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# Exponential stability of uncertain Takagi-Sugeno fuzzy Hopfield neural networks with time delays

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*Abstract*—In this paper, based on linear matrix inequality (LMI), by using Lyapunov functional theory, the exponential stability criterion is obtained for a class of uncertain Takagi-Sugeno fuzzy Hopfield neural networks (TSFHNNs) with time delays. Here we choose a generalized Lyapunov functional and introduce a parameterized model transformation with free weighting matrices to it, these techniques lead to generalized and less conservative stability condition that guarantee the wide stability region. Finally, an example is given to illustrate our results by using MATLAB LMI toolbox.

*Keywords*—Hopfield neural network; Linear matrix inequality; Exponential stability; Time delay; T-S fuzzy model.

## I. Introduction

IN past few years, the well-known Hopfield neural network has been extensively studied, and successfully applied in many areas such as combinatorial optimization, signal processing and pattern recognition, see e.g.[1,2]. Recently, it has been realized that significant time delays as a source of instability and bad performance may occur in neural processing and signal transmission. Thus, the stability problem of Hopfield neural networks has became interesting and many sufficient conditions have been proposed to guarantee the asymptotic or exponential stability for the neural networks with various type of time delays, see for examples [3]-[6]. In practical systems, analysis of a mathematical model is usually an important work for a control engineer as to control a system. However, the mathematical model always contains some uncertain elements, these uncertainties may be due to additive unknown internal or external noise, environmental influence, poor plant knowledge, reduced-order models, uncertain or slowly varying parameters. Therefore, under such imperfect knowledge of the mathematical model, seeking to design a robust control such that the system responses can meet desired properties is an important topic in system theory. Hence, robust stability analysis for uncertain time-delay systems have been the focus of much research in recent years [6]-[10]. Fuzzy systems in the form of the Takagi-Sugeno (T-S) model [11] have attracted rapidly growing interest in recent years [12,13]. TS fuzzy systems are nonlinear systems described by a set of IF-THEN rules. It has shown that the T-S model can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. Some nonlinear dynamic systems can be approximated by the overall fuzzy linear T-S models for the purpose of stability analysis [12,13]. Originally, Tanaka and his colleagues have provided a sufficient condition for the quadratic stability of the T-S fuzzy systems in the sense of Lyapunov in a series of papers [14,15] by considering a Lyapunov function of the sub-fuzzy systems of the T-S fuzzy systems.

Based on the above discussions, we shall generalize the ordinary T-S fuzzy models to express a class of Hopfield neural network with time delays. The main purpose of this paper is to study the exponential stability results of TSFHNNs in terms of LMIs. The main advantage of the LMI based approaches is that the LMI stability conditions can be solved numerically using MATLAB LMI toolbox [16] which implements the state of art interior-point algorithms [17]. We also provide a numerical example to demonstrate the effectiveness of the proposed stability results.

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## II. Preliminaries

Consider the following uncertain Hopfield neural networks with time delays

$$\begin{cases} \dot{u_i}(t) = -(a_i + \Delta a_i)u_i(t) + \sum_{j=1}^{n}(w_{ij} + \Delta w_{ij}) \\ \qquad\qquad \times F_j(u_j(t-\tau)) + I_i, \\ u_i(t) = \phi_i(t), \quad i = 1, 2, \cdots, n. \end{cases} \tag{1}$$

where $u_i(t)$ is the activations of the $i^{th}$ neurons, positive constants $a_i$ denote the rates with which the cell $i$ reset its potential to the resting state when isolated from the other cells and inputs, $\phi_i(s)$ is the initial condition, and $\phi_i(s)$ is bounded and continuously differential on $[-\tau, 0]$. $w_{ij}$ is the connection weights at the time $t$, $I_i$ denote the external inputs, $\tau$ is the unknown time delay, $F_j(\cdot)$ is the neuron activation functions of $j^{th}$ neurons.

Throughout this paper, we make the following assumption: (**A**) There exist positive numbers $L_q$ such that

$$0 \leq \frac{f(x) - f(y)}{x - y} \leq L_q, q = 1, 2, \cdots, n,$$

for all $x, y \in R, x \neq y$ and denote $L = diag\{L_1, L_2, \cdots, L_n\}$.

Suppose (**A**) holds, then it is clear the conditions of Lemma 2 in [18] hold for the functions $f(\cdot)$, therefore, similarly to the proof of Theorem 1 in [18], we can obtain that the equilibrium point of the system (1) is exist and unique.

Assume that $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ is the equilibrium point of system (1), then we will shift the equilibrium point to the origin by the transformation $x_i(t) = u_i(t) - u_i^*$, $f_j(x_j(t))$

$= F_j(u_j(t)) - F_j(u_j^*)$. Then system (1) is transformed as

$$\dot{x_i}(t) = -(a_i + \Delta a_i)x_i(t) + \sum_{j=1}^{n}(w_{ij} + \Delta w_{ji}) \times f_j(u_j(t-\tau)), \quad i = 1, 2, \cdots, n.$$

For convenience, we can write it in the form

$$\dot{x}(t) = -(A + \Delta A)x(t) + (W + \Delta W)f(x(t-\tau)), \qquad (2)$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^n; A = diag\{a_1, a_2, \cdots, a_n\}, a_i > 0, i = 1, 2, \cdots, n; W \in R^{n\times n}; f(x(t)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot)), \cdots, f_n(x_n(\cdot))] \in R^n$, $f(0) = 0$; $\Delta A$ and $\Delta W$ represent the parameter uncertainties.

In this section, we will consider a Hopfield neural network with time delays, which is represented by a T-S fuzzy model composed of a set of fuzzy implications and each implications is expressed as a linear system model.

The continuous fuzzy system was proposed to represent a nonlinear system [11]. The system dynamics can be captured by a set of fuzzy rules which characterize local correlation in the state space. Each local dynamic described by the fuzzy IF-THEN rule has the property of linear input-output relation. Based on the T-S fuzzy model concept, a general class of T-S fuzzy Hopfield neural networks with time delays is considered here. The model of Takagi-Sugeno fuzzy Hopfield neural networks with time delays is described as follows.

**Plant Rule** $k$:
**IF** $\{\theta_1(t)$ is $M_{k1}\}$ and$\cdots$and $\{\theta_r(t)$ is $M_{kr}\}$,
**THEN**

$$\dot{x}(t) = -(A_k + \Delta A_k)x(t) + (W_k + \Delta W_k)f(x(t-\tau)), \quad (3)$$

where $\theta_i(t), (i = 1, 2, \cdots, r)$ are known variables. $M_{kl}(k \in 1, 2, \cdots, m, l \in 1, 2, \cdots, r)$ is the fuzzy set and $m$ is the number of model rules. The parameter uncertainties $\Delta A_k, \Delta W_k$ are time varying matrices with appropriate dimensions, which are defined as follows:

$$[\Delta A_k \quad \Delta W_k] = M_k F(t)[E_1 k \quad E_2 k],$$

where $E_{1k}, E_{2k}, M_k$ are known constant matrices of appropriate dimensions and $F(t)$ is an known time varying matrix with Lebegue measurable elements bounded by

$$F^T(t)F(t) \le I,$$

where $I$ is the identity matrix with appropriate dimension.

By inferring from the fuzzy models, the final output of TSFHNNs is obtained by

$$\begin{aligned}\dot{x}(t) &= \sum_{k=1}^{m}\{\omega_k(\theta(t))/\sum_{k=1}^{m}\omega_k(\theta(t))\}\{-(A_k + \Delta A_k)x(t) \\ &\quad +(W_k + \Delta W_k)f(x(t-\tau))\} \\ &= \sum_{k=1}^{m}\eta_k(\theta(t))\{-(A_k + \Delta A_k)x(t) \\ &\quad +(W_k + \Delta W_k)f(x(t-\tau))\}. \end{aligned} \qquad (4)$$

The weight and averaged weight of each fuzzy rule are denoted by

$$\omega_k(\theta(t)) = \prod_{l=1}^{t} M_{kl}(\theta(t)),$$
$$\eta_k(\theta(t)) = \omega_k(\theta(t))/\sum_{k=1}^{m}\omega_k(\theta(t)),$$

respectively. Then term $M_{kl}(\theta(t))$ is grade membership of $\theta_l(t)$ in $M_{kl}$. We assume that

$$\omega_k(\theta(t)) \ge 0, k \in \{1, 2, \cdots, m\}, \ \sum_{k=1}^{m}\eta_k(\theta(t)) = 1, \forall\ t > 0.$$

**Definition 2.1** For system (1) and every $\phi_i \in C([-\tau, 0]; R^n)$, the trivial solution is globally exponentially stable in the mean square with convergence rate $\gamma$ for all admissible uncertainties, If there exist $\gamma > 0$ and $\chi(\gamma) > 0$ such that

$$\| x(t) \|^2 \le \chi(\gamma)e^{-2\gamma t}, \ \forall\ t > 0.$$

**Lemma 2.1** (Schur complement [17]).The LMI

$$\begin{bmatrix} Q(y) & S(y) \\ S^T(y) & R(y) \end{bmatrix} < 0$$

is equivalent to

$$R(y) < 0, Q(y) - S(y)R^{-1}(y)S^T(y) < 0,$$

where $Q(y) = Q^T(y), R(y) = R^T(y)$, and $S(y)$ depend affinely on $y$.

**Lemma 2.2**([19]) Given matrices $Q = Q^T, H, E$ and $R = R^T$ of appropriate dimensions, then

$$Q + HFE + E^T F^T H^T < 0$$

for all $F$ satisfying $F^T F \le R$, if and only if there exist an $\varepsilon > 0$ such that

$$Q + \varepsilon HH^T + \varepsilon^{-1}E^T RE^T < 0.$$

**Lemma 2.3** For matrices $P \in R_{n\times n}, M \in R_{n\times k}, N \in R_{l\times n}$ and $F \in R_{k\times l}$ with $P > 0, \|F\| \le 1$, and scalar $\varepsilon > 0$, one has the following

$(i)$ $(MFN)^T P + P(MFN) \le \varepsilon PMM^T P + \varepsilon^{-1}N^T N$.
$(ii)$ $if\ P - \varepsilon MM^T > 0, then (A + MFN)^T P^{-1}(A + MFN) \le A^T(P - \varepsilon MM^T)^{-1}A + \varepsilon^{-1}N^T N$.

## III. Main Result

In this section, we shall obtain a exponential stability criterion for uncertain fuzzy Hopfield neural networks with time varying delays.

**Theorem 3.1** Under the assumption (A), the equilibrium point of system (4) is exponential stability if there exist symmetric positive definite matrices $P > 0, Q > 0, R > 0, S > 0, X > 0, Y > 0$, symmetric matrices $N_i, S_{ij}, (i = 1, \cdots, 4, i < j < 4)$ and a scalar $\varepsilon$ such that the following conditions hold for $k = 1, 2, \cdots, m$,

$$\Omega = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & N_1 \\ * & S_{22} & S_{23} & S_{24} & N_2 \\ * & * & S_{33} & S_{34} & N_3 \\ * & * & * & S_{44} & N_4 \\ * & * & * & * & e^{-2\alpha\tau}R \end{bmatrix} \ge 0, \qquad (5)$$

$$\Sigma_k = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & 0 \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & 0 \\ * & * & * & \Sigma_{44} & \Sigma_{45} & 0 \\ * & * & * & * & \Sigma_{55} & 0 \\ * & * & * & * & * & \Sigma_{66} \end{bmatrix} < 0, \quad (6)$$

where

$$\begin{aligned}
\Sigma_{11} =& 2\alpha P - PA_k - A_k^T P + L^T X L + 2YL + N_1 \\
&+ N_1^T + \tau S_{11} + \varepsilon E_{1k}^T E_{1k}, \\
\Sigma_{12} =& -N_1 + N_2 + \tau S_{12}, \Sigma_{13} = -Y + N_3 + \tau S_{13}, \\
\Sigma_{14} =& PW_k + N_4 + \tau S_{14} + \varepsilon E_{1k}^T E_{2k}, \\
\Sigma_{15} =& 0, \Sigma_{16} = PM_k, \Sigma_{22} =, \Sigma_{23} = -N_3 + \tau S_{23}, \\
\Sigma_{24} =& -N_4 + \tau S_{24}, \Sigma_{25} = 0, \Sigma_{33} = Q - X + \tau S_{33}, \\
\Sigma_{34} =& \tau S_{34}, \Sigma_{35} = 0, \Sigma_{44} = -e^{-2\alpha\tau} Q + \tau S_{44} + \varepsilon E_{2k}^T E_{2k}, \\
\Sigma_{45} =& 0, \Sigma_{55} = S, \Sigma_{66} = \varepsilon I.
\end{aligned}$$

*Proof:* Consider the following Lyapunov functional

$$V(t) = V_1(t) + V_2(t) + V_3(t),$$

where

$$\begin{aligned}
V_1(t) &= e^{2\alpha t} x^T(t) P x(t), \\
V_2(t) &= \int_{t-\tau}^{t} e^{2\alpha s} f^T(x(s)) Q f(x(s) ds, \\
V_3(t) &= \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} e^{2\alpha s} \dot{x}^T(s) R \dot{x}(s) ds d\theta\theta.
\end{aligned}$$

Calculate derivative of $V(t)$ along the trajectories of the system (4), then

$$\begin{aligned}
\dot{V}_1(t) &= 2\alpha e^{2\alpha t} x^T(t) P x(t) + e^{2\alpha t} \dot{x}^T(t) P x(t) \\
&\quad + e^{2\alpha t} x^T(t) P \dot{x}(t) \\
&= e^{2\alpha t} x^T(t) 2\alpha P x(t) + 2 e^{2\alpha t} \sum_{k=1}^{m} \eta_k(\theta(t)) x^T(t) \\
&\quad \times P\{[-(A_k + \Delta A_k) x(t) \\
&\quad + (W_k + \Delta W_k) f(x(t-\tau))]\} \\
&= e^{2\alpha t}\{x^T(t) 2\alpha P x(t) + \sum_{k=1}^{m} \eta_k(\theta(t))\{x^T(t) 2P \\
&\quad \times [-(A_k + \Delta A_k) x(t) \\
&\quad + x^T(t) 2(W_k + \Delta W_k) f(x(t-\tau))]\}\} \\
&\le e^{2\alpha t} \sum_{k=1}^{m} \eta_k(\theta(t))\{x^T(t) 2\alpha P x(t) \\
&\quad + x^T(t) 2P[-(A_k + \Delta A_k) x(t) \\
&\quad + x^T(t) 2P(W_k + \Delta W_k) f(x(t-\tau))]\}, \\
\dot{V}_2(t) &= e^{2\alpha t} f^T(x(t)) Q f(x(t) - e^{2\alpha(t-\tau)} f^T(x((t-\tau))) \\
&\quad \times Q f(x((t-\tau)) \\
&= e^{2\alpha t}\{f^T(x(t)) Q f(x(t) - f^T(x((t-\tau))) e^{-2\alpha\tau} \\
&\quad \times Q f(x((t-\tau))\}, \\
\dot{V}_3(t) &= \tau^2 e^{2\alpha t} \dot{x}^T(t) R \dot{x}(t) - \tau \int_{t-\tau}^{t} e^{2\alpha t} \dot{x}^T(t) R \dot{x}(t) ds.
\end{aligned}$$

By using Jensen's inequality [20], we have

$$\begin{aligned}
& -\tau \int_{t-\tau}^{t} e^{2\alpha t} \dot{x}^T(s) R \dot{x}(s) ds \\
\le\ & -e^{2\alpha(t-\tau)} \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big]^T R \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big] \\
=\ & -e^{2\alpha t} \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big]^T (e^{-2\alpha\tau} R) \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big].
\end{aligned}$$

So

$$\begin{aligned}
\dot{V}_3(t) &\le \tau^2 e^{2\alpha t} \dot{x}^T(t) R \dot{x}(t) - e^{2\alpha t} \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big]^T \\
&\quad \times (e^{-2\alpha\tau} R) \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big] \\
&\le e^{2\alpha t} \Big\{\dot{x}^T(t) \tau^2 R \dot{x}(t) - \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big]^T \\
&\quad \times (e^{-2\alpha\tau} R) \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big]\Big\}.
\end{aligned}$$

Finally, let $S = \tau^2 R$, then

$$\begin{aligned}
\dot{V}(t) \le\ & e^{2\alpha t} \sum_{k=1}^{m} \eta_k(\theta(t)) \Big\{x^T(t) 2\alpha P x(t) \\
& + x^T(t) 2P[-(A_k + \Delta A_k) x(t) \\
& + x^T(t)(W_k + \Delta W_k) f(x(t-\tau))] \\
& + f^T(x(t)) Q f(x(t)) - f^T(x((t-\tau))) e^{-2\alpha\tau} \\
& \times Q f(x((t-\tau))) + \dot{x}^T(t) \tau^2 R \dot{x}(t) \\
& - \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big]^T (e^{-2\alpha\tau} R) \Big[\int_{t-\tau}^{t} \dot{x}(s) ds\Big] \\
& - f^T(x(t)) X f(x(t)) - 2x^T(t) Y f(x(t)) \\
& + f^T(x(t)) X f(x(t)) + 2x^T(t) Y f(x(t))\Big\}.
\end{aligned}$$

Noting that $X$ and $Y$ are positive definite matrices and using Assumption (A), we can get

$$x^T(t) Y f(x(t)) \le x^T(t) Y L x(t), \quad (7)$$

$$f^T(x(t)) X f(x(t)) \le x^T(t) L^T X L x(t). \quad (8)$$

According to Leibniz-Nowton formula, for any matrices $N_i, (i = 1, \cdots, 4)$, the following equations hold

$$\begin{aligned}
& 2[x^T(t) N_1 + x^T(t-\tau) N_2 + f^T(x(t)) N_3 \\
& + f^T(x(t-\tau)) N_4] \times [x(t) - x(t-\tau) \\
& - \int_{t-\tau}^{t} \dot{x}(s) ds] = 0.
\end{aligned} \quad (9)$$

On the other hand, for any appropriately dimensional matrices $S_{ij}, (i = 1, \cdots, 4, i \le j \le 4)$ the following equation also hold:

$$\begin{bmatrix} x(t) \\ x(t-\tau) \\ f(x(t)) \\ f(x(t-\tau)) \end{bmatrix}^T \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} \\ \Upsilon_{12}^T & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} \\ \Upsilon_{13}^T & \Upsilon_{23}^T & \Upsilon_{33} & \Upsilon_{34} \\ \Upsilon_{14}^T & \Upsilon_{24}^T & \Upsilon_{34}^T & \Upsilon_{44} \end{bmatrix}$$

$$\times \begin{bmatrix} x(t) \\ x(t-\tau) \\ f(x(t)) \\ f(x(t-\tau)) \end{bmatrix} = 0, \tag{10}$$

where $\Upsilon_{ij} = \tau(S_{ij} - S_{ji}), i = 1, \cdots, 4; i \leq j \leq 4$.

Then we add the terms on the left sides of equations (9)-(10) to $\dot{V}(x(t))$, consider (7)-(8) and by the fact that for any $r \geq 0$ and any function $f(t)$, then

$$\int_{t-r}^{t} f(t)ds = rf(t).$$

Therefore, $\dot{V}(x(t))$ can be expressed as follows:

$$\dot{V}(x(t)) \leq \xi_1^T(t)\Pi_k\xi_1(t) - \tau\int_{t-r}^{t} \xi_2^T(t)\Omega\xi_2(t)ds,$$

where

$$\xi_1 = [x^T(t)\ x^T(t-\tau)\ f^T(x(t))\ f^T(x(t-\tau))\ \dot{x}^T(t)]^T,$$
$$\xi_2 = [x^T(t)\ x^T(t-\tau)\ f^T(x(t))\ f^T(x(t-\tau))\ \dot{x}^T(s)]^T,$$

and

$$\Pi_k = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} & \Pi_{15} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} \\ * & * & * & \Pi_{44} & \Pi_{45} \\ * & * & * & * & \Pi_{55} \end{bmatrix}, \tag{11}$$

where

$$\begin{aligned}
\Pi_{11} &= 2\alpha P - PA_k - A_k^T P + P\Delta A_k + \Delta A_k^T P \\
&\quad + L^T XL + 2YL + N_1 + N_1^T + \tau S_{11}, \\
\Pi_{12} &= -N_1 + N_2 + \tau S_{12}, \Pi_{13} = -Y + N_3 + \tau S_{13}, \\
\Pi_{14} &= PW_k + P\Delta W_k + N_4 + \tau S_{14}, \Pi_{15} = 0, \\
\Pi_{22} &= -N_2^T - N_2 + \tau S_{22}, \Pi_{23} = -N_3 + \tau S_{23}, \\
\Pi_{24} &= -N_4 + \tau S_{24}, \Pi_{25} = 0, \Pi_{33} = Q - X + \tau S_{33}, \\
\Pi_{34} &= \tau S_{34}, \Pi_{35} = 0, \Pi_{44} = -e^{-2\alpha\tau}Q + \tau S_{44}, \\
\Pi_{45} &= 0, \Pi_{55} = S.
\end{aligned}$$

Define

$$\Gamma_k = \begin{bmatrix} \Gamma_{11} & \Pi_{12} & \Pi_{13} & \Gamma_{14} & \Pi_{15} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & \Pi_{35} \\ * & * & * & \Pi_{44} & \Pi_{45} \\ * & * & * & * & \Pi_{55} \end{bmatrix},$$
$$\Theta_k^T = \begin{bmatrix} M_k^T P & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Lambda_k = \begin{bmatrix} E_{1k} & 0 & 0 & E_{2k} & 0 \end{bmatrix},$$

where $\Gamma_{11} = 2\alpha P - PA_k - A_k^T P + L^T XL + 2YL + N_1 + N_1^T + \tau S_{11}, \Gamma_{14} = PW_k + N_4 + \tau S_{14}$,
then

$$\Pi_k = \Gamma_k + \Theta_k F(t)\Lambda_k + \Lambda_k^T F^T(t)\Theta_k^T.$$

By Lemma 2.3, we have

$$\Theta_k F(t)\Lambda_k + \Lambda_k^T F^T(t)\Theta_k^T \leq \varepsilon^{-1}\Theta_k\Theta_k^T + \varepsilon\Lambda_k^T\Lambda_k.$$

So

$$\Pi_k \leq \Gamma_k + \varepsilon^{-1}\Theta_k\Theta_k^T + \varepsilon\Lambda_k^T\Lambda_k.$$

By Lemma 2.1, if $\sum_k$ and $\Omega$ are defined in (5)-(6), then $\dot{V}(x(t)) < 0$, which indicates form the Lyapunov stability theory, that the dynamics of the fuzzy Hopfield neural network (4) is exponential stability, which completes the proof. ■

In the previous part, we developed a exponential stability analysis approach for uncertaintain fuzzy Hopfield neural network with time varying delays based on LMI. From Theorem 3.1, we can obtain a stability criterion for the fuzzy Hopfield neural network with time varying delays without uncertains.

**Theorem 3.2** Under the assumption (A), the equilibrium point of system (4) with $\Delta A_k = \Delta W_k = 0$ is exponential stability if there exist symmetric positive definite matrices $P > 0, Q > 0, R > 0, S > 0, X > 0, Y > 0$, symmetric matrices $N_i, S_{ij}, (i = 1, \cdots, 4, i < j < 4)$ such that the following conditions hold for $k = 1, 2, \cdots, m$,

$$\Omega = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & N_1 \\ * & S_{22} & S_{23} & S_{24} & N_2 \\ * & * & S_{33} & S_{34} & N_3 \\ * & * & * & S_{44} & N_4 \\ * & * & * & * & e^{-2\alpha\tau}R \end{bmatrix} \geq 0, \tag{12}$$

$$\Sigma_k = \begin{bmatrix} \Xi_{11} & \Sigma_{12} & \Sigma_{13} & \Xi_{14} & \Sigma_{15} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} \\ * & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} \\ * & * & * & \Xi_{44} & \Sigma_{45} \\ * & * & * & * & \Sigma_{55} \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned}
\Xi_{11} &= 2\alpha P - PA_k - A_k^T P + L^T XL + 2YL + N_1 \\
&\quad + N_1^T + \tau S_{11}, \\
\Xi_{14} &= PW_k + N_4 + \tau S_{14}, \\
\Xi_{44} &= -e^{-2\alpha\tau}Q + \tau S_{44}.
\end{aligned}$$

*Proof:* If $\Delta A_k = \Delta W_k = 0$, its proof is similar to Theorem 3.1, we omit it here. ■

**Remark** When $r = 1$, system (4) is simplified to the general Hopfield neural networks with time delays and uncertainties. Recently , some papers have been studied the Hopfield neural networks with time delays . Thus our results make more general case of those result in the literature.

## IV. An example

Consider the Plant rule 2 with $m = 2$. The T-S fuzzy model of fuzzy Hopfield neural network with uncertainties is of the following form:

**Plant Rules:**

**Rule 1:** IF $\{\theta_1(t)$ are $M_{k1}\}$ THEN

$$\dot{x}(t) = -(A_1 + \Delta A_1(t))x(t) + (W_1 + \Delta W_1(t))f(x(t-\tau)),$$

**Rule 2:** IF $\{\theta_2(t)$ are $M_{k2}\}$ THEN

$$\dot{x}(t) = -(A_2 + \Delta A_2(t))x(t) + (W_2 + \Delta W_2(t))f(x(t-\tau)),$$

with $f(x) = \tanh(x)$. The membership functions for rule 1 and rule 2 are $M_{k1} = \frac{1}{e^{-2\theta_1(t)}}, M_{k2} = 1 - M_{k1}$,

$$A_1 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.7 \end{bmatrix}, W_1 = \begin{bmatrix} -1 & 0.4 \\ 0 & -0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & 1.8 \end{bmatrix}, W_1 = \begin{bmatrix} 1 & -0.8 \\ 0.4 & 0.5 \end{bmatrix}.$$
$$E_{1k} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}, E_{2k} = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$
$$F_k = \begin{bmatrix} \sin(t) & -\cos(t) \\ -\cos(t) & -\sin(t) \end{bmatrix}, M_k = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix},$$

Obviously assumption (A) is satisfied, $L = \mathrm{diag}(1, 1)$. By using MATLAB LMI toolbox, we solve the LMIs (5)-(6) for $\varepsilon > 0$ and $\tau = 2$ the feasible solutions are

$$P = \begin{bmatrix} -0.1500 & -0.0376 \\ -0.0376 & 0.0197 \end{bmatrix}, Q = \begin{bmatrix} -4.2931 & -0.0644 \\ -0.0644 & -5.3808 \end{bmatrix},$$
$$R = \begin{bmatrix} -3.7689 & 0.0014 \\ -0.0014 & -2.5709 \end{bmatrix}, S = \begin{bmatrix} -1.7461 & -0.6834 \\ -0.6834 & -1.7461 \end{bmatrix}.$$

If $\Delta A_k = \Delta W_k = 0$, by using MATLAB LMI toolbox, we solve the LMIs (12)-(13) for $\varepsilon > 0$ and $\tau = 2$ the feasible solutions are

$$P = \begin{bmatrix} -0.6869 & 2.5702 \\ 2.5702 & 114.1880 \end{bmatrix}, Q = \begin{bmatrix} 61.4404 & 3.2001 \\ 3.2001 & 54.8536 \end{bmatrix},$$
$$R = \begin{bmatrix} 49.2576 & 0.3889 \\ 0.3889 & 50.3812 \end{bmatrix}, S = \begin{bmatrix} 30.9177 & 0.8834 \\ 0.8834 & 33.0755 \end{bmatrix}.$$

Therefore, the concerned neural networks with time delays is exponentially stable.

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## REFERENCES

[1] J. Hopfield, Neural networks and physical systems with emergent collect computational abilities. Proc. Natl. Acad. Sci. USA; 1982; 2554-2558 .
[2] J. Hopfield, Neurons with graded response have collective computational proporties like those of two-state neurons. Proc. Natl. Acad. Sci. USA; 1984; 3088-3092.
[3] S. Chen, Q. Zhang, C. Wang, Existence and stability of equilibria of the continuous-time Hopfield neural network. Journal of Computational and Applied Mathematics 2004; 169; 117-125.
[4] S. Hu, X. Liao, X. Mao, Stochastic Hopfield neural networks. J. Phys. A: Math. Gen. 2004; 9; 47-53.
[5] H. Huang, D. Ho, J. Lam, Stochastic stability analysis of fuzzy Hopfield neural networks with Time-Varying Delays. IEEE Trans. Circuits Syst. II, Exp. Briefs 2005; 52; 251- 255.
[6] Z. Wang, H. Shu, J. Fang, X. Liu. Robust stability for stochastic Hopfield neurarl networks with time delays. Nonlinear Anal. Real World. Appl. 2006, 7: 1119-1128.
[7] X. Mao, N. Koroleva, A. Rodkina, Robust stability of uncertain stochastic delay differential equations. Systems Control lett. 1998; 35; 325-336.
[8] K. Tanaka, T. Ikede, H. Wang, Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H control theory, andlinear matrix inequalities. IEEE Trans. Fuzzy Systems 1996; 4; 1-13.
[9] Z. Wang, S. Lauria, J. Fang, X. Liu, Exponential stability of uncertain stochastic neural networks with mixed time delays. Chaos Solitons Fractals 2007; 32; 62-72.
[10] Z. Wang, H. Qiao, Robust filtering for bilinear uncertain stochastic discrete-time systems. IEEE Trans. Signal Process. 2002; 50(3); 560-567.
[11] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control. IEEE Trans. Syst. Man, Cybern. 1985; 15; 116 - 132.
[12] Y. Cao, P. Frank, Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi-Sugeno fuzzy models. Fuzzy Sets and Systems 2001; 124; 213-229.
[13] T. Takagi, M. Sugeno, Stability analysis andd esign of fuzzy control systems. Fuzzy Sets and Systems 1993; 45; 135-156.
[14] K. Tanaka, T. Ikede, H. Wang, An LMI approach to fuzzy controller designs basedon the relaxed stability conditions Proceedings of the IEEE international conference on fuzzy systems. Barcelona Spain; 1997; 171-176.
[15] K. Tanaka, T. Ikede, H. Wang, Fuzzy regulators and fuzzy observers: Relaxedstability conditions and LMI-based design. IEEE Trans. Fuzzy Systems 1998; 6; 250-265.
[16] P. Gahinet, Nemirovski A, Laub A, Chilali M. LMI control toolbox user's guide. The Mathworks; Massachusetts; 1995.
[17] B. Boyd, L. Ghoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory. SIAM; philadephia; 1994.
[18] H. Zhao, Global asymptotic stability of Hopfield neural network involving distributed delays. Neural Networks. 2004; 17; 4753.
[19] L. Xie, Output feedback H control of systems with parameter uncertainty. Internat. J. control 1996; 63; 741-759.
[20] K. Gu, V. Kharitonov, J. Chen, Stability of time delay systems. Birkhuser; Boston; 2003.