

# Finite Element Investigation of Transmission Conditions for Non-Monotonic Temperature Interphases

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**Abstract**—Imperfect transmission conditions modeling a thin reactive 2D interphases layer between two dissimilar bonded strips have been extracted. In this paper, the soundness of these transmission conditions for heat conduction problems are examined by the finite element method for a strong temperature-dependent source or sink and non-monotonic temperature distributions around the faces..

**Keywords**—Imperfect interface; Transmission conditions; Finite element analysis; Interphase

## I. INTRODUCTION

THIN interphases are nowadays an important part of technological processes and components [1]. An inhomogeneous structure obtained is such a way that may exhibit a wider variety of thermal and mechanical properties. As an example, adhesive layers allow for joining materials with essentially different properties at very high quality. On the other hand, finite element modeling of composite with thin interphases is still a difficult numerical task as it requires high in homogeneity of the constructed mesh which can lead to a loss of accuracy and even numerical instability. In the case of constant heat conductivity, the problem has been completely solved [2], where a general approach was developed independent of the range of the heat conductivity of the thin interphase. Transmission conditions for 2D heat conduction problems without reaction were investigated by [3, 4]. Later, various imperfect transmission conditions with thin reactive heat-conducting interphases for constant temperature were examined by [5, 6]. In the scope of this paper, imperfect transmission condition applied to a non-monotonic temperature distributions and a reactive thin-resistant layer in a hybrid model structure (see Fig. 1) with linear temperature dependence of the source.

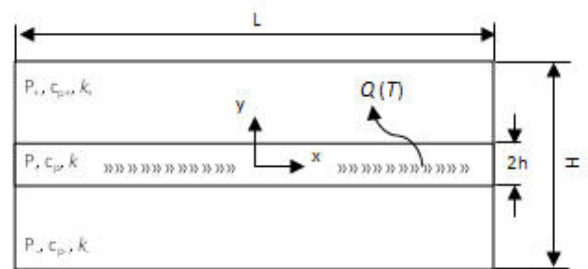


Fig. 1 Schematic representation of the hybrid model problem

## II. FINITE ELEMENT SOLUTION

A plane problem is considered for modeling a dissimilar body. The bonded materials from the top and the bottom part of the dissimilar strip possess the isotropic parameters  $\rho_+$ ,  $c_{p+}$ ,  $k_+$  and  $\rho_-$ ,  $c_{p-}$ ,  $k_-$  (mass density, specific heat and conductivity, respectively). The interphase is assumed to be isotropic with  $\rho$ ,  $c_p$ ,  $k$ . The commercial finite element code MSC Marc is used for the simulation of the thermal behavior of the modeling thin adhesive layer between two adherents. Both adherents reveal constant material properties for all simulations. The source or sink formulation is implemented by means of a special user subroutine (flux) written in FORTRAN. In the simulation, the interphase layer has a thickness of  $2h = H/100 = 0.01$  while the length of all components is equal to  $L = 10$ . Further details of the finite element mesh can be found in [7]. Numerical simulations have been made for the same aluminum adherents which reveal a constant conductivity of  $k_{\pm} = 237 \text{ W/(m.k)}$ , a mass density of  $\rho_{\pm} = 2698.8 \text{ Kg/m}^3$  and a specific heat of  $c_{\pm} = 898.2 \text{ j/(kg.K}^\circ\text{)}$ .

## III. BOUNDARY CONDITION OF PROBLEM

Figures 2.a and 2.b show three cases of boundary which can be applied to the boundary segments (b.s.). The cases give temperature distribution at the top ( $y = +H/2$ ) and bottom ( $y = -H/2$ ) part of surface along x-axis. The first case is constant temperature distribution (CTD) which appears along the x-axis. The second case shows linear temperature distribution (LTD) and third case will be parabolic temperature distribution (PTD).

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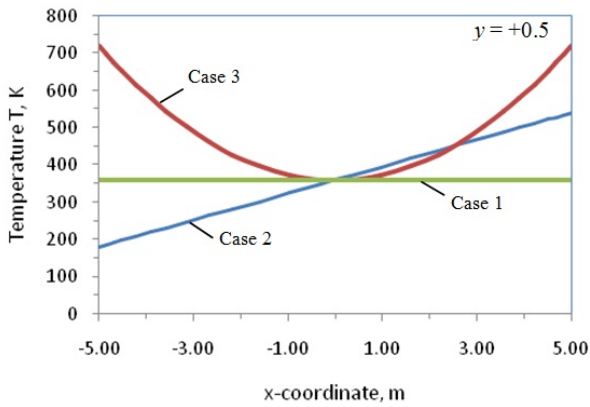


Fig. 2.a Boundary Condition (for,  $y = +0.5$ )

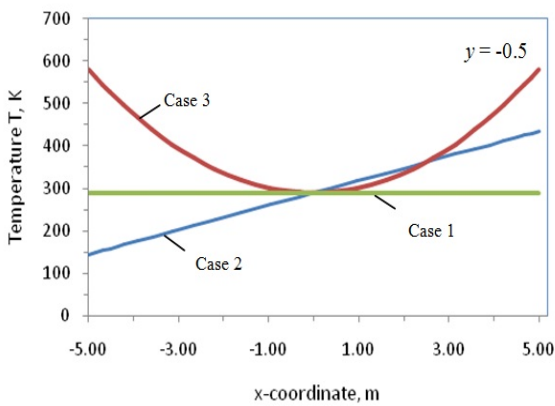


Fig. 2.b Boundary Condition (for,  $y = -0.5$ )

The thin interphase is assumed to be made of an epoxy resin ( $k = 0.2 \text{ W}/(\text{m}\cdot\text{k})$ ,  $\rho = 1200 \text{ Kg}/\text{m}^3$ ,  $c = 790 \text{ j}/(\text{kg}\cdot\text{K}^\circ)$ ) which exhibit temperature-dependent source and sink and forms:

$$Q = Q_0 \cdot T \quad (1)$$

where

$$Q_0 = +1000.$$

This problem refers to a steady-state solution where boundary conditions material properties are chosen.

#### IV. RESULTS AND DISCUSSION

As shown in previous paper [5] in the case of a linear temperature dependency, the first transmission condition (1<sup>st</sup> TC) can be obtained as

$$q_+^2(x, +h) - q_-^2(x, -h) = -kQ_0(T_+^2(x, +h) - T_-^2(x, -h)) \quad (2)$$

The second transmission condition (2<sup>nd</sup> TC) has also been shown in other paper [6] for the case of source ( $Q > 0$ ) as

$$\arcsin \frac{T_+ \sqrt{kQ_0}}{\sqrt{q_+^2 + kQ_0 T_+^2}} - \arcsin \frac{T_- \sqrt{kQ_0}}{\sqrt{q_-^2 + kQ_0 T_-^2}} = \pi + 2h \sqrt{\frac{Q_0}{k}} \cdot \text{sign}(q_-) \quad (3)$$

where the results are evaluated in x-direction at the upper and lower interfaces. Figure 3 shows x-component of heat flux along x-axis in the upper interface ( $y = +0.005$ ) where there are three cases of temperature distribution while a heat source ( $Q_0 > 0$ ) inserts along the interphase. The results are shown for the y-component of heat flux along x-axis at the upper interface ( $y = +0.005$ ) in Fig. 4.

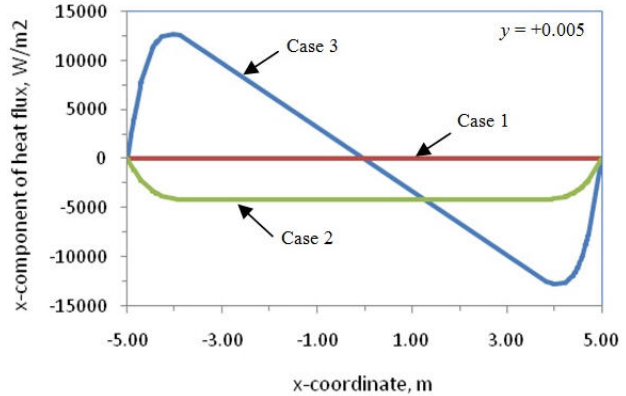


Fig. 3: x-Comp of heat flux upper interface in  $Q = 10^3 \cdot T$

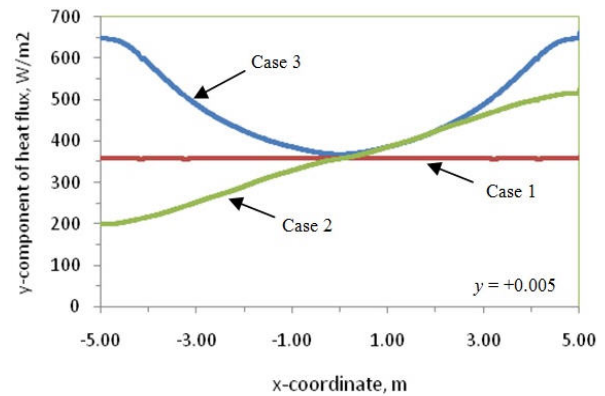


Fig. 4: y-Comp of heat flux upper interface in  $Q = 10^3 \cdot T$

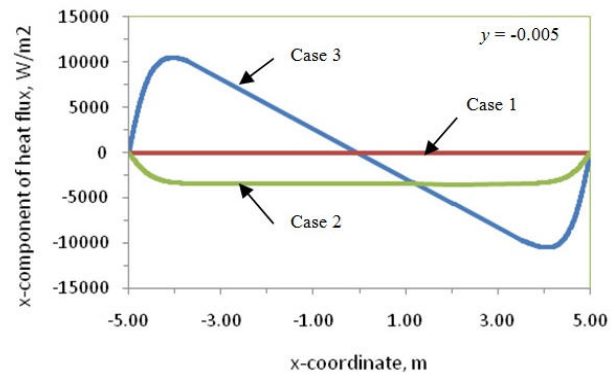


Fig. 5: x-Comp of heat flux lower interface in  $Q = 10^3 \cdot T$

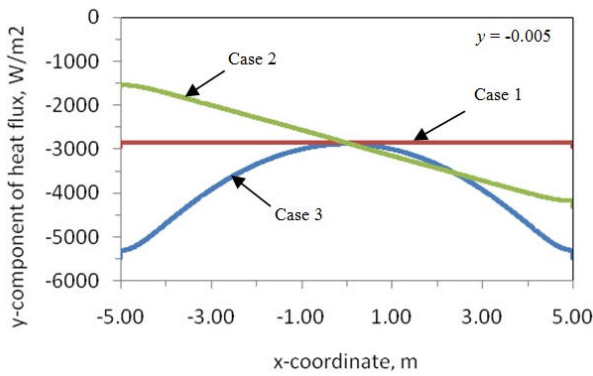


Fig. 6:  $y$ -Comp of heat flux lower interface in  $Q=10^3.T$

Figures 5 and 6 show the  $x$  and  $y$ -components of heat flux along  $x$ -axis at the lower interface ( $y = -0.005$ ) where non-monotonic temperature distributions are inserted in the interface.

TABLE I

VERIFICATION OF THE TRANSMISSION CONDITIONS VALIDITY ALONG THE  $X$ -AXIS FOR *FIRST CASE* (CTD) WITH VALUE OF THE TEMPERATURE-DEPENDENT SOURCE OR SINK, CF. EQNS. (2) - (3)

Coordinate	LHS	RHS	error
<b>1<sup>st</sup> TC</b>			
X = -5	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
X = -3	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
X = -1	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
X = 0	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
X = 1	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
X = 3	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
X = 5	-8.51287573*10 <sup>6</sup>	-8.518489254*10 <sup>6</sup>	10 <sup>-4</sup>
<b>2<sup>nd</sup> TC</b>			
X = -5	-2.436062015	-2.434485872	10 <sup>-4</sup>
X = -3	-2.436062015	-2.434485872	10 <sup>-4</sup>
X = -1	-2.436062015	-2.434485872	10 <sup>-4</sup>
X = 0	-2.436062015	-2.434485872	10 <sup>-4</sup>
X = 1	-2.436062015	-2.434485872	10 <sup>-4</sup>
X = 3	-2.436062015	-2.434485872	10 <sup>-4</sup>
X = 5	-2.436062015	-2.434485872	10 <sup>-4</sup>

Tables I, II and III present the verification of the transmission conditions (Eqns. (2)-(3)) along the  $x$ -axis by independently extracting the right and left hand side of the equations from FEM evaluation. The absolute value of the error has been obtained by calculating the difference of the LHS and RHS relating this difference to the RHS of the respective transmission conditions.

V. CONCLUSION

Let us summarize all the results obtained in this paper and the following our work compare the two stronger source with parabolic temperature distributions into the interphase are shown.

TABLE II

VERIFICATION OF THE TRANSMISSION CONDITIONS VALIDITY ALONG THE  $X$ -AXIS FOR *SECOND CASE* (LTD) WITH VALUE OF THE TEMPERATURE-DEPENDENT SOURCE OR SINK, CF. EQNS. (2) - (3)

Coordinate	LHS	RHS	error
<b>1<sup>st</sup> TC</b>			
X = -5	-2.45294906*10 <sup>6</sup>	-2.438721964*10 <sup>6</sup>	10 <sup>-3</sup>
X = -3	-4.16658084*10 <sup>6</sup>	-4.146826328*10 <sup>6</sup>	10 <sup>-3</sup>
X = -1	-6.84237198*10 <sup>6</sup>	-6.810330244*10 <sup>6</sup>	10 <sup>-3</sup>
X = 0	-8.51215802*10 <sup>6</sup>	-8.471887016*10 <sup>6</sup>	10 <sup>-3</sup>
X = 1	-1.03637179*10 <sup>7</sup>	-1.031301473*10 <sup>7</sup>	10 <sup>-3</sup>
X = 3	-1.43934095*10 <sup>7</sup>	-1.431812642*10 <sup>7</sup>	10 <sup>-3</sup>
X = 5	-1.82236907*10 <sup>7</sup>	-1.811799332*10 <sup>7</sup>	10 <sup>-3</sup>
<b>2<sup>nd</sup> TC</b>			
X = -5	-2.421826546	-2.434485872	10 <sup>-3</sup>
X = -3	-2.421583097	-2.434485872	10 <sup>-3</sup>
X = -1	-2.420852752	-2.434485872	10 <sup>-3</sup>
X = 0	-2.420122406	-2.434485872	10 <sup>-3</sup>
X = 1	-2.419148611	-2.434485872	10 <sup>-3</sup>
X = 3	-2.41768792	-2.434485872	10 <sup>-3</sup>
X = 5	-2.417201023	-2.434485872	10 <sup>-3</sup>

TABLE III

VERIFICATION OF THE TRANSMISSION CONDITIONS VALIDITY ALONG THE  $X$ -AXIS FOR *THIRD CASE* (PTD) WITH VALUE OF THE TEMPERATURE-DEPENDENT SOURCE OR SINK, CF. EQNS. (2) - (3)

Coordinate	LHS	RHS	error
<b>1<sup>st</sup> TC</b>			
X = -5	-2.93172394*10 <sup>7</sup>	-2.949292377*10 <sup>7</sup>	10 <sup>-3</sup>
X = -3	-1.59158960*10 <sup>7</sup>	-1.514607587*10 <sup>7</sup>	10 <sup>-3</sup>
X = -1	-9.31199157*10 <sup>6</sup>	-9.370086109*10 <sup>6</sup>	10 <sup>-3</sup>
X = 0	-8.60788625*10 <sup>6</sup>	-8.663331577*10 <sup>6</sup>	10 <sup>-3</sup>
X = 1	-9.31199157*10 <sup>6</sup>	-9.370086109*10 <sup>6</sup>	10 <sup>-3</sup>
X = 3	-1.59158960*10 <sup>7</sup>	-1.514607587*10 <sup>7</sup>	10 <sup>-3</sup>
X = 5	-2.93172394*10 <sup>7</sup>	-2.949292377*10 <sup>7</sup>	10 <sup>-3</sup>
<b>2<sup>nd</sup> TC</b>			
X = -5	-2.420756233	-2.434485872	10 <sup>-3</sup>
X = -3	-2.420521874	-2.434485872	10 <sup>-3</sup>
X = -1	-2.420212255	-2.434485872	10 <sup>-3</sup>
X = 0	-2.420102564	-2.434485872	10 <sup>-3</sup>
X = 1	-2.420212255	-2.434485872	10 <sup>-3</sup>
X = 3	-2.420521874	-2.434485872	10 <sup>-3</sup>
X = 5	-2.420756233	-2.434485872	10 <sup>-3</sup>

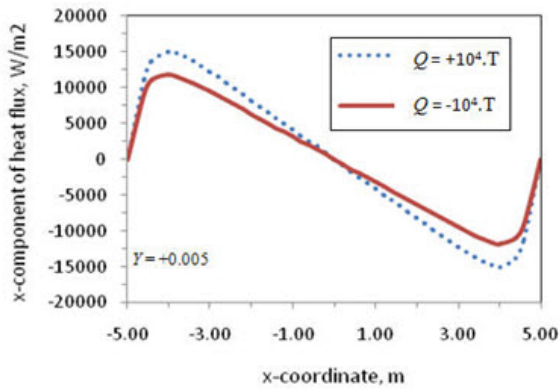


Fig. 7a x-component of heat flux with parabolic temperature distribution at the upper interface

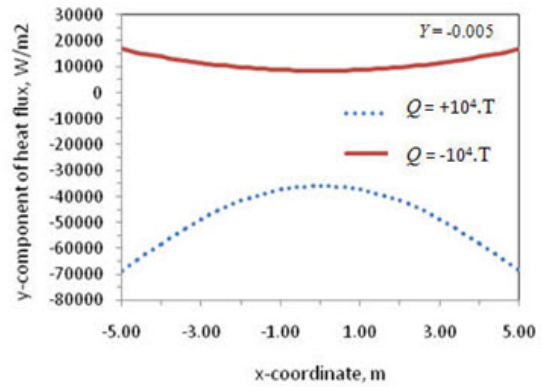


Fig. 8b y-component of heat flux with parabolic temperature distribution at the lower interface

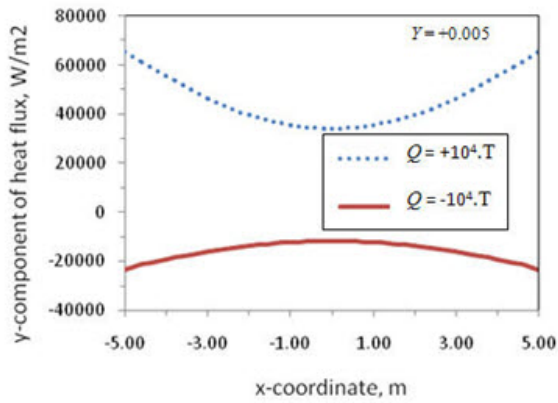


Fig. 7b y-component of heat flux with parabolic temperature distribution at the upper interface

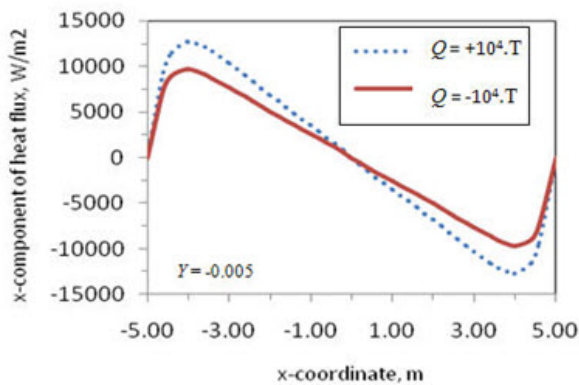


Fig. 8a x-component of heat flux with parabolic temperature distribution at the lower interface

In the presented work, it has been shown that the transmission conditions are not altered for constant temperature distributions along  $x$ -axis. On the other hand, the transmission conditions are altered in the non-monotonic temperature distributions along  $x$ -axis.

Figures 7 and 8 are compared the parabolic temperature distributions for the stronger sources along  $x$  in the upper and lower interface.

In the scope of this paper, the non-monotonic temperature distributions have been studied in the interphase. The investigation of non-monotonic source distributions into the interphase is reserved for our future research work.

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