

Flexural Strength and Ductility Improvement of NSC beams

Jun Peng and Johnny Ching Ming Ho

Abstract—In order to calculate the flexural strength of normal-strength concrete (NSC) beams, the nonlinear actual concrete stress distribution within the compression zone is normally replaced by an equivalent rectangular stress block, with two coefficients of α and β to regulate the intensity and depth of the equivalent stress respectively. For NSC beams design, α and β are usually assumed constant as 0.85 and 0.80 in reinforced concrete (RC) codes. From an earlier investigation of the authors, α is not a constant but significantly affected by flexural strain gradient, and increases with the increasing of strain gradient till a maximum value. It indicates that larger concrete stress can be developed in flexure than that stipulated by design codes. As an extension and application of the authors' previous study, the modified equivalent concrete stress block is used here to produce a series of design charts showing the maximum design limits of flexural strength and ductility of singly- and doubly- NSC beams, through which both strength and ductility design limits are improved by taking into account strain gradient effect.

Keywords—Concrete beam, Ductility, Equivalent concrete stress, Normal strength, Strain gradient, Strength

I. INTRODUCTION

TO design the flexural strength of reinforced concrete (RC) beams, the nonlinear concrete stress distribution in compression zone is usually replaced by an equivalent rectangular stress block [2, 8, 9, 17, 18, 19], whose width represents the equivalent concrete stress developed under flexure, denoted by $\alpha f'_c$ (f'_c = concrete cylinder strength), where $\alpha \leq 1.0$, and height represents the depth of the equivalent stress block, denoted by βc (c = neutral axis depth), where $\beta \leq 1.0$. The method of using an equivalent rectangular stress block for concrete in compression has been commonly adopted in many of the current RC design codes [1, 4, 16]. In these codes, α and β are taken as 0.85 and 0.85 (0.80 for ECS) respectively, which are constant. The value of $\alpha = 0.85$ is actually the ratio of the ultimate strength of NSC columns tested under concentric axial load to their respective concrete cylinder strength [2, 9].

An earlier flexural strength comparison of the theoretical flexural strengths calculated using the codes with the experimentally measured strength by the authors [11] reveals that the specified value of $\alpha = 0.85$ in various RC codes could only predict accurately the flexural strength of NSC columns subjected to high and ultra-high axial load levels, but is too

conservative for NSC beams and columns subjected to low or medium axial load level. Since the flexural strength underestimation is different for beams and columns, which are subjected to different strain gradient (defined as the ratio of ultimate concrete strain to neutral axis depth), it is believed that the value of α as well as the concrete stress developed in flexure, should also depend on the strain gradient. In the event that $\alpha = 0.85$, which was obtained from testing NSC columns under pure axial load without strain gradient [2, 3], is adopted for flexural strength calculation, it would underestimate the equivalent stress and hence flexural strength of the members.

The authors have conducted an experimental study on the effect of strain gradient on the maximum concrete compressive stress that can be developed under flexure in NSC columns [10]. From the test results, it was found that the relationship between α and strain gradient can be represented by a tri-linear curve. As a continued study, this paper will utilise the previously proposed values of α and β to investigate the flexural performance of NSC beams in terms of the limits of flexural strength and ductility that can be designed simultaneously. A set of design charts will be produced showing the design limits of NSC beams with strain gradient effect considered. It will be verified from the charts that the design limit of NSC beams can be improved significantly after the strain gradient effects has been considered.

II. EXPERIMENTAL SCHEME

A. Test Setup and Specimen Details

A total of 9 inverted T-shape square column specimens (in 2 different groups) with concrete cylinder strength from 41 to 55 MPa were fabricated and tested under concentric and eccentric axial loads as well as horizontal loads. The specimens within each group were of identical cross-section properties and materials' strength. In each group, one specimen was tested under concentric load (zero strain gradient), while the rest of them were tested under eccentric axial load (small strain gradient) or horizontal load (large strain gradient). The cross section of the specimens is $400 \times 400 \text{ mm}^2$. The height of columns is 1400 mm and the length of supporting beams is 1500 mm. The testing regions for specimens subjected to concentric and eccentric axial loads are in the middle 800 mm of the column height, while the testing regions is located within 800 mm in the column from the beam-column interface for horizontally loaded specimens. Table I lists the properties of the column specimens together with their loading eccentricities.

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The uni-axial stress-strain behaviour of concrete in each group is taken as that of the concentrically loaded specimen. On the other hand, the concrete stress-strain curve developed in flexure was obtained by modifying the concrete stress-strain curve obtained from the concentrically loaded specimens, such that theoretical axial load and moment match with the obtained experimental values. To investigate the effects of different extent of strain gradient, the loading eccentricities in columns varied from 50 to 140 mm. For investigating the effects of larger strain gradient, the specimens were tested under combined axial and horizontal loads. The test setup for these three types of specimens is shown in Fig 1.

B. Loading Procedure

For specimens subjected to concentric load, a 20 mm steel plate was installed on top of the column to ensure a smooth contact surface for loading application. For specimens subjected to eccentric load, a guided steel roller was installed at prescribed eccentricity on top of the aforementioned steel plate. In all specimens, the loading was applied in a displacement-controlled manner with a rate of 0.36 mm/min for concentric or eccentric specimens and 0.5 mm/min for horizontal specimens. All the data from the instrumentation were recorded by a data logger. The loading process would stop after the applied load had reached the maximum value and then dropped below 80% of the maximum value.



Fig. 1 Test setup

TABLE I
COLUMN SPECIMENS PROPERTIES

Specimen code	Longitudinal steel		f_c' (MPa)		Eccentricity (mm)
	f_y (MPa)	E_s (GPa)	28 th day	Testing day	
RC51-0.75-CON				54.8	0
RC51-0.75-ECC-1				54.8	50
RC51-0.75-ECC-2	517	192	51.0	54.8	140
RC51-0.75-HOR-1				53.3	---
RC51-0.75-HOR-2				53.3	---
RC41-0.75-CON				41.0	0
RC41-0.75-ECC-1				43.7	100
RC41-0.75-ECC-2	529	202	41.0	41.9	140
RC41-0.75-HOR				41.0	---

III. EXPERIMENTAL RESULTS

A. Concentric Column specimens

Fig 2 shows the measured axial load-column displacement curves and concrete stress-strain curves respectively in the primary and secondary axes for concentrically loaded (CON) column specimens. The concrete stress-strain curve obtained here acts as the uni-axial compression behavior of concrete within one group and will be adopted to derive the equivalent concrete stress block parameters for other specimens subjected to flexure of the same group later.

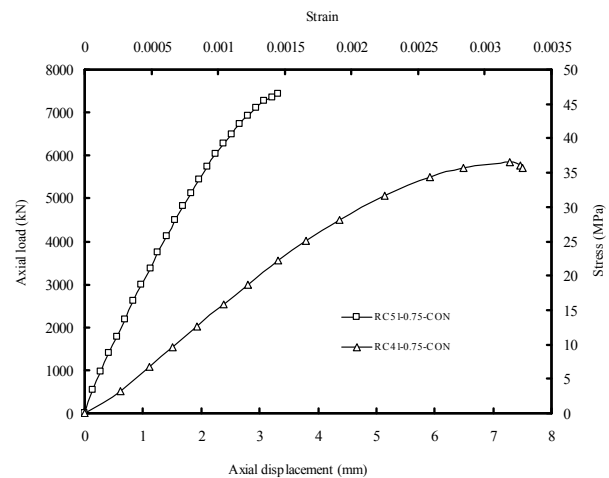
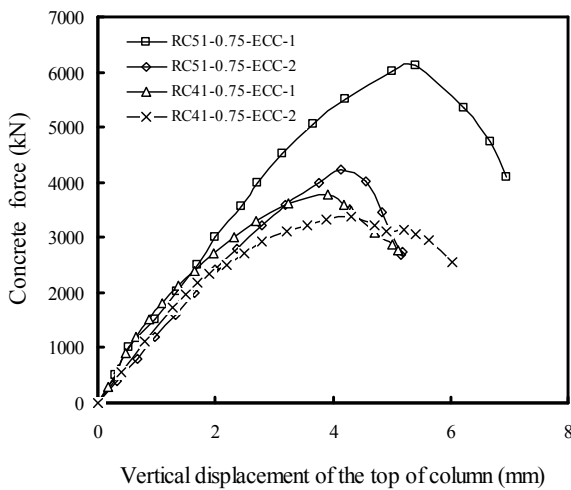


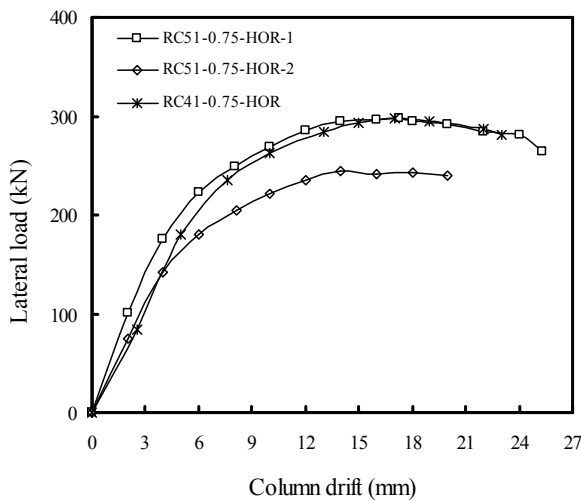
Fig. 2 Load-displacement and stress-strain curves of CON columns

B. Eccentric/Horizontal Column specimens

Fig 3 plots the obtained axial concrete force-column displacement and horizontal load-column drift curves respectively for eccentrically loaded (ECC) and horizontally loaded (HOR) specimens.



(a) Axial force-column displacement curves of ECC columns



(b) Horizontal load-column drift curves of HOR columns

Fig. 3 Axial force-column displacement and horizontal load-column drift curves

IV. DERIVATION OF EQUIVALENT STRESS BLOCK PARAMETERS

A. Derived Values of Target Parameters

The derivation of equivalent stress block parameters is based on axial force and moment equilibriums through modifying the experimentally obtained uni-axial concrete stress-strain curve of concentrically loaded column specimens. The theoretical axial load and moment calculated using the equivalent rectangular stress block can be matched with the respective measured values in the experiment as the following equations:

$$P_e = \alpha \beta f_c' b c + \sum_{i=1}^n f_{si} A_{si} \quad (1)$$

$$M_e = \alpha \beta f_c' b c \left(\frac{h}{2} - \frac{\beta}{2} c \right) + \sum_{i=1}^n f_{si} A_{si} \left(\frac{h}{2} - d_i \right) \quad (2)$$

where P_e is measured axial load of eccentrically loaded specimens in the experiment and the expression on the right-hand side represents the theoretical axial load, M_e is the measured moment of eccentrically or horizontally loaded specimen and the expressions on the right-hand side represent the theoretical moment.

The neutral axis depth c in the above equations is evaluated based on the modified concrete stress-strain curve obtained from the concentrically loaded specimens by solving Eqs. (3), (4) and (5):

$$P_e = \int_{A_c} k_3 \sigma_c(\varepsilon) dA_c + \sum_{i=1}^n f_{si} A_{si} \quad (3)$$

$$M_e = \int_{A_c} k_3 \sigma_c(\varepsilon) \left(\frac{h}{2} - c + x \right) dA_c + \sum_{i=1}^n f_{si} A_{si} \left(\frac{h}{2} - d_i \right) \quad (4)$$

$$\varepsilon = \frac{x}{c} \varepsilon_{cu} \quad (5)$$

where $\sigma_c(\varepsilon)$ is the concrete stress-strain curve obtained from the concentrically loaded specimens, k_3 is the ratio of the maximum concrete stress developed under flexure to that in uni-axial condition.

The calculated equivalent stress block parameters from this study are shown in Table II. From the table, it is observed that: (1) The value of α for the eccentrically/horizontally loaded specimens subjected to strain gradient is larger than that of the corresponding concentrically loaded specimens. Therefore, the strain gradient would have beneficial effect on the equivalent concrete stress developed in flexural RC members. (2) The value of α for the eccentrically/horizontally loaded specimens increases as the strain gradient increases until reaching a maximum value. (3) The average value of α obtained for the concentrically loaded columns is about 0.828, which is very close to the current design value of $\alpha = 0.85$ stipulated in various RC design codes [1, 4, 16]. (4) The values of β are insensitive to the extent of strain gradient.

The derived equivalent rectangular stress block parameters listed in Table II are plotted against strain gradient factor d/c in Figure 4. From this figure, it is found that the value of α remains constant at low strain gradient, then increases significantly at moderate strain gradient extent till reaches a upper bound value. Based on this, equation (6) correlates α with strain gradient factor d/c in a tri-linear formula. Also in Figure 4, the value of β

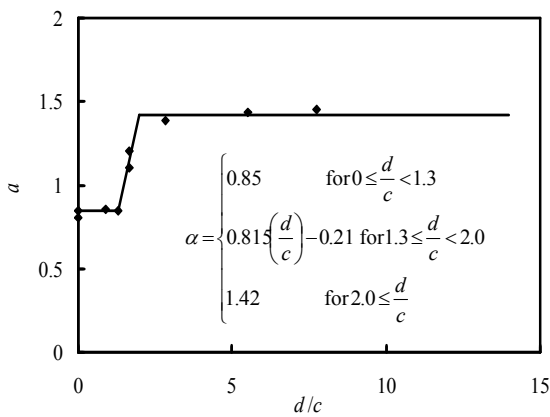
remains relatively constant no matter how strain gradient changes. Since β is found independent to strain gradient and similar to those specified in current codes, thus it is proposed in this study as constant in Eq. (7). The derived values of α and β in this study have the similar trend to those of the authors' early study [10] and some data regions that were not covered by the early study were tested in this study.

$$\alpha = \begin{cases} 0.85 & \text{for } 0 \leq \frac{d}{c} < 1.3 \\ 0.815\left(\frac{d}{c}\right) - 0.21 & \text{for } 1.3 \leq \frac{d}{c} < 2.0 \\ 1.42 & \text{for } 2.0 \leq \frac{d}{c} \end{cases} \quad (6)$$

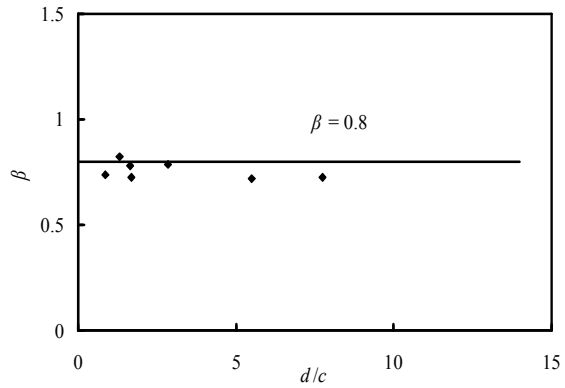
$$\beta = 0.80 \quad (7)$$

TABLE II
DERIVED EQUIVALENT STRESS BLOCK PARAMETERS

Specimen code	α	β	ϵ_{cu}	d/c
RC51-0.75-CON	0.850	---	0.0024	0.0
RC41-0.75-CON	0.805	---	0.0030	0.0
Average	0.828	---	0.0027	0.0
RC51-0.75-ECC-1	0.856	0.738	0.0035	0.873
RC51-0.75-ECC-2	1.206	0.723	0.0029	1.670
RC51-0.75-HOR-1	1.436	0.720	0.0030	5.515
RC51-0.75-HOR-2	1.455	0.726	0.0031	7.744
RC41-0.75-ECC-1	0.845	0.821	0.0031	1.291
RC41-0.75-ECC-2	1.100	0.778	0.0032	1.655
RC41-0.75-HOR	1.387	0.785	0.0034	2.844
Average	---	0.756	0.0032	---



(a) α against strain gradient



(b) β against strain gradient

Fig. 4 Derived parameters plotted against strain gradient

B. Verification of the Proposal

The proposed formulae of Eqs. (6) and (7) for equivalent stress block parameters are validated by using them to predict the flexural strength of beams tested by other researchers [5, 7, 12, 13, 14, 15], then the predicted strengths are compared with the measured values and the theoretical values predicted by RC design codes. From the comparison, the proposed strength has the best agreement with the measured strength than RC codes, and the accuracy is improved by 5% [10].

V. APPLICATION ON STRENGTH AND DUCTILITY DESIGN OF NSC BEAMS

In this study, the tri-linear relationship of concrete stress developed in flexure with strain gradient effect considered, will be adopted to produce a set of strength-ductility diagrams. RC beams with concrete strength of 30, 40 and 50MPa will be evaluated and the results will be compared with those obtained without strain gradient considered. The maximum design limits for flexural strength and ductility will be improved significantly for NSC RC beams when the equivalent stress block is modified according to the proposal from this study with strain gradient effect is taken into account.

A. Singly-reinforced beams

Singly-reinforced NSC beams with various tension steel ratio (ρ_t) and concrete strength are calculated using the proposed values of α and β . The flexural strength of singly-reinforced beams can be evaluated by Eqs. (8) and (9), where α and β are proposed by Eqs. (6) and (7):

$$\alpha\beta f_c' bc = f_{st} \rho_t bd \quad (8)$$

$$M = \alpha\beta f_c' bc(d - 0.5\beta c) \quad (9)$$

The flexural ductility can be measured by Eq. (10), which was previously proposed by the author [6]:

$$\mu = 10.7 (f_{co})^{-0.45} ((\rho_t - \rho_c) / \rho_{bo})^{-1.25} \quad (10)$$

The obtained flexural strength-ductility curves for singly-reinforced beams (compression steel ratio $\rho_c = 0$) are plotted in Figure 5. The curves in the figure show the maximum flexural strength and ductility that can be achieved by singly RC beams simultaneously with the specified concrete strength. The corresponding tension steel ratio can be read from the intermediate lines. The advantage of the chart is, for a given design requirement of strength and ductility, the possible combination of concrete strength and tension steel ratio can be obtained rapidly from the graph, which enables both strength and ductility design of singly-reinforced NSC beams in just one step.

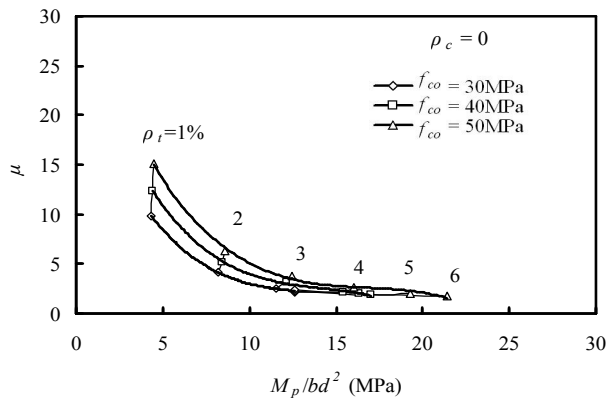


Fig. 5 Strength-ductility design chart for singly-reinforced beams

B. Doubly-reinforced beams

Doubly-reinforced NSC beams with various tension steel ratio (ρ_t), concrete strength and different compression steel ratio (ρ_c) are calculated using the proposed values of α and β . The flexural strength of doubly-reinforced beams can be evaluated by Eqs. (11) and (12), where α and β are proposed by Eqs. (6) and (7):

$$\alpha\beta f_c'bc + f_{sc}\rho_cbd = f_{st}\rho_tbd \quad (11)$$

$$M = \alpha\beta f_c'bc(d - 0.5\beta c) + f_{sc}\rho_cbd(d - d') \quad (12)$$

The flexural ductility can be measured by Eq. (13), which was previously proposed by the author [6]:

$$\mu = 10.7 (f_{co})^{-0.45} ((\rho_t - \rho_c) / \rho_{bo})^{-1.25} (1 + 95.2(f_{co})^{-1.1}(\rho_c / \rho_t)^3) \quad (13)$$

The strength-ductility curves of doubly-reinforced beams with concrete strengths of 30, 40 and 50MPa, compression steel ratios of 0.5%, 1% and 2% and tension steel ratios of 2-6% are

plotted in Figure 6 for design purpose. The design limits of strength and ductility that can be achieved by the doubly-reinforced beams with specified cross section properties are shown in these figures. For a given design requirement of strength and ductility, the possible combination of concrete strengths, tension and compression steel ratios can be obtained directly from these charts, which enable both strength and ductility design of doubly-reinforced NSC beams in just one step.

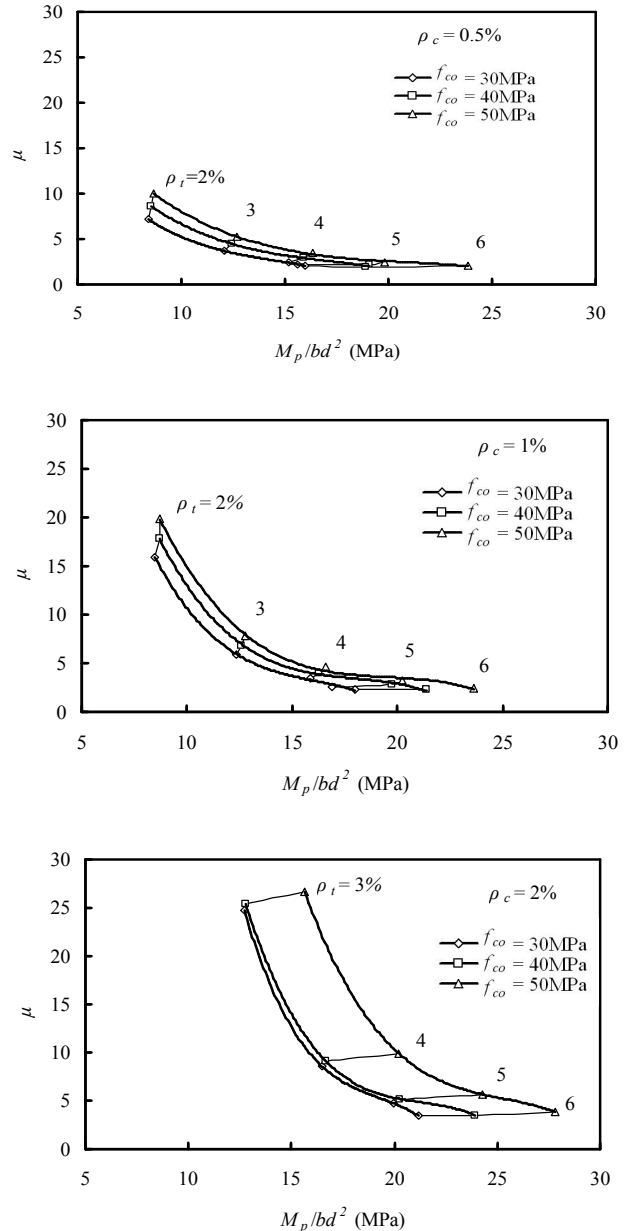


Fig. 6 Strength-ductility design charts for doubly-reinforced beams

C. Comparison with design codes

The difference of flexural strength and ductility evaluation of NSC beams by RC design codes and the proposed method is investigated by plotting the strength-ductility charts using those two approaches in the same figure, as shown in Figure 7. The concrete strengths are selected as 30 and 50MPa, compression steel ratios are 0 and 1%, and tension steel ratios vary from 1% to 6%.

From Figure 7, the following conclusions can be drawn: (1) The evaluation of strength-ductility obtained based on various design codes is very similar, which is because the equivalent rectangular stress block parameters for NSC stipulated in these codes are very close to each other due to ignorance of strain gradient effect. (2) The strength-ductility curves with strain gradient effect considered are located on the upper right-hand side of other curves without strain gradient effect considered, which means that there is improvement on strength-ductility performance, especially for singly reinforced beams. (3) Given the same flexural strength or ductility design requirement, the consideration of strain gradient effect can improve significantly the limit of ductility or strength that the beams can achieve, which means both strength and ductility design limits can be improved simultaneously.

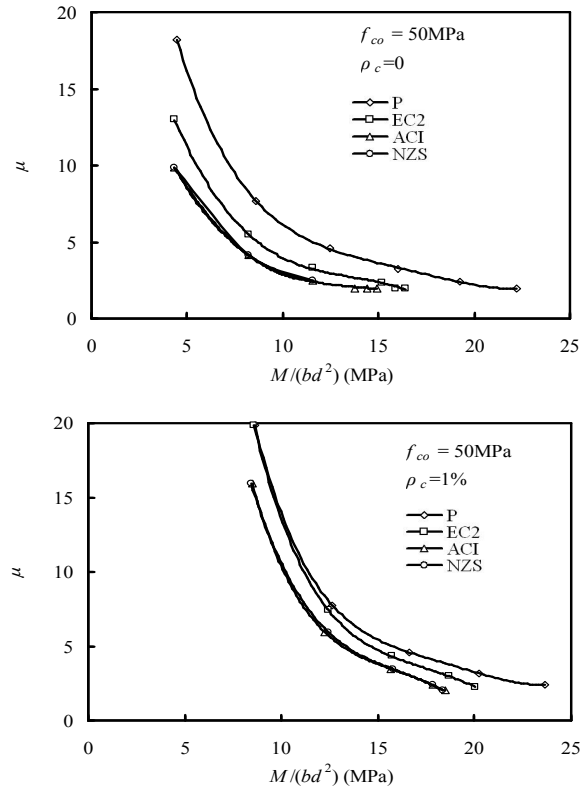
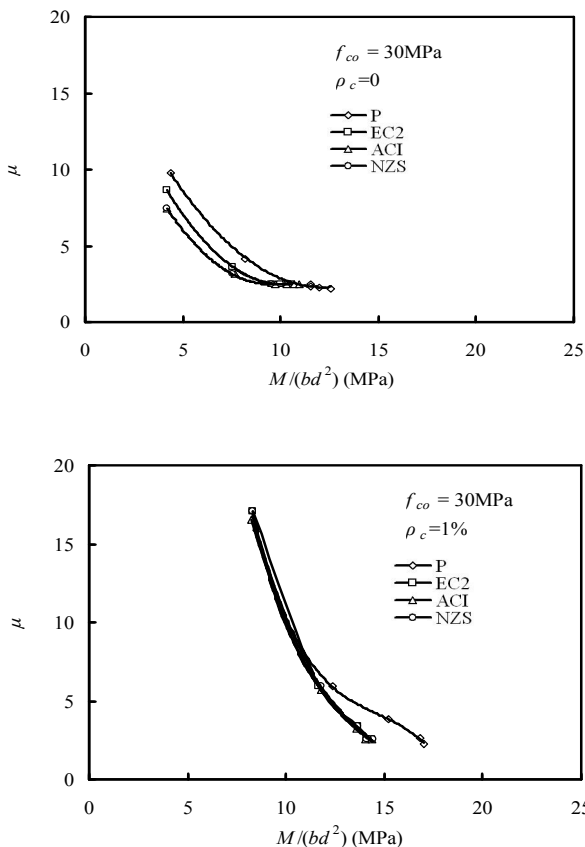


Fig. 7 Strength-ductility design charts comparison

VI. CONCLUSION

The actual concrete stress distribution developed under flexural RC members is nonlinear and is normally replaced by an equivalent rectangular concrete stress block for design purpose in many RC design codes. The simplified stress block is usually represented by two coefficients of α and β , which are assumed to be dependent only on concrete strength for NSC by those codes. However, from an early investigation by the authors, it was found that the parameter of α is not only dependent on concrete strength but also on flexural strain gradient, while β was found to be insensitive to strain gradient.

In order to verify the findings in the early study by the authors, another two groups of RC column specimens with different concrete strengths, which are higher than the early study ranging from 41 to 55MPa, were fabricated and tested for this study. The columns in each group were cast from the same batch of concrete and had identical property. In each group, one specimen was subjected to concentric load while others were subjected to eccentric or horizontal loads. The concentrically loaded column served as reference specimen subjected to no flexural strain gradient from which the concrete stress-strain curve under uni-axial compression was obtained. The concrete stress-strain curve developed in the eccentric or horizontal specimen subjected to flexural strain gradient was derived based on axial force and moment equilibriums, from which the target parameters α and β were calculated.

The findings in the previous study of the authors were verified in this study, and the obtained data were regressed to propose the design equations of α and β by incorporating flexural strain gradient. The formulae of α is represented by a tri-linear equation with strain gradient considered while that for β remains constant. The proposal was validated by using them predict flexural strength of RC members tested by other researchers.

As extension and application of this study, the proposed values of α and β were used to plot design charts of strength-ductility of NSC singly and doubly reinforced beams with specified concrete strengths, compression steel ratios and tension steel ratios, through which both strength and ductility design can be achieved at the same time. And the proposed strength-ductility curves were also compared with those calculated by RC codes, it was observed that the design limits for strength and ductility were improved simultaneously for NSC beams.

ACKNOWLEDGMENT

The research grant from Seed Funding Programme for Basic Research (account code 10208226) of The University of Hong Kong (HKU) for the work presented herein is gratefully acknowledged. The authors gratefully thank the Department of Civil and Structural Engineering, The Hong Kong Polytechnic University (PolyU), where the loading tests were conducted. Also, supports from the technical staff in the structural laboratory of PolyU and the Department of Civil Engineering, HKU, are greatly appreciated.

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