

Feasibility Study of Distributed Lightless Intersection Control with Level 1 Autonomous Vehicles

Bo Yang, Christopher Monterola

Abstract—Urban intersection control without the use of the traffic light has the potential to vastly improve the efficiency of the urban traffic flow. For most proposals in the literature, such lightless intersection control depends on the mass market commercialization of highly intelligent autonomous vehicles (AV), which limits the prospects of near future implementation. We present an efficient lightless intersection traffic control scheme that only requires Level 1 AV as defined by NHTSA. The technological barriers of such lightless intersection control are thus very low. Our algorithm can also accommodate a mixture of AVs and conventional vehicles. We also carry out large scale numerical analysis to illustrate the feasibility, safety and robustness, comfort level, and control efficiency of our intersection control scheme.

Keywords—Intersection control, autonomous vehicles, traffic modelling, intelligent transport system.

I. INTRODUCTION

MANAGEMENT of the traffic at the urban road intersections is of paramount importance to achieve safe and efficient traffic flows in the cities. Optimization of the signalized intersection control will continue to be of most importance in the foreseeable future in most parts of the world. A single traffic intersection can be more efficiently controlled with either pre-signal systems [1] or with readily available sensing technologies, so that the traffic light can adapt to the real time demand of the intersection [2], [3]. Multi-layer agent control of a system of intersections in either a centralized or decentralized way has been intensively studied [4]–[10]. More innovative approaches where intersection manager sends out individual traffic signals to each vehicle are also reported [11]–[13], so that the traffic flow across the intersection can be optimized with the sequence-based protocols. On the other hand, the advancement in smart vehicles, in particular the driverless vehicles with the communication and self-control capabilities, opens up opportunities that can shift the paradigm of the intersection control. Many researches have been focusing on lightless (with no traffic light) intersection control with AV [14], [15]. In these proposals, all the incoming vehicles relay information on their positions, velocities as well as their intentions to a central system; such a central system sends out instructions to each of these vehicles, based on that information and an optimized algorithm [16], [17]. While such schemes are highly efficient as compared to the signalized intersection control, it is not desirable to require near 100% market penetration of

very intelligent AV. Factors such as technological, legal, and cost issues can hamper the adoption of AV by the mainstream commuters in the next decade or so. With a sophisticated centralized control agent at the intersection, it is possible that we do not require the vehicles to be equipped with the sensing capabilities, thereby lowering the technological barrier of the AV. However, the infrastructure of such a central agent can be expensive to build and difficult to scale during peak hours with large vehicle densities. A centralized control can also make it vulnerable to disruptions and security attacks. The requirement of the urban traffic to consist mainly of highly intelligent vehicles, and the centralized intersection control, are two major challenges of the lightless intersection traffic management.

In this paper, we try to deal with these two issues by asking this fundamental question: what is the minimal level of intelligence that is required of the vehicles, so that we can move beyond the traffic light and control the intersection with high efficiency? The National Highway Traffic Safety Administration (NHTSA) of the United States proposed a formal five-level classification of the AV [18]. The majority of vehicles on the road nowadays are at Level 0, while current technologies are more or less mature up to smart vehicles at Level 2, at which the vehicle can be equipped with adaptive cruise control and perform lane keeping (changing) as well as parking. The scheme proposed by us, on the other hand, only requires AV at Level 1 or above. Specifically, the vehicle only needs to be able to throttle or brake automatically based on the algorithms we have designed, in addition to the ability to communicate with the infrastructure at the intersection. Such “feet-off” smart vehicles can be easily achieved with the current technology.

In our scheme, the control of the vehicles approaching the intersection is also fully distributed or decentralized. Every vehicle computes and decides its own dynamics independently, based on specific information about its environment. Other decentralized intersection control proposed in the literature [19]–[21] focus on avoiding collisions in the intersection region via communication and negotiation between vehicles [21]. In contrast, our scheme does not focus on the right of way, and the collision is avoided at the intersection by the proper “repulsive interactions” between the vehicles. Such “repulsive interaction” already exists between any two consecutive vehicles in the same lane, since each driver needs to make sure that a safe distance is kept between his/her vehicle and the vehicle in the front. We extend such interaction to include selected vehicles near the intersection travelling in different directions. This additional

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layer of interactions can be accomplished by the properly tuned adaptive cruise control, or the Intersection Cruise Control (ICC) device. Communications between vehicles and the intersection infrastructure are also kept minimal, and the infrastructure does not control the dynamics of the vehicles.

The present paper will be organized as follows: in Section II, we introduce the main ideas of our lightless intersection control scheme, including the overall design and the architecture of such scheme; in Section III, we detail the algorithm of the intersection control that governs the interactions between vehicles close to the intersection, and how the vehicles respond to such interactions dynamically; in Section IV, we show extensive numerical simulations of our intersection control algorithm; in Section V, we discuss about the technical issues of safety, efficiency, and passenger comfort; in Section VI, we summarize our results and discuss about real world implementation.

II. THE GENERAL SCHEME

In our scheme, when the vehicle installed with an ICC device is within a specific range from the intersection, the device receives relevant information about its environment. We define the part of the road within this range to be the interaction zone. The information sent to the ICC device of the individual vehicles comes from a beacon located at the intersection, replacing the traffic light. The beacon is able to detect the positions and velocities of a few vehicles closest to the intersection, and such is the information broadcasted to all the vehicles within the interaction zone.

For most of the time, the ICC device does not really interfere with the motion of the vehicle. The driver still drives the vehicle in the normal way in the interaction zone, though lane-changing is not allowed in this zone. However, under certain conditions (as will be specified in Section III), the ICC device will override the driver's action and decelerate the vehicle momentarily. Such deceleration is deemed necessary to ensure a safe passage through the intersection. There are thus two major functions of the ICC devices. The first function is information relay, by either receiving information sent by the beacon at the intersection, or by communicating with nearby vehicles about the relative positions and velocities. This function is switched on automatically whenever the vehicle is within the interaction zone and cannot be manually switched off. The second function is to slow down the vehicle when necessary. The slowdown reflects the "repulsive interaction" between vehicles close to the intersection. The driver, however, can choose to take back the control of the ICC anytime, but in the absence of human interference the ICC will guide the vehicle to move across the intersection, after which it will automatically switch off. We will discuss more in details about the effect of human drivers overriding the ICC devices in Section VI.

The ICC device only needs to perform a very small set of simple tasks as compared to the full-fledged AV, or even the commercially available vehicles equipped with the advanced adaptive cruise controls. The ICC device does not have to be capable of any sensing capabilities. In principle, the active

driver is still the person, who will be monitoring the road conditions and steering the wheel, and deciding whether or not to override the ICC in uncommon situations. The ICC devices only control the acceleration and deceleration of the vehicle, with no control over the steering.

III. CONTROL ALGORITHMS

The intersection control algorithm is only applied to vehicles close to the intersection (see Fig. 1). The interaction zone starts at a distance $L_s + L_c$ measured upstream from the intersection and it is divided into two parts: the caution zone, which starts right from the intersection with a distance of L_c , and the synchronization zone, which starts from the end of the caution zone. To illustrate the algorithm as simply as possible, we look at the simplest intersection where the two road sections are of the single lane and unidirectional. We also ignore the turning behaviours at the intersection.

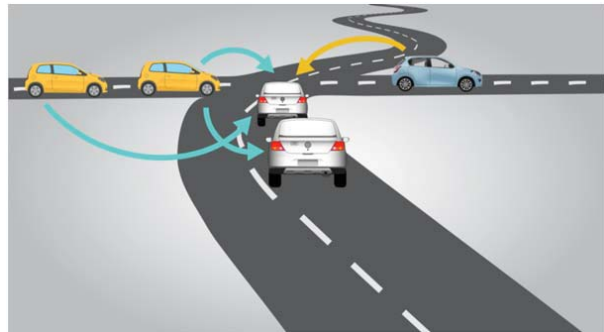


Fig. 1 For vehicles travelling from south to north (the silver vehicles), only the two closest to the intersection will interact with vehicles travelling in the other direction (the yellow and blue vehicles). The arrows indicate the vehicles interacting with the silver vehicles. For the south most silver vehicle, only one vehicle will affect its dynamics, while for the silver vehicle closest to the intersection, all three vehicles travelling in the other direction affect its dynamics

For the vehicles outside of the interaction zone, the human driver is in charge and driving the vehicle as if there is green light at the intersection. Such human driving behaviours are modelled by the popular intelligent driver model [22]. We now describe the behaviours of the ICC device in the interaction zone, so as to completely define the motion of the vehicle. The human driver is still in control of the vehicle, but the ICC device may enforce deceleration momentarily based on the algorithm. The deceleration is instituted mainly to prevent possible collisions at the intersection.

Due to the distributed nature of the control algorithm, let us choose a random vehicle and denote it as Vehicle A. Let Vehicle B be the leading vehicle approaching the intersection in the other direction, and Vehicle C follows immediately after Vehicle B. Vehicle A' is right in front of Vehicle B, and it has already passed the intersection. The interaction between Vehicles A and A', B, C depend on their respective time headway to the intersection $t_A = l_A / v_A$, $t_B = l_B / v_B$, $t_C = l_C / v_C$, $t_{A'} = l_{A'} / v_{A'}$, where l_A , $l_{A'}$, l_B , l_C are the distances of the vehicle to the intersection respectively (and $l_{A'}$ is negative since the

vehicle has passed the intersection); $v_A, v_{A'}, v_B, v_C$ are the velocities respectively. We also define $\tau_B = l_{safe} / v_B, \tau_C = l_{safe} / v_C, \tau_{A'} = (l_{A'} + l_{safe}) / v_{A'}$. Both l_{safe} here and t_{safe} (which will be used later) are the safety parameters that we can tune to guarantee safety and at the same time balance between efficiency and comfort. In particular, l_{safe} depends on the length of the vehicle and the width of the lane. One can choose it to be slightly larger than the sum of car length and lane width.

If there are more than one vehicles in front of Vehicle A that are also approaching the intersection, the ICC device will not decelerate the vehicle; the driver just needs to follow the vehicle in the front. If there is only one vehicle in the front approaching the intersection, the ICC device will decelerate Vehicle A if the following criterion is satisfied:

$$t_A > t_B \text{ and } t_A - t_B < \tau_B + t_{safe} \quad (1)$$

If Vehicle A is the foremost vehicle in the lane approaching the intersection, the ICC device will decelerate the vehicle if any one of the following three criteria is satisfied:

$$t_A > t_B \text{ and } t_A - t_B < \tau_B + t_{safe} \quad (2)$$

$$t_A > t_C \text{ and } t_A - t_C < \tau_C + t_{safe} \quad (3)$$

$$\tau_{A'} > 0 \text{ and } \tau_{A'} < t_A \quad (4)$$

The criteria (1)-(4) make sure Vehicle A will not have any potential collisions with other vehicles at the intersection, and when the time headway differences are large enough between vehicles heading towards the intersection from different directions, the vehicle also tends to accelerate (since the human driver will follow the vehicle in the front), leading to very efficient traffic flow across the intersection.

When one or more of the conditions of (1)-(4) are satisfied, the ICC device will decelerate the vehicle with the magnitude that depends on the vehicle's position and velocity. Such dependence can be further optimized by taking into consideration of factors such as safety, passenger comfort as well as intersection control efficiency.

IV. NUMERICAL SIMULATIONS

For a simple illustration, we first use a very simple deceleration profile that does not depend on the velocity of the vehicles. When one or more of (1)-(4) are satisfied, the ICC will decelerate the vehicle at 2 ms^{-1} if the vehicle is in the synchronization zone, and 5 ms^{-1} if the vehicle is in the caution zone. Due to the distributed nature of our intersection control, for vehicles to arrive at the intersection without colliding with each other, we need to make sure that each vehicle will not crash into any other vehicle that is passing through the intersection, i.e. the front bumper of all other vehicles that have already reached the intersection should be sufficiently far away from the intersection. We can control and guarantee such conditions by tuning the safety parameters l_{safe} and t_{safe} .

With larger l_{safe} and t_{safe} , the gap between vehicles at the intersection is also greater, implying more secure passage across the intersection. On the other hand, the safety parameters implicitly quantify the strength of interaction between vehicles close to the intersection. If both l_{safe} and t_{safe} are zero, the traffic in two directions is completely decoupled. The stronger the interaction, the greater the effective perturbation it induces to the chain of vehicles travelling in the same lane. If the density of the traffic is in the metastable regime [23], traffic congestions or the stop-and-go waves are more likely to occur with greater perturbation, leading to a reduction of the intersection capacity and the efficiency of the intersection control.

From an optimization point of view, we thus would like to minimize both l_{safe} and t_{safe} , while at the same time guaranteeing the safe passage of vehicles across the intersection. We evaluate the intersection capacity of the control by inserting in vehicles continuously into the traffic system, and monitoring the maximum inflow, beyond which the traffic congestions will occur (see Fig. 2). The vehicles are inserted randomly with a fixed average inflow. No incidents of collisions have ever been observed numerically even with the very simple deceleration profile we described, showing strong evidence of the robustness of our algorithm.

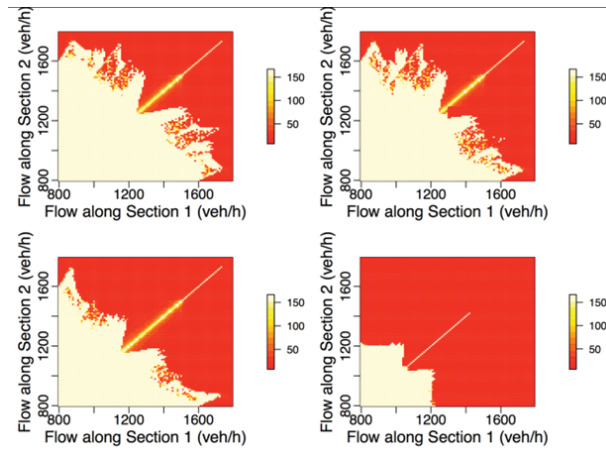


Fig. 2 The x-axis is the average inflow of the traffic travelling from west to east, while the y-axis is the average inflow of the traffic from south to north. The color scale here shows the time it takes for the congestion to develop because of the intersection. Note that the white region shows no congestion after numerical simulation of more than three hours, indicating the free flow phase with no accumulation of vehicles around the intersection. The four plots are the same numerical simulation of the lightless intersection control with different values for the safety parameters. Top left: $l_{safe} = 9 \text{ m}, t_{safe} = 0.2 \text{ s}$; Top right: $l_{safe} = 11 \text{ m}, t_{safe} = 0.2 \text{ s}$; Bottom left: $l_{safe} = 9 \text{ m}, t_{safe} = 0.4 \text{ s}$; Bottom right: $l_{safe} = 9 \text{ m}, t_{safe} = 0.8 \text{ s}$. The numerical simulations are performed by random insertion of the vehicles while keeping the average inflow fixed

It is clear in Fig. 2 that increasing either l_{safe} or t_{safe} makes the intersection control less efficient, since congestions are more likely to occur at lower inflow of the traffic. On the other

hand, the gaps between vehicles passing through the intersection are larger, implying safer trips for the drivers.

The simulations above do not consider the comfort level of the drivers; in particular, when the ICC decelerates the vehicle, the acceleration changes abruptly with very large jerk. We can remedy this by enforcing a “comfortable” jerk profile as discussed in [24]. The sinusoidal jerk profile can be easily implemented in our algorithm, and the resulting phase diagram is shown in Fig. 3.

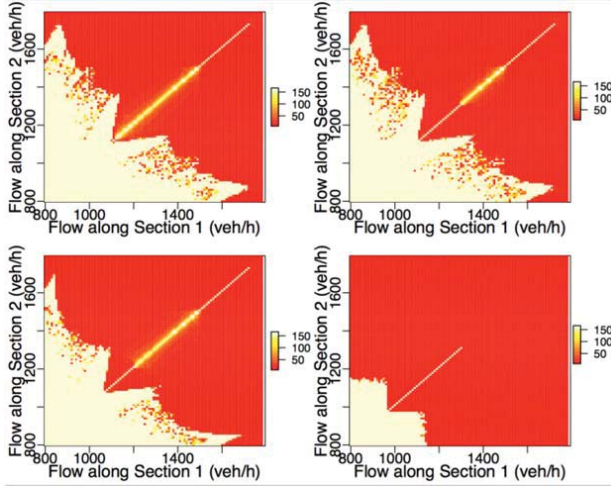


Fig. 3 The same plot as the one in Fig. 2, but with the jerk satisfying (5), and with different safety parameters. Top left: $l_{\text{safe}} = 9 \text{ m}$, $t_{\text{safe}} = 0.3\text{s}$; Top right: $l_{\text{safe}} = 11 \text{ m}$, $t_{\text{safe}} = 0.2\text{s}$; Bottom left: $l_{\text{safe}} = 9 \text{ m}$, $t_{\text{safe}} = 0.4\text{s}$; Bottom right: $l_{\text{safe}} = 9 \text{ m}$, $t_{\text{safe}} = 0.8\text{s}$. The numerical simulations are performed by random insertion of the vehicles while keeping the average inflow fixed

Formally, when the ICC device activates the dynamical control, it actually institutes an acceleration profile of:

$$a(t) = a(0) + \frac{\Delta a}{2} \left(1 \pm \cos\left(\frac{\pi}{\Delta t} t\right) \right) \quad 0 \leq t \leq \Delta t \quad (5)$$

where $a(0)$ and $a(\Delta t)$ are the accelerations before and after the big change instituted by the ICC device, and $\Delta a = a(\Delta t) - a(0)$. The time interval Δt is chosen so that the maximum absolute value of the jerk will not exceed 20 ms^{-2} . Such restrictions imply vehicles do not reach the desired acceleration instantaneously, affecting the capability of avoiding a potential collision. This, however, can be compensated by increasing l_{safe} and t_{safe} , at the cost of reducing the efficiency of the intersection control.

The implementation of the jerk profile of (5), while increasing the driver comfort level, does reduce the efficiency of the intersection control, as the capacity of the intersection control is reduced. For $l_{\text{safe}} = 9 \text{ m}$ and $t_{\text{safe}} = 0.2 \text{ s}$, implementing (5) will cause a 0.01% chance of collisions at the intersection, while instantaneous change of the acceleration is completely safe. For $l_{\text{safe}} = 9 \text{ m}$ and $t_{\text{safe}} = 0.4 \text{ s}$, $l_{\text{safe}} = 9 \text{ m}$ and $t_{\text{safe}} = 0.8 \text{ s}$, $l_{\text{safe}} = 11 \text{ m}$ and $t_{\text{safe}} = 0.2 \text{ s}$, no collisions occur even if the jerk is constrained by (5).

However, the red region (congested phase) is larger in Fig. 3, as compared the corresponding plots in Fig. 2.

V. SAFETY, COMFORT AND EFFICIENCY

The algorithm for the lightless intersection control has two freely tunable parameters l_{safe} and t_{safe} ; the deceleration instituted by the ICC device can also be optimized. Optimization of the algorithm needs to guarantee safety, at the same time providing sufficient driving comfort with no excessive jerk during acceleration or deceleration. Lastly, the self-organized traffic flow needs to be much more efficient as compared to the signalized intersection control.

The numerical simulation has shown that even for very simple ICC deceleration, one can tune l_{safe} and t_{safe} to ensure safety and balance between comfort and efficiency. Note that we have stringent criteria in determining the safety of the algorithm, including random insertion of the vehicles and generous space allowance between vehicles at the intersection. More than one million simulations, each simulating a real-time traffic flow of three hours, are shown in Figs. 2 and 3, with no collisions between vehicles.

Comparing Fig. 3 with Fig. 2, one can also see that by making the trips more comfortable by restricting the jerk profile, the efficiency of the intersection control is reduced, reflecting the compromise between efficiency and comfort. A more comfortable trip also requires larger values of l_{safe} and t_{safe} to maintain the same amount of spaces between vehicles at the intersection.

One should note that even with relatively large l_{safe} and t_{safe} , and relatively high comfort level, the intersection capacity with lightless intersection control is always much superior as compared to the signalized traffic control. It is well known that the outflow of the traffic queueing at the red light is characteristic [22], [25], providing a theoretical upper limit of the intersection capacity for any traffic light control. Here in our illustration, the upper limit is 1200 veh/h. If the inflow of the traffic from both directions at the intersection is similar, each vehicle will have an equal chance of encountering a red light or a green light; thus for traffic light control, the intersection capacity in generally will never exceed 600 veh/h. All numerical simulations in Figs. 2 and 3 show much better intersection control capacity with various different parameters in the control algorithm.

VI. SUMMARY AND OUTLOOK

In this paper, we study the feasibility of an interaction based lightless intersection control scheme, focusing on the issue of safety, driver comfort as well as efficiency. We show that the algorithm can be easily tuned by two safety parameters, so that the avoidance of the collisions at the intersection can be guaranteed, and the balance between driver comfort and intersection control efficiency can be easily adjusted. Our extensive numerical simulations show strong evidence that all three factors can be accommodated, and a highly efficient, self-organized traffic flow can be resulted from our control scheme.

While we only consider the simplest case in this paper for the purpose of illustration and due to the length limitation, our algorithm can be easily extended to multilane traffic with turning allowed. It can also allow a mixture of conventional vehicles and vehicles equipped with ICC devices. Detailed analysis of more complicated scenarios will be presented elsewhere.

From the point of view of the actual implementation, our algorithm has the advantage of very low technological barrier, since it only requires vehicles that can throttle and brake by themselves, and no sensing capabilities are required. The lightless intersection control scheme can also coexist with the signalized traffic control. Depending on the traffic flow patterns within a city, such coexistence can either be arranged location-wise, or for a single intersection, different control schemes can be used at different time of the day (e.g. peak hours vs. off-peak hours).

The human driver is in charge all the time and can override ICC whenever needed, and this is potentially significant from the legal perspective. In general, however, the ICC device should be treated as the traffic light, whose instructions (ICC taking over the deceleration of the vehicle) should be obeyed as part of the traffic rules. Thus the ICC device should only be overridden if there are justifiable reasons; otherwise, such action is similar to disobeying the traffic lights. There can be a visible indicator on the vehicle indicating if the ICC device is active or if it is overridden, which makes it easier for fellow drivers and the traffic police to detect any violations. The ICC devices only interfere with driving momentarily, so the overall effect on the conventional driving experience is also minimal.

Further research is needed to optimize the control algorithm for more complicated intersections, and for rare events such as accidents and rogue drivers. We believe the capability and design of our scheme potentially make it easy to be tested and implemented in the near future.

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