

# Fault Detection of Pipeline in Water Distribution Network System

Shin Je Lee, Go Bong Choi, Jeong Cheol Seo, Jong Min Lee and Gibaek Lee

**Abstract**—Water pipe network is installed underground and once equipped, it is difficult to recognize the state of pipes when the leak or burst happens. Accordingly, post management is often delayed after the fault occurs. Therefore, the systematic fault management system of water pipe network is required to prevent the accident and minimize the loss. In this work, we develop online fault detection system of water pipe network using data of pipes such as flow rate or pressure. The transient model describing water flow in pipelines is presented and simulated using MATLAB. The fault situations such as the leak or burst can be also simulated and flow rate or pressure data when the fault happens are collected. Faults are detected using statistical methods of fast Fourier transform and discrete wavelet transform, and they are compared to find which method shows the better fault detection performance.

**Keywords**—fault detection, water pipeline model, fast Fourier transform, discrete wavelet transform.

## I. INTRODUCTION

**W**ATER supply network is a large system to distribute water to house and industry. This network is installed underground and entangled complicatedly. Therefore, it is difficult to recognize faults such as leak or burst when the faults happen. Also, since the number of sensors to measure pressure or flow rate of the pipe is limited, accurate fault detection is hardly possible. As longer as the fault detection is delayed, economical and social losses are bigger and bigger. Thus, the detection techniques are required to minimize the massive loss when the faults occur. In this work, we develop a mathematical model of pipelines and simulate to collect the pressure and flow rate data when the leak is created. Then, we present the fault detection techniques, fast Fourier transform (FFT) and discrete wavelet transform (DWT) based on statistical methods. These techniques are applied to the simulated data with the leak to show the fault detection performance. We also apply the proposed detection schemes to real data obtained from a small area. This paper is organized follows. Section 2 shows the procedure of modeling of pipelines and simulation results. Then, fast Fourier transform and wavelet transform are mathematically explained at Section 3. Section 4 presents the results of fault detection with the simulated data and real data using the proposed methods. Finally, it is followed by Conclusion.

S. J. Lee, G. B. Choi and J. M. Lee are with the School of Chemical and Biological Engineering, Seoul National University, Seoul 151-744, South Korea.

J. C. Seo is with Samchully Corporation, Seoul 150-885, South Korea.

G. Lee is with the Department of Chemical and Biological Engineering, Korea National University of Transportation, Chungju, Chungbuk 380-720, South Korea.

## II. MATHEMATICAL MODEL

### A. Model of Pipeline

Modeling transients in pipelines and pipe networks is presented by Wylie [1]. Unsteady state flow in a pipeline can be described by the principles of conservation of mass and momentum. The simplified continuity and motion equations are in the form of

$$\frac{\partial H}{\partial t} + V \frac{\partial H}{\partial x} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \quad (1)$$

$$\frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{V}{gA} \frac{\partial Q}{\partial x} + \frac{\partial H}{\partial x} + \frac{fQ|Q|}{2gDA^2} = 0 \quad (2)$$

where  $H$  is hydraulic head,  $Q$  is volumetric flow rate,  $V$  is mean velocity of the flow,  $g$  is gravitational acceleration,  $a$  is wave speed in a conduit,  $f$  is friction factor,  $D$  is pipe diameter,  $A$  is cross-sectional area,  $x$  is distance along the pipe, and  $t$  is time. Wave speed can be calculated from

$$a = \sqrt{\frac{\frac{K}{\rho}}{1 + \frac{K}{E} \frac{D}{e} \phi}} \quad (3)$$

where  $K$  is bulk modulus of elasticity of the fluid,  $e$  is pipe wall thickness,  $E$  is Young's modulus of elasticity of the conduit walls,  $\rho$  is fluid density, and  $\phi$  is parameter depending on the pipe anchoring. The transient model is partial differential equation (PDE) and it requires a particular solution scheme to convert the PDE to a set of ODEs.

### B. Method of Characteristics

The method of characteristics (MOC) is one of the most popular techniques for solving the above partial differential equation [2]. The governing pipe flow equations are written as

$$L_1 = \frac{\partial H}{\partial t} + V \frac{\partial H}{\partial x} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \quad (4)$$

$$L_2 = \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{V}{gA} \frac{\partial Q}{\partial x} + \frac{\partial H}{\partial x} + \frac{fQ|Q|}{2gDA^2} = 0 \quad (5)$$

A linear combination of Equations (4) and (5) is derived as

$$\lambda L_1 + L_2 = \frac{1}{gA} \left[ \frac{\partial Q}{\partial t} + (V + \lambda a^2) \frac{\partial Q}{\partial x} \right] + \lambda \left[ \frac{\partial H}{\partial t} + (V + \frac{1}{\lambda}) \frac{\partial H}{\partial x} \right] + \frac{fQ|Q|}{2gDA^2} = 0 \quad (6)$$

The time derivatives of  $Q$  and  $H$  are expressed as

$$\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{dx}{dt} \frac{\partial Q}{\partial x} \quad (7)$$

$$\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{dx}{dt} \frac{\partial H}{\partial x} \quad (8)$$

respectively. The time derivative of  $x$  can be derived as

$$\frac{dx}{dt} = V + \lambda a^2 = V + \frac{1}{\lambda} \quad (9)$$

comparing the bracketed terms of Equation (6) and Equations (7) and (8). Then, the PDE (6) is transformed to the ODE

$$\frac{1}{gA} \frac{dQ}{dt} + \lambda \frac{dH}{dt} + \frac{fQ|Q|}{2gDA^2} = 0 \quad (10)$$

From the second and third terms of Equation (9), the following is obtained

$$\lambda = \pm \frac{1}{a} \quad (11)$$

and using this, the time derivative of  $x$  reduces to

$$\frac{dx}{dt} = V \pm a \quad (12)$$

The flow velocity,  $V$ , can be neglected in water pipes, then Equation (12) becomes

$$\frac{dx}{dt} = \pm a \quad (13)$$

Equation (13) defines two straight lines, called characteristics. The characteristic associated with  $+a$  is referred to as the  $C^+$  characteristic, and the  $C^-$  characteristic is associated with  $-a$ . Substituting corresponding values of  $\lambda$  into Equation (10) leads to two pairs of equations which are identified as  $C^+$  and  $C^-$ .

$$C^+ : \begin{cases} \frac{a}{gA} \frac{dQ}{dt} + \frac{dH}{dt} + \frac{fQ|Q|a}{2gDA^2} = 0 \\ \frac{dx}{dt} = +a \end{cases} \quad (14)$$

$$C^- : \begin{cases} \frac{a}{gA} \frac{dQ}{dt} - \frac{dH}{dt} + \frac{fQ|Q|a}{2gDA^2} = 0 \\ \frac{dx}{dt} = -a \end{cases} \quad (15)$$

Equations (14) and (15) are called compatibility equations and used to solve for points in the  $xt$  plane. The solution of compatibility equations is achieved by integration using an approximation of the friction term (the third term on the left hand side). A linear approximation of the flow between end points gives

$$C^+ : \frac{a}{gA} (Q_P - Q_A) + (H_P - H_A) + \frac{f\Delta x}{2gDA^2} Q_P |Q_A| = 0 \quad (16)$$

$$C^- : \frac{a}{gA} (Q_P - Q_B) + (H_P - H_B) + \frac{f\Delta x}{2gDA^2} Q_P |Q_B| = 0 \quad (17)$$

The simultaneous solution of two compatibility equations yields the conditions at a particular time and position in the  $xt$  plane designated by point  $P$ , given the conditions at a previous time step (points  $A$  and  $B$ ), as shown in Fig. 1. Solving for  $H_P$  simplifies Equations (16) and (17) to

$$C^+ : H_P = C_P - BQ_P \quad (18)$$

$$C^- : H_P = C_M + BQ_P \quad (19)$$

where  $C_P$  and  $C_M$  are constants defined at the previous step

$$C_P = H_A + Q_A(B - R|Q_A|) \quad (20)$$

$$C_M = H_B - Q_B(B - R|Q_B|) \quad (21)$$

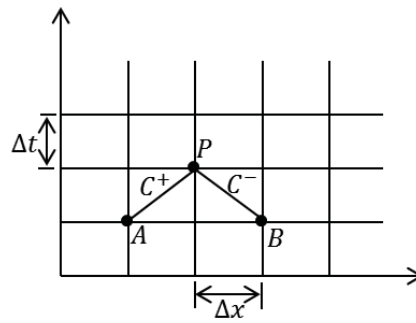


Fig. 1. Charateristics grid

$B$  and  $R$  are written as

$$B = \frac{a}{gA} \quad (22)$$

$$R = \frac{f\Delta x}{2gDA^2} \quad (23)$$

respectively.  $H_P$  can be solved using Equations (18) and (19)

$$H_P = \frac{C_P + C_M}{2} \quad (24)$$

$Q_P$  can then be calculated from Equation (18) or (19)

$$Q_P = \frac{C_P - H_P}{B} \quad (25)$$

$$Q_P = \frac{H_P - C_M}{B} \quad (26)$$

At ends of the single pipe, only one of the compatibility equations is available. Thus, the boundary conditions of the pipeline are used to specify  $Q_P$  or  $H_P$  at ends. The boundary equation for the leak is presented in the next section.

### C. Leakage Boundary

An orifice equation that describes the leak has the following form

$$Q_L = C_d A_0 \sqrt{2gH_L} \quad (27)$$

where  $Q_L$  and  $H_L$  are the flow through the orifice and a hydraulic head at the leak, respectively,  $C_d$  and  $A_0$  is an orifice discharge coefficient and the cross-section area of the orifice, respectively. A mass balance equation for the leakage node is given as

$$Q_u - Q_d - Q_L = 0 \quad (28)$$

where  $Q_u$  and  $Q_d$  are the upstream and downstream flows, respectively, derived from Equations (25) and (26) with  $H_L$  instead of  $H_P$ .  $H_L$  can be found by substituting modified ones of Equations (25) and (26), and Equation (27) into Equation (28).  $Q_u$  and  $Q_d$  can then be obtained using  $H_L$ .

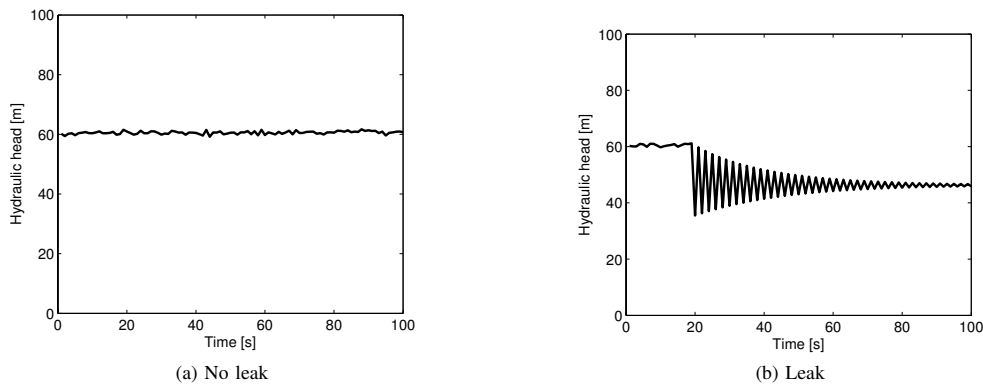


Fig. 2. Hydraulic heads without leak (a) and with leak (b)

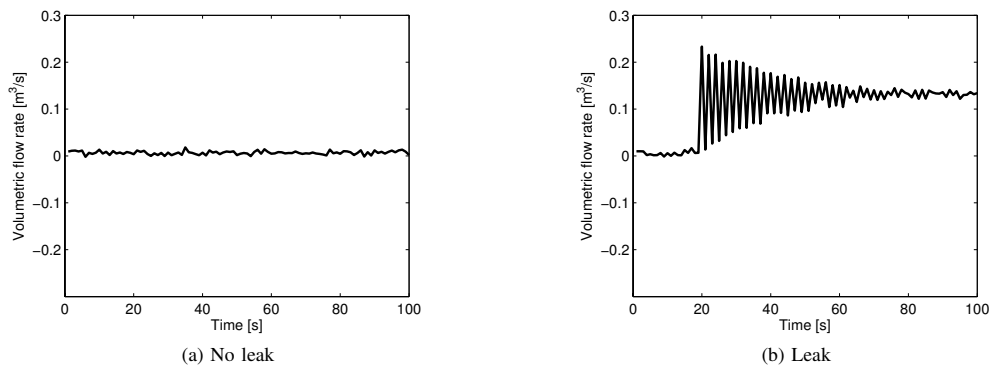


Fig. 3. Volumetric flow rates without leak (a) and with leak (b)

#### D. Simulation Results

The simulation results from the presented transient model are shown. The pipe diameter is 1.22 m and the pipe length is 100 m discretized by 1 so that it has 100 points. The simulation is implemented for 100 s. Eventually, it has 10000 nodes in the  $xt$  plane. We show the hydraulic head and volumetric flow rate of the single pipe when having the leak or not. The leak happens on 30 m point of the pipe at 20 s. Fig. 2 shows the hydraulic heads without the leak and with the leak. When the leak occurs, the hydraulic head drops about 20 m and converges on 46 m. Fig. 3 shows the volumetric flow rates of cases without the leak and with the leak. The volumetric flow rate rises to 0.2 m³/s and converges to 0.14 m³/s.

### III. FAULT DETECTION TECHNIQUES

We here present the fault detection techniques, fast Fourier transform (FFT) and wavelet transform (WT). The mathematical algorithms of these methods are explained in this section.

#### A. Fast Fourier Transform

The Fourier transform is a reversible, linear transform with many important properties. For any function  $f(t)$ , the Fourier transform can be denoted  $F(s)$ , where  $f(t)$  is the time-domain

signal and  $F(s)$  is the frequency-domain signal. The Fourier transform is defined by

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-2\pi i s t} dt, \quad i = \sqrt{-1} \quad (29)$$

The continuous Fourier transform (CFT) converts a time-domain signal of infinite duration into a continuous spectrum composed of an infinite number of sinusoids. We usually deal with signals that are discretely sampled at constant intervals and of finite duration. Only a finite number of sinusoids is needed for such data and the discrete Fourier transform (DFT) is appropriate. The DFT of  $N$  uniformly sampled data points  $x_n$  (where  $n = 0, \dots, N - 1$ ) is defined by

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi i n k / N}, \quad k = 0, \dots, N - 1 \quad (30)$$

The continuous variable  $s$  is replaced by the discrete variable  $k$ . The DFT is usually computed by an efficient algorithm known as the fast Fourier transform (FFT). The key advantage of the FFT over the DFT is that the operational complexity decreases from  $O(N^2)$  for the DFT to  $O(N \log_2 N)$  for the FFT. When comparing the speeds obtained by computing the FFTs and DFTs for length  $10^3$  and  $10^6$  points, the difference

is quite clear.

$$\begin{aligned} \text{speed improvement} \quad (N = 10^3) \\ \propto \frac{N^2}{N \log_2 N} \approx \frac{10^3}{10} \approx 10^2 \end{aligned} \quad (31)$$

$$\begin{aligned} \text{speed improvement} \quad (N = 10^6) \\ \propto \frac{N^2}{N \log_2 N} \approx \frac{10^6}{20} \approx 5 \times 10^4 \end{aligned} \quad (32)$$

We can notice the fault by comparing the spectrum of original signals using the FFT [4].

### B. Wavelet Transform

The wavelet transform (WT) decomposes a given signal into its frequency components like the Fourier transform, but differs in giving a global representation of the signal. The WT provides a representation in both time and frequency. A mathematical definition of the continuous wavelet transform (CWT) follows [4]

$$CWT_{\Psi} f(a, b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t) \Psi^* \left( \frac{t-b}{a} \right) dt \quad (33)$$

where  $\Psi(t)$  is the mother wavelet and  $a$  and  $b$  are dilating and translating coefficients, respectively. The asterisk denotes a complex conjugate and the multiplication of  $|a|^{-1/2}$  is for energy normalization. The WT decomposes the signal into different scales with different levels of resolution by dilating the mother wavelet. One drawback of the CWT is that the representation of the signal is redundant, since  $a$  and  $b$  are continuous over  $\mathbf{R}$  where  $\mathbf{R}$  is the set of real numbers. A discrete wavelet function (DWT) solves the drawback [3]. A wavelet function  $\psi(t) \in L^2(\mathbf{R})$  has the following basic properties

$$\int_{\mathbf{R}} \psi(t) dt = 0 \quad \text{and} \quad \int_{\mathbf{R}} \psi^2(t) dt = 1 \quad (34)$$

We also require a scaling function  $\phi \in L^2(\mathbf{R})$  that satisfies

$$\int_{\mathbf{R}} \phi(t) dt \neq 0 \quad \text{and} \quad \int_{\mathbf{R}} \phi^2(t) dt = 1 \quad (35)$$

Let  $\mathbf{Z}$  denote the set of all integers. Starting from choices of  $\phi(t)$  and  $\psi(t)$  and using dilations of  $\eta = 2^{j_0}$  and  $\eta = 2^j$  with translations of  $\tau = k$  for  $j_0, j, k \in \mathbf{Z}$ , we can construct an orthonormal basis for the space  $L^2(\mathbf{R})$  consisting of the scaling functions  $\{\phi_{j_0, k}(t) = 2^{j_0/2} \phi(2^{j_0} t - k)\}$  and the wavelet functions  $\{\psi_{j, k}(t) = 2^{j/2} \psi(2^j t - k)\}$ . Then, any target function  $f(t)$  can be expressed as

$$f(t) = \sum_{k \in \mathbf{Z}} c_{j_0, k} \phi_{j_0, k}(t) + \sum_{j=j_0}^{\infty} \sum_{k \in \mathbf{Z}} d_{j, k} \psi_{j, k}(t) \quad (36)$$

where the coefficients

$$c_{j_0, k} = \int_{\mathbf{R}} f(t) \phi_{j_0, k}(t) dt \quad (37)$$

$$d_{j, k} = \int_{\mathbf{R}} f(t) \psi_{j, k}(t) dt \quad (38)$$

are defined as the inner product of  $f(t)$  with the basis functions  $\phi_{j_0, k}(t)$  and  $\psi_{j, k}(t)$ , respectively, and called approximation coefficients and detailed coefficients, respectively. When DWT

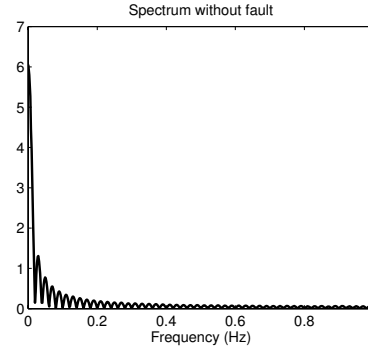


Fig. 4. FFT of hydraulic head without fault

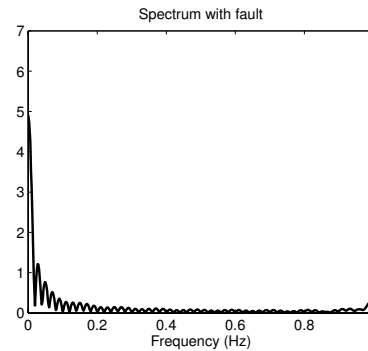


Fig. 5. FFT of hydraulic head with fault

is applied to a data set of  $N$ , the DWT transforms  $N$  data points into  $N$  wavelet coefficients. The original data can be expressed as a linear sum of products of wavelet coefficients and their corresponding basis functions, as in Equation (36). Daubechies' wavelet, Morlet wavelet, and Harr wavelet are popularly used for the wavelet and scaling functions. The DWT can be used to detect the fault by observing the wavelet coefficients [5].

## IV. RESULTS AND DISCUSSION

We present the results of fault detection using the proposed detection schemes, the FFT and the DWT. We first apply the techniques to the simulated data shown in Section 2. Real data of pressure of pipe network are also used to detect faults to show that the presented schemes work well for actual situations.

### A. Fault Detection with Simulated Data

We first compare the spectrum of the simulated hydraulic heads without and with the fault using the FFT. The spectrum are shown in Fig. 4 and Fig. 5. The spectrum with the fault differ from the one without the fault around 0 Hz and 1 Hz. The faults can be noticed by comparing the spectrum of original signals, however, it does not give any information about the time of the fault. Now we present the results of the DWTs of the simulated hydraulic heads without and with the fault as shown in Fig. 6 and Fig. 7. When the pipeline has

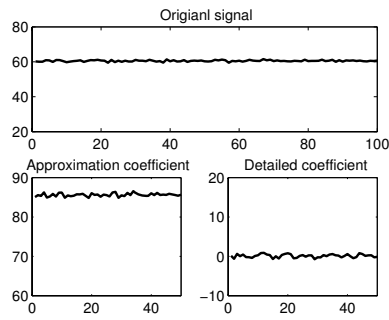


Fig. 6. DWT of hydraulic head without fault

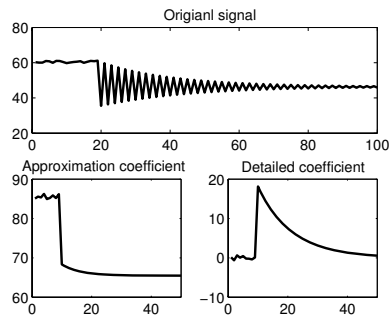
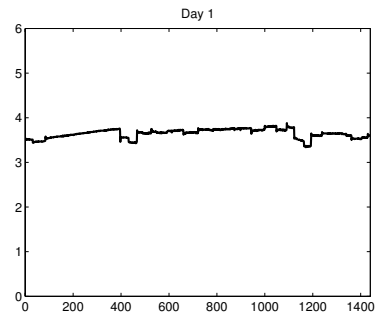


Fig. 7. DWT of hydraulic head with fault

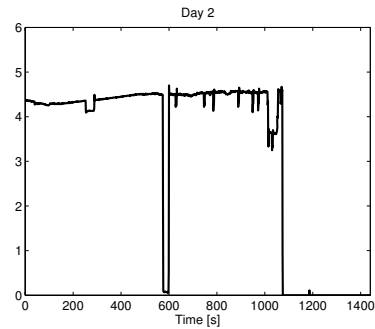


Fig. 8. Pressure changes of Area S for Day 1 and Day 2

the fault, the wavelet coefficients are changed sharply as in Fig. 7. Thus, the faults are easily recognized by observing the wavelet coefficients. In addition, the DWT gives information about the time of the fault. The simulated fault happens at 20 s and the wavelet coefficients change rapidly about 10 s because of the Nyquist frequency. We can estimate the time of the fault using this information.

**B. Fault Detection with Real Data**

Fig. 8 shows pressure changes of Area S for Day 1 and Day 2. The pressure is measured every minutes for 24 hours, thus, it has the total data points of 1440. We can infer a severe fault for Day 2 from the fact that the pressure at 600 s drops rapidly and goes zero after 1100 s. The FFTs of pressure data are shown in Fig. 9. The spectrum of pressures for Day 1 and Day 2 are different each other around 0 Hz. Although the difference of the spectrum is recognized, it is not clear to detect the serious fault. Now we apply the DWT to the pressure data and obtain results as shown in Fig. 10 and Fig. 11. The fault detection of the DWT is quite obvious by observing detailed coefficients for Day 1 and Day 2. Peaks in the detailed coefficient for Day 2 represent the faults of the pipe at 600 s and 1100 s, noting that the time scale is scaled down the half of the whole duration due to the Nyquist frequency.

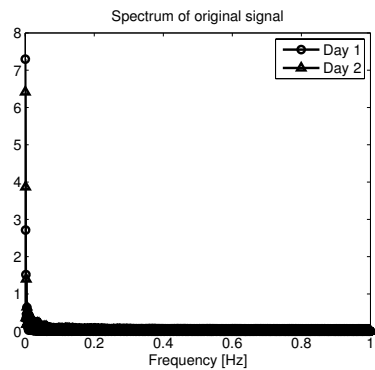


Fig. 9. FFTs of pressures of Area S

water pipe network system. First of all, the transient model for the pipeline is developed and simulated to collect the data of the hydraulic head and volumetric flow rate of the pipe when the fault is generated. The presented detection techniques show good performance by detecting the fault. In general, the DWT shows better performance than the FFT. These schemes are also applied to real data obtained from Area S for two days, resulting in good detection performance for the faults occurred on Day 2.

**V. CONCLUSION**

In this work, we propose fault detection techniques, the fast Fourier transform and the discrete wavelet transform, for the

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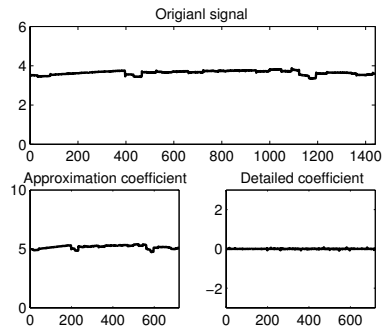


Fig. 10. DWT of pressure of Area S for Day 1

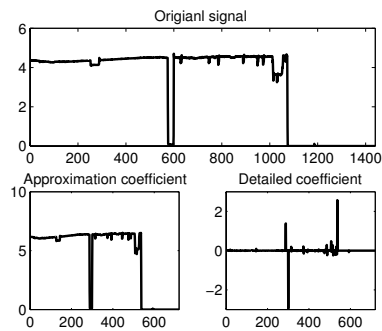


Fig. 11. DWT of pressure of Area S for Day 2

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**Go Bong Choi** received the B.S degree in chemical and biological engineering from Seoul National University, Seoul, South Korea in 2011. He is working toward Ph.D. degree in chemical and biological engineering at Seoul National University, Seoul, South Korea.

**Jeong Cheol Seo** received the B.S, M.S., and Ph.D. degrees in chemical engineering from Seoul National University, Seoul, South Korea in 1991, 1993, and 1997, respectively. He is currently a director of Samchully Corporation, Seoul, South Korea.



**Jong Min Lee** received the B.S degree in chemical engineering from Seoul National University, Seoul, South Korea in 1996 and the Ph.D. degree in chemical and biomolecular engineering from the Georgia Institute of Technology in 2004, Atlanta. He is currently an assistant professor of the School of Chemical and Biological Engineering at Seoul National University, Seoul, South Korea.



**Giback Lee** received the B.S, M.S., and Ph.D. degrees in chemical engineering from Seoul National University, Seoul, South Korea in 1991, 1993, and 1997, respectively. He is currently a professor of the Department of Chemical and Biological Engineering, Korea National University of Transportation, Chungju, South Korea.



**Shin Je Lee** received the B.S degree in chemical and biomolecular engineering from Sogang University, Seoul, South Korea in 2011. She is working toward the M.S. degree in chemical and biological engineering at Seoul National University, Seoul, South Korea.