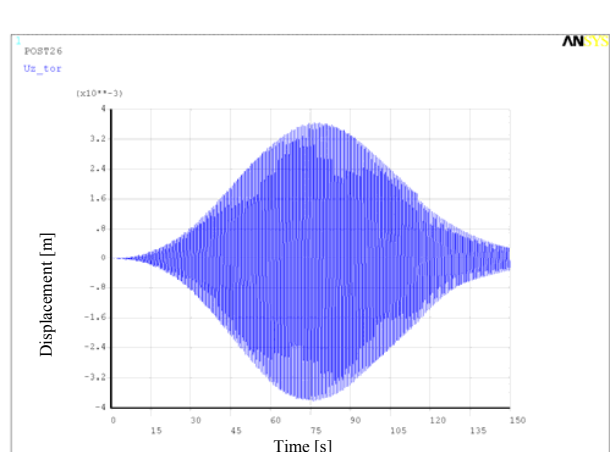Fig. 35 Dynamic vertical displacement U_z, near pylon p_1, Variant No. 3

Fig. 38 Displacements difference for U_{z_l} and U_{z_p} , Variant No. 3
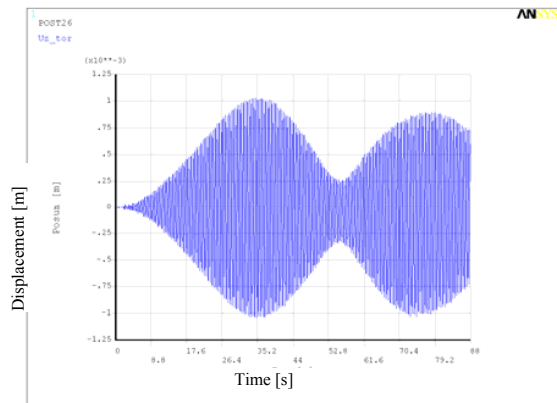
Fig. 39 Displacements difference for U_{z_l} and U_{z_p} , Variant No. 6

Figures 36 and 37 show the displacement response of the points near the left and right deck railway in the middle of the span for variant 2 (fig. 36) and variant 3 (fig. 37). The graph in fig. 38 represents the difference of responses in fig. 37. In fig. 38, a clear torsional vibration with significant vertical response differences is perceptible. The response difference from the graph in fig. 36 is insignificant. In fig. 39, a higher torsional vibration shape response corresponding to 2.003 Hz for variant 6 can be seen. The graph is finished in the moment of pedestrian leaving the deck.

VIII. CONCLUSION

Horizontal displacements of all variants were insignificant and a criterion of 10 mm limit transverse displacement was fulfilled. The evaluation of all variants was performed for maximum acceleration in vertical direction. According to (3), the acceleration limit is proportional to the excitation frequency square root. The worst load case was, as expected, the variant 9 – running with step frequency of 2.286 Hz. For this load case, the limit acceleration is a_{lim} = 0.76 m.s^{-2}. As it can be seen in figures 24 and 25, this criterion is fulfilled for the whole deck in the whole solved time, the peak acceleration amplitude is equal to A_z = 0.23 m.s^{-2}. These acceleration level areas are small and on the majority of the deck surface, the peak acceleration values are three times lower. For walking variants evaluation, the acceleration amplitude in vertical direction was A_z = 0.14 m.s^{-2}.

The bridge has been put into service at the end of 2005. The bridge shall not only do its proper function but it shall present a nice architectural work, too. To attain the latter, the bridge has been designed so that the suspension cable lines passing like free over the pylon seats dominate the skyline of the work.

REFERENCES

[1] Bachmann, H. Lively Footbridges a Real Challenge. *Proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges*, Paris, France, November 20-22, 2002, pages 18-30.
[2] Dallard, P., Fitzpatrick, T., Flint, A., Low, A., Ridsdill Smith, R., Willford, M. and Roche, M. *London Millennium Bridge: Pedestrian-Induced Lateral Vibration.* ASCE
[3] Design Manual for Road and Bridges: Loads for Highway Bridges: BD 37/01, Highway Agency, London, February, 2002.
[4] Eriksson, P. E. *Vibration of Low-Frequency Floors—Dynamic Forces and Response Prediction*, PhD Thesis, Unit for Dynamics in Design, Chalmers University of Technology, Goteborg, Sweden, 1994.
[5] Eurocode, Basis of Structural Design - prAnnex A2. EN1990: 2002. European Committee for Standardization, Brussels, Belgium 2002.
[6] Eurocode 5, Design of Timber Structures Part 2: Bridges, EN1995- 2: 2004, European Committee for Standardization, Brussels, Belgium 2004.
[7] Grundmann, H. Kreuzinger, H., Schneider, M. *Dynamic calculations of footbridges,* Bauingenieur 68 (1993) 215–225
[8] ISO, *Bases for design of structures Serviceability of buildings and pedestrian walkways against vibration*, ISO/CD 10137, International Stadardization Organization, Geneva, Switzerland, 2005.
[9] Ohlsson, S. V., *Floor Vibration and Human Discomfort*, PhD Thesis, Chalmers University of Technology, Goteborg, Sweden, 1982 (in English).
[10] SETRA, Footbridges, Assessment of vibrational behaviour of footbridges under pedestrian loading, *Technical guide SETRA*, Paris, France 2006.
[11] Young, P. *Improved floor vibration prediction methodologies*, ARUP Vibration Seminar, October 4, 2001.
[12] Wheeler, J. E. Prediction and control of pedestrian induced vibration in footbridges, *ASCE Journal of the Structural Division 108* (ST9) (1982) 2045–2065.
[13] Zivanovic, S., Pavic, A., and Reynolds, P. Vibration serviceability of footbridges under human-induced excitation: a literature review. *Journal og Sound and Vibration 279* (2005).