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# Existence of Solution of Nonlinear Second Order Neutral Stochastic Differential Inclusions with Infinite Delay

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*Abstract*—The paper is concerned with the existence of solution of nonlinear second order neutral stochastic differential inclusions with infinite delay in a Hilbert Space. Sufficient conditions for the existence are obtained by using a fixed point theorem for condensing maps.

*Keywords*—Mild solution, Convex multivalued map, Neutral stochastic differential inclusions.

## I. Introduction

LET $K$ be a separable Hilbert space, let $(\Omega, \mathfrak{F}, \mathfrak{F}_t, P)$ be a complete probability space furnished with a complete family of right continuous increasing $\sigma$ algebras $\{\mathfrak{F}_t\}$ satisfying $\mathfrak{F}_t \subset \mathfrak{F}$ for $t \geq 0$. Suppose $w(t)$ is a given $K$-valued , $\mathfrak{F}_t$ adapted Brownian motion with a finite trace nuclear covariance operator $Q \geq 0$. We are interested in the existence of solution of nonlinear second order stochastic differential inclusions with infinite delay

$$\begin{cases} d[x'(t) - g(t, x_t)] \in Ax(t)dt + F(t, x_t)dw(t), \\ \qquad\qquad\qquad\qquad\qquad t \in J = [0, b], \\ x_0 = \phi \in \mathfrak{B}_h, x'(0) = \eta, \qquad t \in J_0 = (-\infty, 0], \end{cases} \tag{1}$$

where $\phi$ is $\mathfrak{F}_o$ measurable and $A$ is the infinitesimal generator of a strongly continuous cosine family $\{C(t), t \in R\}$, the state $x(\cdot)$ takes values in Hilbert space $H$ with the norm $|\cdot|$, $F : J \times \mathfrak{B}_h \to 2^{L(K,H)}$ is a bounded closed, convex-valued mulivalued map, $g : J \times \mathfrak{B}_h \to H$ is continuous, the histories $x_t : (-\infty, 0] \to H$, $x_t(\theta) = x(t + \theta), \theta \leq 0$ belong to the space $\mathfrak{B}_h$. For $\sigma_1, \sigma_2 \in L(K, H)$, define $\ll \sigma_1, \sigma_2 \gg = tr(\sigma_1 Q \sigma_2^*)$, where $\sigma_2^*$ is the adjoint of the operator $\sigma_2$, $Q \in L_n^+(K)$. $L(K, H)$ furnished with the scalar product $\ll \cdot, \cdot \gg$ is a pre-Hilbert space. The completion of $L(K, H)$ with respect to the topology induced by the norm $\| \cdot \|_2$, where $\| \sigma \|_2^2 = \ll \sigma, \sigma \gg_2$, is a Hilbert space.

At first, we present the abstract phase space $\mathfrak{B}_h$. Assume that $h : (-\infty, 0] \to (0, +\infty)$ is continuous function with $l = \int_{-\infty}^0 h(s)ds < +\infty$. Define

$$\begin{aligned} \mathfrak{B}_h = \{\varphi : (-\infty, 0] \to H : &\text{ for any } a > 0, (E|\varphi(\theta)|^p)^{\frac{1}{p}} \\ &\text{is a bounded and measurable function on } [-a, 0] \\ &\text{and } \int_{-\infty}^0 h(s) \sup_{s \leq \theta \leq 0} (E|\varphi(\theta)|^p)^{\frac{1}{p}} ds < +\infty\}. \end{aligned}$$

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If $\mathfrak{B}_h$ is endowed with the norm

$$||\phi||_{\mathfrak{B}_h} = \int_{-\infty}^0 h(s) \sup_{s \leq \theta \leq 0} (E|\phi(\theta)|^p)^{\frac{1}{p}} ds, \ \ \forall \phi \in \mathfrak{B}_h,$$

then $(\mathfrak{B}_h, ||.||_{\mathfrak{B}_h})$ is a Banach space.

Stochastic differential equations have received much attention in many areas of science including finance, engineering and social science. The problems of existence of functional differential equations and inclusions have been extensively studied, for example [1 -4]. M.Benchohra, and S.K Ntouyas [5,6] discussed the nonlocal cauchy problems and impulsive multivalued semilinear neutral functional differential and integrodifferential inclusions in Banach Spaces. In [6, 7], Balasubbramaniam discussed the existence of solutions of functional stochastic differential inclusions with the help of some fixed-point theorems. Since many systems arising from realistic models heavily depend on histories (i.e., there is the effect of infinite delay on state equations) [8], there is a real need to discuss partial functional differential systems with infinite delay. So in the present paper, we will concentrate on the case with infinite delay and establish sufficient conditions for the existence of systems(1) by relying on a fixed-point theorem for condensing maps due to Martelli [9].

## II. Preliminaries

Let$J_1 = (-\infty, b]$ and $C(J_1, H)$ is the space of all continuous $H$-valued stochastic processes $\{\xi(t) : t \in J_1\}$.

Let $(E, \| \cdot \|)$ be a Banach space. A multivalued map $\mathfrak{J} : E \to 2^E$ is convex (closed)-valued,if $\mathfrak{J}(x)$ is convex(closed) for all $x \in$ E. $\mathfrak{J}$ is bounded on bounded set if $\mathfrak{J}(B) = \cup_{x \in B} \mathfrak{J}(x)$ is bounded in E for any bounded set $B$ of $E$; i.e,

$$\sup_{x \in B} \sup\{\|y\| \in \mathfrak{J}(x)\} < \infty.$$

$\mathfrak{J}$ is called upper semicontinuous ($Usc$) on $E$, if for each $x_* \in E$, the set $\mathfrak{J}(x_*)$ is nonempty, closed subset of E, and if for each open set $B$ of $E$ containing $\mathfrak{J}(x_*)$, there exists an open neighborhood V of $x_*$ such that $\mathfrak{J}(V) \subseteq B$.

$\mathfrak{J}$ is said to be completely continuous if $\mathfrak{J}(B)$ is relatively compact, for every bounded subset $B \subseteq E$.

If the multivalued map $\mathfrak{J}$ is completely continuous with nonempty compact values, then $\mathfrak{J}$ is $Usc$ if and only if $\mathfrak{J}$ has a closed graph(i.e.,$x_n = x_*,\ y_n = y_*,\ y_n \in \mathfrak{J}x_n\ imply\ \ y_* \in \mathfrak{J}x_*$).

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Let $BCC(E)$ denote the set of all the set of all nonempty, bounded ,closed and convex subsets of $E$. For more detail on multivalued maps see the books of Deimling [10], Hu and Papageorgiou [11].

An upper semicontinuous map $H : E \to E$ is said to be condensing if for any subset $B \subseteq E$ with $\alpha(B) \neq 0$, we have $\alpha(H(B)) < \alpha(B)$, where $\alpha$ denotes the Kuratowski measure of noncompactness. It is easy to see that s completely continuous multivalued map is a condensing map.

We say that a family $\{C(t) : t \in R\}$ of operators in $B(E)$ is a strongly continuous cosine family if

(i) $C(0) = I$ ($I$ is the identity operator in $E$),

(ii) $C(t+s) + C(t-s) = 2C(t)C(s)$ for all $s, t \in R$,

(iii) the map $t \longmapsto C(t)x$ is strongly continuous for each $x \in E$.

The strongly continuous sine family $\{S(t) : t \in R\}$, associated to the given strongly continuous cosine family $\{C(t) : t \in R\}$, is defined by

$$S(t)x = \int_0^t C(s)xds, \ x \in E, \ t \in R.$$

The infinitesimal generator $A : E \to E$ of a cosine family $\{C(t) : t \in R\}$ is defined by

$$Ax = \frac{d^2}{dt^2}C(t)x|_{t=0}.$$

For more details on strongly continuous cosine and sine families, we refer the reader to the books of Goldstein [12] and to the papers of Fattorini [13,14] and Travis and Webb [15,16].

The key tool in our approach is following fixed-point theorem.

**Theorem 1**(Martelli [9]). Let $E$ be a Banach space and $N : E \to BCC(E)$ a condensing map. If the set

$$\Omega = \{x \in E : \lambda x \in Nx, \text{for some } \lambda > 1\}$$

is bounded, then $N$ has s fixed point.

## III. Main Result

In the following, we shall apply Theorem 1 to study the existence of solution of system(1).

**Definition 1**. A function $x : (-\infty, b] \to H$ is called a mild solution of system(1) if the following holds: $\phi$ be $\mathfrak{F}_0$ measurable $H$-valued stochastic processes $x_0 = \phi \in \mathfrak{B}_h$ on $(-\infty, 0]$ and the integral equation

$$\begin{cases} x(t) = C(t)\phi(0) + S(t)[\eta - g(0,\phi)] \\ \qquad + \int_0^t C(t-s)g(s,x_s)ds \\ \qquad + \int_0^t S(t-s)f(s)dw(s), \text{ for a.e. } t \in J, \\ x_0 = \phi \in \mathfrak{B}_h, \quad t \in J_0. \end{cases} \tag{2}$$

is satisfied, where

$$f \in S_{F,x} = \{f \in L^2(J,H) : f(t) \in F(t,x_t), \text{ for a.e } t \in J\}.$$

To investigate the existence of solution of system (1), we use the following hypotheses:

$(H_1)$ $A$ is the infinitesimal generator of a strongly continuous and bounded cosine family $\{C(t) : t \in J\}$. Assume that $C(t)$ is compact and there exists constant $M_1 > 0$ such that $M_1 = \sup\{|C(t)| : t \in J\}$.

$(H_2)$ There exists constants $c_1 \geq 0$ and $c_2 \geq 0$ such that

$$E|g(t,u)|^p \leq c_1 \parallel u \parallel^p_{\mathfrak{B}_h} + c_2, \quad t \in J, \ u \in \mathfrak{B}_h.$$

$(H_3)$ $F : J \times \mathfrak{B}_h \to BCC(H); (t,\phi) \to F(t,\phi)$ is measurable with respect to t for each $\phi \in \mathfrak{B}_h$, $Usc$ with respect to $\phi$ for each $t \in J$, and for each fixed $\phi \in \mathfrak{B}_h$, the set

$$S_{F,\phi} = \{f \in L^2(J,H) : f(t) \in F(t,\phi), \text{ for a.e } t \in J\}$$

is nonempty.

$(H_4)$ The operator $G$ with values $(G(x))(t) = g(t,x_t), t \in J$, $G$ is completely continuous in $C(J,H)$ and for any bounded set $V \subseteq C(J,H)$, the set $\{t \to g(t,x_t) : x \in V\}$ is equicontinuous in $C(J,H)$.

$(H_5)$ $E \parallel F(t,\phi) \parallel^p_2 = \sup\{E \parallel v \parallel^p_2 : v \in F(t,\phi)\} \leq p(t)\psi(\parallel \phi \parallel^p_{\mathfrak{B}_h})$, $t \in J$, $p \in L^2(J,R^+)$, $\phi \in \mathfrak{B}_h$, and $\psi : [0,+\infty) \to (0,+\infty)$ is a continuous nondecreasing function, and the following inequality holds:

$$\int_0^b \bar{m}(s)ds < \int_N^{+\infty} \frac{1}{1+g(s)+\psi(g(s))}ds,$$

where

$$\begin{aligned} \bar{m}(t) &= \max\{4^{p-1}b^{\frac{p}{2}-1}M_1^p c_1, 4^{p-1}b^{\frac{p}{2}-1}M_1^p c_2, \\ &\quad 4^{p-1}b^{\frac{3p}{2}-1}M_1^p p(t)\} \end{aligned}$$

$$g(s) = (ls^{\frac{1}{p}} + ||\phi||_{\mathfrak{B}_h})^p, l = \int_{-\infty}^0 h(s)ds < +\infty,$$

$$\begin{aligned} N &= 4^{p-1}M_1^p E|\phi(0)|^p + 8^{p-1}M_1^p b^p (E|\eta|^p \\ &\quad + c_1||\phi||^p_{\mathfrak{B}_h} + c_2). \end{aligned}$$

**Lemma 1**(Lasota and Opial[17]). Let $I$ be a compact real interval and $E$ be a Banach space. Let $F$ be a multivalued map satisfying $(H_3)$ and let $\Gamma$ be a linear continuous mapping from $L^2(I,E) \to C(I,E)$. Then the operator

$$\Gamma \circ S_F : C(I,E) \to BCC(C(I,E)),$$

$$x \to (\Gamma \circ S_F)(x) = \Gamma(S_{F,x}))$$

is a closed graph operator in $C(I,E) \times C(I,E)$.

$C(J_1,H)$ is the space of all continuous $H$-valued stochastic processes $\{\xi(t) : t \in J_1\}$. Now we consider the space $\mathfrak{B}_b$, let $\mathfrak{B}_b = \{x : x \in C((-\infty,b],H), x_0 = \phi \in \mathfrak{B}_h\}$ , Let $\parallel \cdot \parallel$ be a seminorm in $\mathfrak{B}_b$ defined by

$$\parallel x \parallel_b = \parallel x_0 \parallel_{\mathfrak{B}_h} + \sup_{s\in[0,b]} (E|x(s)|^p)^{\frac{1}{p}}, \quad x \in \mathfrak{B}_b.$$

**Lemma 2**. Suppose $x \in \mathfrak{B}_b$, then for $t \in J$, $x_t \in \mathfrak{B}_h$, Moreover

$$lE^{\frac{1}{p}}|x(t)|^p \leq \parallel x_t \parallel_{\mathfrak{B}_h} \leq l \sup_{s\in[0,t]} (E|x(s)|^p)^{\frac{1}{p}} + \parallel x_0 \parallel_{\mathfrak{B}_h},$$

where $l = \int_{-\infty}^0 h(s)ds < +\infty$.

Proof: For any $t \in [0, a]$, we have

$$\begin{aligned}
\| x_t \|_{\mathfrak{B}_h} &= \int_{-\infty}^{0} h(s) \sup_{\theta\in[s,0]} E^{\frac{1}{p}} |x_t(\theta)|^p ds \\
&= \int_{-\infty}^{-t} h(s) \sup_{\theta\in[s,0]} E^{\frac{1}{p}} |x_t(\theta)|^p ds \\
&\quad + \int_{-t}^{0} h(s) \sup_{\theta\in[s,0]} E^{\frac{1}{p}} |x_t(\theta)|^p ds \\
&= \int_{-\infty}^{-t} h(s) \sup_{\theta_1\in[t+s,t]} E^{\frac{1}{p}} |x(\theta_1)|^p ds \\
&\quad + \int_{-t}^{0} h(s) \sup_{\theta_1\in[t+s,t]} E^{\frac{1}{p}} |x(\theta_1)|^p ds \\
&\leq \int_{-\infty}^{-t} h(s) [\sup_{\theta_1\in[t+s,0]} E^{\frac{1}{p}} |x(\theta_1)|^p \\
&\quad + \sup_{\theta_1\in[0,t]} E^{\frac{1}{p}} |x(\theta_1)|^p] ds \\
&\quad + \int_{-t}^{0} h(s) \sup_{\theta_1\in[0,t]} E^{\frac{1}{p}} |x(\theta_1)|^p ds \\
&= \int_{-\infty}^{-t} h(s) \sup_{\theta_1\in[t+s,0]} E^{\frac{1}{p}} |x(\theta_1)|^p ds \\
&\quad + \int_{-\infty}^{0} h(s) ds \times \sup_{\theta_1\in[0,t]} E^{\frac{1}{p}} |x(\theta_1)|^p \\
&\leq \int_{-\infty}^{-t} h(s) \sup_{\theta_1\in[s,0]} E^{\frac{1}{p}} |x(\theta_1)|^p ds \\
&\quad + l \sup_{\theta_1\in[0,t]} E^{\frac{1}{p}} |x(\theta_1)|^p \\
&\leq \int_{-\infty}^{0} h(s) \sup_{\theta_1\in[s,0]} E^{\frac{1}{p}} |x(\theta_1)|^p ds \\
&\quad + l \sup_{s\in[0,t]} E^{\frac{1}{p}} |x(s)|^p \\
&= \int_{-\infty}^{0} h(s) \sup_{\theta_1\in[s,0]} E^{\frac{1}{p}} |x_0(\theta_1)|^p ds \\
&\quad + l \sup_{s\in[0,t]} E^{\frac{1}{p}} |x(s)|^p \\
&= l \sup_{s\in[0,t]} E^{\frac{1}{p}} |x(s)|^p + \| x_0 \|_{\mathfrak{B}_h} .
\end{aligned}$$

Since $\phi \in \mathfrak{B}_h$, then $x_t \in \mathfrak{B}_h$. Moreover

$$\| x_t \|_{\mathfrak{B}_h} = \int_{-\infty}^{0} h(s) \sup_{\theta\in[s,0]} E^{\frac{1}{p}} |x_t(\theta)|^p ds$$

$$\geq E^{\frac{1}{p}} |x_t(0)|^p \int_{-\infty}^{0} h(s) ds = l E^{\frac{1}{p}} |x(t)|^p.$$

The proof is complete.

Now, consider the mutivalued map $\mathfrak{L} : \mathfrak{B}_b \to 2^{\mathfrak{B}_b}$ defined by $\mathfrak{L}x$ the set of $\rho \in \mathfrak{B}_b$ such that

$$\rho(t) = \begin{cases} \phi(t), \quad t \in (-\infty, 0], \\ C(t)\phi(0) + S(t)[\eta - g(0, \phi)] + \int_0^t C(t-s) g(s, x_s) ds \\ + \int_0^t S(t-s) f(s) dw(s), \quad t \in J, \end{cases} \qquad (3)$$

where $f \in S_{F,x}$.

We shall show that the operator $\mathfrak{L}$ has fixed points,which are then a solution of system (1). For $\phi \in \mathfrak{B}_h$, we define $\bar{\phi}$ by

$$\bar{\phi}(t) = \begin{cases} \phi(t), \quad -\infty < t \leq 0 \\ C(t)\phi(0), \quad 0 \leq t \leq b \end{cases}$$

then $\bar{\phi} \in \mathfrak{B}_b$. Set

$$x(t) = y(t) + \bar{\phi}(t), \quad -\infty < t \leq b.$$

It is clear to see that $x$ satisfies (2) if and only if $y$ satisfies $y_0 = 0$ and

$$\begin{aligned}
y(t) &= S(t)[\eta - g(0, \phi)] + \int_0^t C(t-s) g(s, y_s + \bar{\phi}_s) ds \\
&\quad + \int_0^t S(t-s) f(s) dw(s), \quad t \in J.
\end{aligned}$$

Let $\mathfrak{B}_b^0 = \{y \in \mathfrak{B}_b : y_0 = 0 \in \mathfrak{B}_h\}$. For any $y \in \mathfrak{B}_b^0$,

$$\| y \|_b = \| y_0 \|_{\mathfrak{B}_h} + \sup_{s\in[0,b]} E^{\frac{1}{p}} |y(s)|^p = \sup_{s\in[0,b]} E^{\frac{1}{p}} |y(s)|^p.$$

Thus $(\mathfrak{B}_b^0, \| \cdot \|_b)$ is a Banach space. Set $\mathfrak{B}_q = \{y \in \mathfrak{B}_b^0 : \| y \|_b \leq q\}$ for some $q \geq 0$, then $\mathfrak{B}_q \subseteq \mathfrak{B}_b^0$ is uniformly bounded, for any $y \in \mathfrak{B}_q$, from Lemma 2, we have

$$\begin{aligned}
\| y_t + \bar{\phi}_t \|_{\mathfrak{B}_h} &\leq \| y_t \|_{\mathfrak{B}_h} + \| \bar{\phi}_t \|_{\mathfrak{B}_h} \\
&\leq l \sup_{0\leq s\leq t} E^{\frac{1}{p}} |y(s)|^p + ||y_0||_{\mathfrak{B}_h} \\
&\quad + l \sup_{0\leq s\leq t} E^{\frac{1}{p}} |\bar{\phi}(s)|^p + \| \bar{\phi}_0 \|_{\mathfrak{B}_h} \\
&\leq lq + \| \phi \|_{\mathfrak{B}_h} + l \sup_{0\leq s\leq t} |C(s)| E^{\frac{1}{p}} |\phi(0)|^p \\
&\leq l(q + M_1 E^{\frac{1}{p}} |\phi(0)|^p) + \| \phi \|_{\mathfrak{B}_h} = q'.
\end{aligned}$$

Define the multivalued map $\mathfrak{L}_1 : \mathfrak{B}_b^0 \to 2^{\mathfrak{B}_b^0}$ defined by $\mathfrak{L}_1 y$ the set of $\bar{\rho} \in \mathfrak{B}_b^0$ such that

$$\bar{\rho}(t) = \begin{cases} 0, \quad t \in (-\infty, 0], \\ S(t)[\eta - g(0, \phi)] + \int_0^t C(t-s) g(s, y_s + \bar{\phi}_s) ds \\ + \int_0^t S(t-s) f(s) dw(s), \quad t \in J. \end{cases} \qquad (4)$$

**Lemma 3**. If the hypotheses $(H_1) - (H_5)$ satisfied, then $\mathfrak{L}_1 : \mathfrak{B}_b^0 \to 2^{\mathfrak{B}_b^0}$ is a completely continuous multivalued map, Usc with a convex closed value.

Proof. We divide the proof into several steps.

**Step 1**. $\mathfrak{L}_1 y$ is convex for each $y \in \mathfrak{B}_b^0$.

In fact, if $\bar{\rho}_1, \bar{\rho}_2$ belong to $\mathfrak{L}_1 y$, then there exit $f_1, f_2 \in S_{F,y}$ such that for each $t \in J$ we have

$$\begin{aligned}
\bar{\rho}_i(t) &= S(t)[\eta - g(0, \phi)] + \int_0^t C(t-s) g(s, y_s + \bar{\phi}_s) ds \\
&\quad + \int_0^t S(t-s) f_i(s) dw(s), \quad t \in J, \quad i = 1, 2.
\end{aligned}$$

Let $\lambda \in [0, 1]$, we have

$$\begin{aligned}
&(\lambda\bar{\rho}_1 + (1-\lambda)\bar{\rho}_2)(t) \\
&= S(t)[\eta - g(0, \phi)] + \int_0^t C(t-s) g(s, y_s + \bar{\phi}_s) ds \\
&\quad + \int_0^t S(t-s)(\lambda f_1(s) + (1-\lambda) f_2(s)) dw(s).
\end{aligned}$$

Since $S_{F,y}$ is convex, we have $\lambda\bar{\rho}_1 + (1-\lambda)\bar{\rho}_2 \in \mathfrak{L}_1 y$.

**Step 2**. $\mathfrak{L}_1$ maps bounded set into bounded set in $\mathfrak{B}_b^0$. Indeed, it is enough to show that there exists a positive constant $\Lambda$ such that for each $\bar{\rho} \in \mathfrak{L}_1 y, y \in \mathfrak{B}_q = \{y \in \mathfrak{B}_b^0 : \| y \|_b \leq q\}$ one has $\| \bar{\rho} \|_b \leq \Lambda$. If $\bar{\rho} \in \mathfrak{L}_1 y$, then there exist $f \in S_{F,y}$, such that for each $t \in J$,

$$\begin{aligned}\bar{\rho}(t) &= S(t)[\eta - g(0,\phi)] + \int_0^t C(t-s)g(s, y_s + \bar{\phi}_s)ds \\ &\quad + \int_0^t S(t-s)f(s)dw(s), \quad t \in J.\end{aligned}$$

By $(H_1) - (H_4)$ we have for $t \in J$,

$$\begin{aligned}
& E|\bar{\rho}(t)|^p \\
= \; & E|S(t)[\eta - g(0,\phi)] + \int_0^t C(t-s)g(s, y_s + \bar{\phi}_s)ds \\
& + \int_0^t S(t-s)f(s)dw(s)|^p \\
\leq \; & 3^{p-1}E|S(t)[\eta - g(0,\phi)]|^p \\
& + 3^{p-1}E|\int_0^t C(t-s)g(s, y_s + \bar{\phi}_s)ds|^p \\
& + 3^{p-1}E||\int_0^t S(t-s)f(s)dw(s)||_2^p \\
\leq \; & 3^{p-1}M_1^p b^p(2^{p-1}E|\eta|^p + 2^{p-1}E|g(0,\phi)|^p) \\
& + 3^{p-1}b^{p-1}\int_0^t E|C(t-s)g(s, y_s + \bar{\phi}_s)|^p ds \\
& + 3^{p-1}E\{\int_0^t [|S(t-s)|^p ||f(s)||_2^p]^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
\leq \; & 3^{p-1}M_1^p b^p(2^{p-1}E|\eta|^p + 2^{p-1}(c_1||\phi||_{\mathfrak{B}_h}^p + c_2)) \\
& + 3^{p-1}b^{p-1}M_1^p \int_0^t (c_1||y_s + \bar{\phi}_s||_{\mathfrak{B}_h}^p + c_2)ds \\
& + 3^{p-1}(M_1 b)^p\{\int_0^t [E||f(s)||_2^p]^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
\leq \; & 3^{p-1}M_1^p b^p(2^{p-1}E|\eta|^p + 2^{p-1}(c_1||\phi||_{\mathfrak{B}_h}^p + c_2)) \\
& + 3^{p-1}b^{p-1}M_1^p \int_0^t (c_1 q'^p + c_2)ds \\
& + 3^{p-1}(M_1 b)^p \sup_{x\in[0,q']} \psi(x^p)\{\int_0^t (p(s))^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
\leq \; & 3^{p-1}M_1^p b^p(2^{p-1}E|\eta|^p + 2^{p-1}(c_1||\phi||_{\mathfrak{B}_h}^p + c_2)) \\
& + 3^{p-1}b^p M_1^p(c_1 q'^p + c_2) \\
& + 3^{p-1}(M_1 b)^p \sup_{x\in[0,q']} \psi(x^p) t^{\frac{p}{2}-1}\int_0^t p(s)ds \\
\leq \; & 3^{p-1}M_1^p b^p(2^{p-1}E|\eta|^p + 2^{p-1}(c_1||\phi||_{\mathfrak{B}_h}^p + c_2)) \\
& + 3^{p-1}b^p M_1^p(c_1 q'^p + c_2) \\
& + 3^{p-1}(M_1 b)^p \sup_{x\in[0,q']} \psi(x^p) b^{\frac{p}{2}-1}\int_0^b p(s)ds \\
= \; & \Lambda^p.
\end{aligned}$$

then for each $\bar{\rho} \in \mathfrak{L}_1(\mathfrak{B}_q)$, we have

$$\| \bar{\rho} \|_b \leq \Lambda.$$

**Step 3**. $\mathfrak{L}_1$ maps bounded sets into equicontinuous sets of $\mathfrak{B}_b^0$. Let $0 < t_1 < t_2 \leq b$, for each $y \in \mathfrak{B}_q = \{y \in \mathfrak{B}_b^0 : \| y \|_b \leq q\}$ and $\bar{\rho} \in \mathfrak{L}_1 y$, there exists $f \in S_{F,y}$ such that (4). Thus

$$\begin{aligned}
& E|\bar{\rho}(t_2) - \bar{\rho}(t_1)|^p \\
= \; & E|(S(t_2) - S(t_1))[\eta - g(0,\phi)] \\
& + \int_0^{t_1} [C(t_2-s) - C(t_1-s)]g(s, y_s + \bar{\phi}_s)ds \\
& + \int_{t_1}^{t_2} C(t_2-s)g(s, y_s + \bar{\phi}_s)ds \\
& + \int_0^{t_1} [S(t_2-s) - S(t_1-s)]f(s)dw(s) \\
& + \int_{t_1}^{t_2} S(t_2-s)f(s)dw(s)|^p \\
\leq \; & 5^{p-1}E|[S(t_2) - S(t_1)][\eta - g(0,\phi)]|^p \\
& + 5^{p-1}E|\int_0^{t_1} [C(t_2-s) - C(t_1-s)]g(s, y_s + \bar{\phi}_s)ds|^p \\
& + 5^{p-1}E|\int_{t_1}^{t_2} C(t_2-s)g(s, y_s + \bar{\phi}_s)ds|^p \\
& + 5^{p-1}E||\int_0^{t_1} [S(t_2-s) - S(t_1-s)]f(s)dw(s)||_2^p \\
& + 5^{p-1}E||\int_{t_1}^{t_2} S(t_2-s)f(s)dw(s)||_2^p \\
\leq \; & 5^{p-1}|S(t_2) - S(t_1)|^p(2^{p-1}E|\eta|^p \\
& + 2^{p-1}c_1||\phi||_{\mathfrak{B}_h}^p + 2^{p-1}c_2) \\
& + 5^{p-1}b^{p-1}\int_0^{t_1} |C(t_2-s) - C(t_1-s)|^p \\
& E|g(s, y_s + \bar{\phi}_s)|^p ds \\
& + 5^{p-1}(t_2 - t_1)^{p-1}\int_{t_1}^{t_2} |C(t_2-s)|^p E|g(s, y_s + \bar{\phi}_s)|^p ds \\
& + 5^{p-1}E\{\int_0^{t_1} [|S(t_2-s) - S(t_1-s)|^p ||f(s)||_2^p]^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
& + 5^{p-1}E\{\int_{t_1}^{t_2} [|S(t_2-s)|^p ||f(s)||_2^p]^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
\leq \; & 5^{p-1}|S(t_2) - S(t_1)|^p(2^{p-1}E|\eta|^p \\
& + 2^{p-1}c_1||\phi||_{\mathfrak{B}_h}^p + 2^{p-1}c_2) \\
& + 5^{p-1}b^{p-1}\int_0^{t_1} |C(t_2-s) - C(t_1-s)|^p \\
& (c_1||y_s + \bar{\phi}_s||_{\mathfrak{B}_h}^p + c_2)ds \\
& + 5^{p-1}(t_2 - t_1)^{p-1}\int_{t_1}^{t_2} |C(t_2-s)|^p \\
& (c_1||y_s + \bar{\phi}_s||_{\mathfrak{B}_h}^p + c_2)ds \\
& + 5^{p-1}\{\int_0^{t_1} [|S(t_2-s) - S(t_1-s)|^p E||f(s)||_2^p]^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
& + 5^{p-1}\{\int_{t_2}^{t_1} [|S(t_2-s)|^p E||f(s)||_2^p]^{\frac{2}{p}} ds\}^{\frac{p}{2}} \\
\leq \; & 5^{p-1}|S(t_2) - S(t_1)|^p(2^{p-1}E|\eta|^p \\
& + 2^{p-1}c_1||\phi||_{\mathfrak{B}_h}^p + 2^{p-1}c_2) \\
& + 5^{p-1}b^{p-1}\int_0^{t_1} |C(t_2-s) - C(t_1-s)|^p
\end{aligned}$$

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$$
\begin{aligned}
&(c_1||y_s+\bar{\phi}_s||^p_{\mathfrak{B}_h}+c_2)ds\\
&+5^{p-1}(t_2-t_1)^{p-1}\int_{t_1}^{t_2}|C(t_2-s)|^p\\
&(c_1||y_s+\bar{\phi}_s||^p_{\mathfrak{B}_h}+c_2)ds\\
&+5^{p-1}b^{\frac{p}{2}-1}\int_0^{t_1}|S(t_2-s)-S(t_1-s)|^pE||f(s)||_2^pds\\
&+5^{p-1}(t_2-t_1)^{\frac{p}{2}-1}\int_{t_2}^{t_1}|S(t_2-s)|^pE||f(s)||_2^pds
\end{aligned}
$$

The right-hand side of inequality above is independent of $y \in \mathfrak{B}_q$ and tends to zero as $t_2 \to t_1$. Thus the set $\{\mathfrak{L}_1 y:\ y \in \mathfrak{B}_q\}$ is equicontinuous. (Note that we considered here only the case $0 < t_1 < t_2 \leq b$, since the other case $t_1 < t_2 \leq 0$ or $t_1 \leq 0 \leq t_2 \leq b$ are very simple). As a consequence of the Ascoli-Arzela theorem, it suffice to show that $\mathfrak{L}_1$ maps $\mathfrak{B}_q$ into a precompact set in $H$. Let $0 < t \leq b$ be fixed and let $\epsilon$ be a real number satisfying $0 < \epsilon < t$. For $y \in \mathfrak{B}_q$ we define

$$
\begin{aligned}
\bar{\rho}_\epsilon(t) &= S(t)[\eta-g(0,\phi)]+\int_0^{t-\epsilon}C(t-s)g(s,y_s+\bar{\phi}_s)ds\\
&\quad+\int_0^{t-\epsilon}S(t-s)f(s)dw(s),\quad t\in J.
\end{aligned}
$$

since $C(t)$ and $S(t)$ is compact operators, the set $H_\epsilon(t) = \{\bar{\rho}_\epsilon(t) : \bar{\rho}_\epsilon \in \mathfrak{L}_1 y\}$ is precompact in $H$ for every $\epsilon$, $0 < \epsilon < t$. Moreover, for every $\bar{\rho} \in \mathfrak{L}_1 y$, we have

$$
\begin{aligned}
&E|\bar{\rho}_\epsilon(t)-\bar{\rho}(t)|^p\\
\leq\ & 2^{p-1}E|\int_{t-\epsilon}^t C(t-s)g(s,y_s+\bar{\phi}_s)ds|^p\\
&+2^{p-1}E||\int_{t-\epsilon}^t S(t-s)f(s)dw(s)||_2^p\\
\leq\ & 2^{p-1}\epsilon^{p-1}\int_{t-\epsilon}^t E|C(t-s)g(s,y_s+\bar{\phi}_s)|^pds\\
&+2^{p-1}E\{\int_{t-\epsilon}^t[|S(t-s)|^p||f(s)||_2^p]^{\frac{2}{p}}ds\}^{\frac{p}{2}}\\
\leq\ & 2^{p-1}\epsilon^{p-1}\int_{t-\epsilon}^t|C(t-s)|^pE|g(s,y_s+\bar{\phi}_s)|^pds\\
&+2^{p-1}\{\int_{t-\epsilon}^t[|S(t-s)|^pE||f(s)||_2^p]^{\frac{2}{p}}ds\}^{\frac{p}{2}}\\
\leq\ & 2^{p-1}\epsilon^{p-1}\int_{t-\epsilon}^t|C(t-s)|^pE|g(s,y_s+\bar{\phi}_s)|^pds\\
&+2^{p-1}\epsilon^{\frac{p}{2}-1}\{\int_{t-\epsilon}^t|S(t-s)|^pE||f(s)||_2^pds.
\end{aligned}
$$

Therefore there are precompact sets arbitrarily close to the set $\{\bar{\rho}(t) : \bar{\rho} \in \mathfrak{L}_1 y\}$. So the set $\{\bar{\rho}(t) : \bar{\rho} \in \mathfrak{L}_1\mathfrak{B}_q\}$ is precompact in $H$. Hence, the operator $\mathfrak{L}_1$ is completely continuous.

**Step 4**. $\mathfrak{L}_1$ has a closed graph.

Let $y^{(n)} \to y^*$, $\bar{\rho}_n \in \mathfrak{L}_1 y^{(n)}$ and $\bar{\rho}_n \to \bar{\rho}_*$. We shall prove that $\bar{\rho}_* \in \mathfrak{L}_1 y^*$. Indeed, $\bar{\rho}_n \in \mathfrak{L}_1 y^{(n)}$ means that there exists $f_n \in S_{F,y^{(n)}}$, such that

$$
\begin{aligned}
&\bar{\rho}_n(t)\\
=\ & S(t)[\eta-g(0,\phi)]+\int_0^t C(t-s)g(s,y_s^{(n)}+\bar{\phi}_s)ds\\
&+\int_0^t S(t-s)f_n(s)dw(s),\qquad t\in J.
\end{aligned}
$$

We must prove that there exists $f_* \in S_{F,y^*}$ such that

$$
\begin{aligned}
&\bar{\rho}_*(t)\\
=\ & S(t)[\eta-g(0,\phi)]+\int_0^t C(t-s)g(s,y^*+\bar{\phi}_s)ds\\
&+\int_0^t S(t-s)f_*(s)dw(s),\qquad t\in J.
\end{aligned}
$$

since $g$ is continuous, for $t \in J$,

$$
\begin{aligned}
&\|\ \{\bar{\rho}_n(t)-S(t)[\eta-g(0,\phi)]\\
&-\int_0^t C(t-s)g(s,y_s^{(n)}+\bar{\phi}_s)ds\}-\{\bar{\rho}_*(t)\\
&-S(t)[\eta-g(0,\phi)]-\int_0^t C(t-s)g(s,y_s^*+\bar{\phi}_s)ds\}\ \|\\
&\longrightarrow 0,\quad as\quad n\longrightarrow\infty.
\end{aligned}
$$

Consider the linear continuous operator

$$
\begin{aligned}
&\Gamma : L^p(J,H)\longrightarrow C(J,H),\\
&f\longrightarrow \Gamma(f)(t)=\int_0^t S(t-s)f(s)dw(s).
\end{aligned}
$$

From Lemma 1, it follows that $\Gamma \circ S_F$ is a closed graph. Moreover, we have

$$
\begin{aligned}
&\bar{\rho}_n(t)-S(t)[\eta-g(0,\phi)]\\
&-\int_0^t C(t-s)g(s,y_s^{(n)}+\bar{\phi}_s)ds\in\Gamma(S_{F,y^{(n)}}).
\end{aligned}
$$

Since $y^{(n)} \longrightarrow y^*$, it follows from Lemma 1 that

$$
\begin{aligned}
&\bar{\rho}_*(t)-S(t)[\eta-g(0,\phi)]-\int_0^t C(t-s)g(s,y_s^*+\bar{\phi}_s)ds\\
&=\int_0^t S(t-s)f_*(s)dw(s).
\end{aligned}
$$

for some $f_* \in S_{F,y^*}$.

Therefore $\mathfrak{L}_1$ is a completely continuous multivalued map, $Usc$ with convex closed values.

Now, in order to apply Theorem 1, we introduce a parameter $\lambda > 1$ and consider the following auxiliary problems:

$$
\begin{cases}
x(t)=C(t)\phi(0)+\frac{1}{\lambda}S(t)[\eta-g(0,\phi)]\\
\qquad+\frac{1}{\lambda}\int_0^t C(t-s)g(s,x_s)ds\\
\qquad+\frac{1}{\lambda}\int_0^t S(t-s)f(s)dw(s),\quad for\ \ a.e\ t\in J,\\
x(t)=\phi(t),\quad t\in(-\infty,0],
\end{cases}
\tag{5}
$$

where $f \in S_{F,x}$.

**Lemma 4**. If hypotheses $(H_1)-(H_5)$ are satisfied. Let $x(t)$ be a mild solution of system (5), then there exists a priori bounds $R>0$, such that $\| x_t \|\leq R$, $t\in J$, where $R$ depends only on $b$ and the function $\psi(\cdot)$ and $p(\cdot)$.

Proof. From the system (5), we have

$$\begin{aligned}
& E|x(t)|^p \\
\leq\; & 4^{p-1}E|C(t)\phi(0)|^p+4^{p-1}E|S(t)[\eta-g(0,\phi)]|^p \\
& +4^{p-1}E|\int_0^t C(t-s)g(s,x_s)ds|^p \\
& +4^{p-1}E||\int_0^t S(t-s)f(s)dw(s)||_2^p \\
\leq\; & 4^{p-1}|C(t)|^pE|\phi(0)|^p \\
& +4^{p-1}|S(t)|^p\{2^{p-1}(E|\eta|^p+E|g(0,\phi)|^p)\} \\
& +4^{p-1}b^{p-1}\int_0^t |C(t-s)|^pE|g(s,x_s)|^pds \\
& +4^{p-1}\{\int_0^t [|S(t-s)|^pE||f(S)||_2^p]^{\frac{2}{p}}ds\}^{\frac{p}{2}} \\
\leq\; & 4^{p-1}M_1^pE|\phi(0)|^p+8^{p-1}M_1^pb^p(E|\eta|^p \\
& +c_1||\phi||^p_{\mathfrak{B}_h}+c_2) \\
& +4^{p-1}b^{p-1}M_1^p\int_0^t(c_1||x_s||^p_{\mathfrak{B}_h}+c_2)ds \\
& +4^{p-1}b^{\frac{p}{2}-1}\int_0^t |S(t-s)|^pE||f(s)||_2^pds \\
\leq\; & 4^{p-1}M_1^pE|\phi(0)|^p+8^{p-1}M_1^pb^p(E|\eta|^p \\
& +c_1||\phi||^p_{\mathfrak{B}_h}+c_2) \\
& +4^{p-1}b^{p-1}M_1^p\int_0^t(c_1||x_s||^p_{\mathfrak{B}_h}+c_2)ds \\
& +4^{p-1}b^{\frac{p}{2}-1}M_1^pb^p\int_0^t p(s)\psi(||x_s||^p_{\mathfrak{B}_h})ds \\
& =N+4^{p-1}b^{p-1}M_1^p\int_0^t(c_1||x_s||^p_{\mathfrak{B}_h}+c_2)ds \\
& +4^{p-1}b^{\frac{p}{2}-1}M_1^pb^p\int_0^t p(s)\psi(||x_s||^p_{\mathfrak{B}_h})ds,
\end{aligned}$$

Where

$$\begin{aligned}
N \;=\; & 4^{p-1}M_1^pE|\phi(0)|^p+8^{p-1}M_1^pb^p(E|\eta|^p \\
& +c_1||\phi||^p_{\mathfrak{B}_h}+c_2).
\end{aligned}$$

Thus from Lemma 2, it follows

$$\begin{aligned}
||x_t||_{\mathfrak{B}_h} \;\leq\; & l\sup_{s\in[0,t]}[E|x(s)|^p]^{\frac{1}{p}}+||x_0||_{\mathfrak{B}_h} \\
\leq\; & l[N+4^{p-1}b^{p-1}M_1^p\int_0^t(c_1||x_s||^p_{\mathfrak{B}_h}+c_2)ds \\
& +4^{p-1}b^{\frac{p}{2}-1}M_1^pb^p\int_0^t p(s)\psi(||x_s||^p_{\mathfrak{B}_h})ds]^{\frac{1}{p}} \\
& +||\phi||_{\mathfrak{B}_h}.
\end{aligned}$$

Let $\mu(t)=\sup\{||x_s||_{\mathfrak{B}_h}:0\leq s\leq t\}$, then the function $\mu(t)$ is nondecreasing in J, and we have

$$\begin{aligned}
\mu(t) \;\leq\; & l[4^{p-1}b^{\frac{p}{2}-1}M_1^p\int_0^t(c_1\mu^p(s)+c_2)ds \\
& +4^{p-1}M_1^pb^{\frac{3p}{2}-1}\int_0^t p(s)\psi(\mu^p(s))ds+N]^{\frac{1}{p}} \\
& +||\phi||_{\mathfrak{B}_h}.
\end{aligned}$$

Let $\alpha(t)=4^{p-1}b^{\frac{p}{2}-1}M_1^p\int_0^t(c_1\mu^p(s)+c_2)ds+4^{p-1}M_1^pb^{\frac{3p}{2}-1}\int_0^t p(s)\psi(\mu^p(s))ds+N$, so we have

$$\alpha(0)=N,\qquad \mu(t)\leq l(\alpha(t))^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h},\qquad t\in J,$$

and

$$\begin{aligned}
\alpha'(t) \;=\; & 4^{p-1}b^{\frac{p}{2}-1}M_1^p(c_1\mu^p(t)+c_2) \\
& +4^{p-1}b^{\frac{3p}{2}-1}M_1^pp(t)\psi(\mu^p(t)) \\
\leq\; & 4^{p-1}b^{\frac{p}{2}-1}M_1^p(c_1(l(\alpha(t))^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h})^p+c_2) \\
& +4^{p-1}b^{\frac{3p}{2}-1}M_1^pp(t)\psi(l((\alpha(t))^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h})^p) \\
\leq\; & \bar{m}(t)(1+(l(\alpha(t))^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h})^p \\
& +\psi((l(\alpha(t))^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h})^p))
\end{aligned}$$

Where

$$\begin{aligned}
\bar{m}(t) \;=\; & \max\{4^{p-1}b^{\frac{p}{2}-1}M_1^pc_1,4^{p-1}b^{\frac{p}{2}-1}M_1^pc_2, \\
& 4^{p-1}b^{\frac{3p}{2}-1}M_1^pp(t)\}.
\end{aligned}$$

This implies

$$\frac{\alpha'(t)}{1+g(\alpha(t))+\psi(g(\alpha(t)))}\leq\bar{m}(t),$$

and

$$\int_0^t\frac{\alpha'(s)}{1+g(\alpha(s))+\psi(g(\alpha(s)))}ds\leq\int_0^t\bar{m}(s)ds,$$

That is

$$\begin{aligned}
\int_{\alpha(0)}^{\alpha(t)}\frac{1}{1+g(s)+\psi(g(s))}ds \;\leq\; & \int_0^t\bar{m}(s)ds \\
& <\int_N^{+\infty}\frac{1}{1+g(s)+\psi(g(s))}ds.
\end{aligned}$$

Where $\alpha(0)=N, g(s)=(ls^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h})^p$. This inequality implies there is a constant $\bar{k}$ such that $\alpha(t)\leq\bar{k}$ for every $t\in[0,b]$, hence

$$||x_t||_{\mathfrak{B}_h}\leq\mu(t)\leq l[\alpha(t)]^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h}\leq l\bar{k}^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h}.$$

This shows that exist constant $R=l\bar{k}^{\frac{1}{p}}+||\phi||_{\mathfrak{B}_h}>0$, such that $||x_t||_{\mathfrak{B}_h}\leq R$.

**Theorem 2** Assume that hypotheses $(H_1)-(H_5)$ hold, then system (1) has at least one mild solution on $J_1$.

Proof Let $\Omega = \{y \in \mathfrak{B}_b^0 : \lambda y \in \mathfrak{L}_1 y, for \quad some \quad \lambda > 1\}$. Then for any $y \in \Omega$, we have

$$\begin{aligned} y(t) &= \frac{1}{\lambda} S(t)[\eta - g(0,\phi)] \\ &\quad + \frac{1}{\lambda}\int_0^t C(t-s)g(s, y_s + \bar{\phi}_s)ds \\ &\quad + \frac{1}{\lambda}\int_0^t S(t-s)f(s)dw(s), \end{aligned}$$

which implies the function $x = y + \bar{\phi}$ is a mild solution of system (5), for which we have proved in Lemma 4 that $||x_t||_{\mathfrak{B}_h} \leq R$, $t \in J$, and hence from Lemma 2

$$\begin{aligned} ||y||_b &= ||y_0||_{\mathfrak{B}_h} + \sup_{s\in[0,b]} E^{\frac{1}{p}}|y(s)|^p \\ &= \sup_{s\in[0,b]} E^{\frac{1}{p}}|y(s)|^p \\ &\leq \sup_{s\in[0,b]} E^{\frac{1}{p}}|x(s)|^p + \sup_{s\in[0,b]} E^{\frac{1}{p}}|\bar{\phi}(s)|^p \\ &\leq \sup\{l^{-1}||x_s||_{\mathfrak{B}_h} : \ s \in [0,b]\} \\ &\quad + \sup_{s\in[0,b]} E^{\frac{1}{p}}|C(t)\phi(0)|^p \\ &\leq l^{-1}R + M_1 E^{\frac{1}{p}}|\phi(0)|^p, \end{aligned}$$

which implies $\Omega$ is bounded on J.

Hence, it follows from Lemma 4 and Theorem 1 that the operator $\mathfrak{L}_1$ has a fixed point $y^* \in \mathfrak{B}_b^0$. Let $x(t) = y^*(t) + \bar{\phi}(t)$, $t \in (-\infty, b]$. Then $x$ is a fixed point of the operator $\mathfrak{L}$ which is a mild solution of problem (1).

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