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Evaluating the Capability of the Flux-Limiter Schemes in Capturing the Turbulence Structures in a Fully Developed Channel Flow

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Abstract**—**Turbulence modelling is still evolving, and efforts are on to improve and develop numerical methods to simulate the real turbulence structures by using the empirical and experimental information. The monotonically integrated large eddy simulation (MILES) is an attractive approach for modelling turbulence in high Re flows, which is based on the solving of the unfiltered flow equations with no explicit sub-grid scale (SGS) model. In the current work, this approach has been used, and the action of the SGS model has been included implicitly by intrinsic nonlinear high-frequency filters built into the convection discretization schemes. The MILES solver is developed using the opensource CFD OpenFOAM libraries. The role of flux limiters schemes namely, Gamma, superBee, van-Albada and van-Leer, is studied in predicting turbulent statistical quantities for a fully developed channel flow with a friction Reynolds number, $Re_T = 180$, and compared the numerical predictions with the well-established Direct Numerical Simulation (DNS) results for studying the wall generated turbulence. It is inferred from the numerical predictions that Gamma, van-Leer and van-Albada limiters produced more diffusion and overpredicted the velocity profiles, while superBee scheme reproduced velocity profiles and turbulence statistical quantities in good agreement with the reference DNS data in the streamwise direction although it deviated slightly in the spanwise and normal to the wall directions. The simulation results are further discussed in terms of the turbulence intensities and Reynolds stresses averaged in time and space to draw conclusion on the flux limiter schemes performance in OpenFOAM context.

Keywords**—**Flux limiters, MILES, OpenFOAM, turbulence structures, TVD schemes.

I. Introduction

IN most of the fluid mechanics areas, the understanding of the phenomena is strongly connected to the presence of turbulence. Although many experimental and theoretical studies in the past have significantly contributed in increasing our physical understanding, a predictive and accurate closed theory of turbulent flows has not been established yet and is unlikely to emerge in the foreseeable future. Moreover, even with the large computation facilities, it is not possible to compute high-Reynolds number (Re) turbulent flows directly, by fully resolving all relevant scales of motions in space and time. Instead, at least part of the unsteady turbulent motion must be approximated or mimicked to make these calculations feasible and give reasonable results. The real challenge is to develop simulation models such that, although they may not be explicitly incorporating all dynamic scales, it will still be able to give accurate and reliable results for at least the large energy-containing scales and even for some of the small scales of flow motion.

Classical LES approach has been ranged from using the inherently limited subgrid viscosity formulations, to more sophisticated and accurate dynamic mixed models. Eddy-viscosity models can reproduce the SGS dissipation quite well but not the SGS forces entering the momentum equation, thereby making this approach less suited for complex high (Re) flows which by necessity are usually poorly resolved. More recent efforts have been focused on developing the mixed models to provide more accuracy, by adding the dissipative models to those of higher accuracy. These mixed models produce better predictions, but their applications and usage have been limited because of their implementation complexity and high computational cost relative to performing a coarse DNS case and thus become very expensive for the practical flows of interest at moderate-to-high Re.

Recognizing the disadvantages mentioned in the previous paragraph, a number of researchers tried to get rid of the classical formulations and started using unfiltered flow equations instead of the filtered ones and putting non-oscillatory constraints via non-linear limiters in finite volume formulations, to work as an implicit filter instead of the explicit one in the conventional LES. This included methods such as flux-corrected transport (FCT), the piecewise parabolic method (PPM), and total variation diminishing (TVD) algorithms. The original idea was due to the research of [1], namely, the Monotone Integrated LES approach (MILES), the particular class of ILES strategy based on using monotonicity-preserving methods.

ILES applies the SGS physics capturing via specific features of the non-linear numerical algorithms on which the simulation model is based. The basic idea of the ILES is to use the adaptive (dynamic, non-oscillatory) numeric to capture the inherent small-scale anisotropy of high-Re turbulent flows (e.g., worm vortices, shocks) and also the viscosity independent dissipation characteristic of the inertial range cascade dynamics, ensuring nonlinear stability and positivity where physically needed.

Major properties of any implicit SGS model are specified as following:

1) The selection of the high and low order schemes which are responsible for the model behavior in the smooth and near sharp flow gradient regions respectively;
2) Selection of the flux limiter which controls the blending

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percentage of the schemes depending on the flow region;

3) Achieve the balance between the dissipation and dispersion contributions to the solution, which relies on the design specifications for each numerical method [2].

The open source software OpenFOAM CFD libraries have been used in the present study to develop the MILES solver. The primary objective of this work is to assess the prediction of the flux limiters for the large scale of turbulent structures. The flux limiter namely Gamma, SuperBee, vanLeer and vanAlbada, are compared in terms of the predicted turbulence statistics of fully developed steady state channel flow at a friction Reynolds no. of 180 [3]-[5].

MILES performance has been evaluated previously in many case studies, using FCT and Gamma as flux limiters for canonical flows (homogeneous isotropic turbulence and turbulent channel flow), complex free and wall-bounded flows (rectangular jets and flow past a prolate spheroid), external flows [6]. Also, it has been shown that MILES is capable of predicting the large-scale turbulent structures in axisymmetric jet flow at Re $\sim 10^5$ by using a second order upwind or QUICK scheme in [7]. On the other hand, MILES has been able to produce equally decaying rate of isotropic turbulence with that one of the DNS via a comparison with the energy spectra for both cases [8].

In the following sections, numerical aspects of the developed MILES solver in OpenFOAM are discussed. The different types of flux limiters and their formulation are briefly discussed in computational aspect section. The numerical predictions are presented in the 'results' section. The turbulent statistical quantities obtained using different flux limiters are compared with the available DNS results.

II. Computational Aspects

A. Computational Domain

The computational channel dimensions considered in the numerical simulations are 4π and 2π in streamwise and spanwise direction respectively, with 2 as channel height in wall normal direction, normalized by the channel half height. The periodic boundary conditions are applied in both streamwise and the spanwise directions, while the top and bottom walls of the channel are applied with no slip boundary condition. The computations are discretized into $4.27*10^6$ finite volume cells, with (192 x 139 x 160) mesh divisions in streamwise, normal to the wall and spanwise direction respectively. The mesh distribution is uniform spacing in the streamwise and spanwise directions, while in wall normal directions, an expanding mesh towards the channel center is used such that the first mesh point next to the wall boundary is with $y^+ = 0.05$. The flow bulk velocity is adjusted during the runtime to maintain a friction Reynolds number value of $Re_\tau = 180$. The friction Reynolds number 'Re_τ' is evaluated with characteristic length scale and velocity scales as half channel height 'δ' and wall shear velocity $u_\tau = \sqrt{\tau_w/\rho}$, respectively.

B. Solver

The three-dimensional, time-dependent, incompressible Navier-Stokes equations (NSE) have been solved by using the open source CFD libraries of OpenFOAM.

The most important feature of MILES is that it generates implicitly a nonlinear tensor-valued eddy-viscosity during the convection discretization, which acts predominantly on stabilizing the flow and suppress the unphysical oscillations near sharp velocity gradients. Therefore, in the current study, the NSE for incompressible flow has been solved as accurately as possible by using a particular class of flux-limiting schemes and their associated built-in (or implicit) SGS models.

The implicit SGS model is created by controlling the leading truncation error and this process is limited to high resolution methods for the convection flux to be able to keep second order accuracy in the flow smooth regions. Also, the leading truncation error has to vanish at d → 0 (where d is the mesh size) so that it remains consistent with the NSE and the conventional LES model [2]. The NSE are modified in the developed CFD solver based on the work of [6].

$$\partial_t(V) + \nabla.(V\otimes V) = -\nabla p + \nabla.S + \nabla.\Big[C(\nabla V)^T + (\nabla V)C^T + \chi^2(\nabla V)d\otimes(\nabla V)d + \left[\tfrac{1}{6}\nu\nabla^3 V - \tfrac{1}{8}\nabla^2 V\right](d\otimes d)\Big] \quad (1)$$

where:

$$C = \chi(V\otimes d),$$

$$\chi = (1/2)(1-\Gamma)(\beta^- - \beta^+),$$

$$\beta^{\pm} = \frac{(1/2)\left(v_f.dA \pm |v_f.dA|\right)}{|v_f.dA|}$$

With continuity equation of $\nabla.V = 0$, v_f is the flux function and Γ is a flux limiter.

From (1), It has been noticed that in smooth regions, where $\Gamma = 1$ and C = 0, the leading order truncation error becomes:

$$\tau = \left[\left[\tfrac{1}{6}\nu\nabla^3 V - \tfrac{1}{8}\nabla^2 V\right](d\otimes d)\right] \quad (2)$$

And hence, the implicit SGS stress term defined as:

$$B = C(\nabla V)^T + (\nabla V)C^T + \chi^2(\nabla V)d\otimes(\nabla V)d \quad (3)$$

PimpleFoam solver in OpenFOAM has been as the MILES solver while, Crank-Nicholson scheme has been used for time discretization of all equations.

C. Flux Limiters

The study of Godunov's order barrier theorem, which proved that linear methods are not able to provide non-oscillatory solutions higher than first order [9], these limitations have encouraged most of the researchers to develop new techniques to overcome these limitations, especially those techniques which are able to avoid the spurious or non-physical oscillations at which the sharp velocity gradients exist. TVD schemes were especially attractive for this direction of research.

The main advantage of the TVD schemes is that it ensures capturing of discontinuities or the sharp change in the velocity gradients in the solution. As the second order centered scheme is chosen because of its simplicity and computational speed, but it may smoothen the sharp velocity gradients over many computational cells, so the first order upwind scheme is blended to it in the TVD scheme.

Flux-limiter Γ is the factor that combines a high-order convective flux-function V_f^H which acts well in the smooth flow regions, with a low-order dispersion-free flux-function V_f^L, which well-behaves near the sharp gradients, so as a result from this combination, the total flux-function becomes as:

$$V_f = V_f^H - (1-\Gamma)\left(V_f^H - V_f^L\right) \tag{4}$$

Choosing the suitable flux limiting scheme gives the appropriate interpolation between the selections of V_f^H and V_f^L and allows the possible correction to $(v_f^H - v_f^L)$ in the governing equation without increasing the variation of the solution, i.e. to comply with the physical principles of causality, monotonicity and positivity and thus to preserve the properties of the NSE [6]. The flux limiter Γ has the main effect on the properties of the implicit SGS model such as the selection of the high and low order schemes, the monotonicity and grid spacing. Flux limiters play a major role in turbulence structure capturing in any CFD code. The flux limiter also has the following effects:

1) For smooth parts of a solution, it will act as second order accurate flux-conserved advection.
2) For regions near a jump or a very sharp and sudden velocity gradient, it will switch to first order (i.e. upwind) flux-conserved advection.

Using flux limiters along with an appropriate high-resolution scheme makes the solutions TVD.

Second-order, TVD limiters must satisfy the following criteria [6]:

$$\begin{gathered} r \leq \Gamma(r) \leq 2r, (0 \leq r \leq 1), \\ 1 \leq \Gamma(r) \leq r, (1 \leq r \leq 2), \\ 1 \leq \Gamma(r) \leq 2, (r > 2), \\ \Gamma(r) = 1 \end{gathered} \tag{5}$$

where r is the ratio of consecutive gradients:

$$r = \frac{\delta V_{P-1/2}^n}{\delta V_{P+1/2}^n} = \frac{(V_P^n - V_{P-1}^n)}{(V_{P+1}^n - V_P^n)} \tag{6}$$

The definitions of well-known TVD flux-limiters which have been used in the current study:

1) The vanLeer flux-limiter, [9], with

$$\Gamma = \frac{r+|r|}{1+|r|} \tag{7}$$

2) The superBee flux-limiter, [10], with

$$\Gamma = max\left(0, max\left(min(2r,1), min(r,2)\right)\right) \tag{8}$$

3) The vanAlbada flux-limiter, [6], with

$$\Gamma = \frac{r+r^2}{1+r^2} \tag{9}$$

4) The Gamma flux-limiter, [6], with

$$\Gamma = \frac{1-k}{k} r\left[\theta(r) - \theta\left(r - \frac{k}{1-k}\right)\right] + \theta\left(r - \frac{k}{1-k}\right) \tag{10}$$

where k is a parameter of the scheme such that k ∈ [0,1], and θ is the Heaviside function [3]. Note that when k = 0.5, this scheme becomes TVD, which has been used in the current study.

The various limiters have different switching characteristics and are used according to the particular problem and solution scheme. No specific limiter has been found to work well for all problems, and a separate choice is usually made on a trial and error basis. Therefore, the present study is evaluating the flux-limiter second order TVD schemes in capturing turbulence structures in a fully developed channel flow.

III. Results

The performance of MILES solver as a function of flux-limiter is discussed in the current section. The study of fully developed turbulent channel flow at a friction velocity based on $Re_\tau = 180$, compared with DNS results from [3]-[5]. The turbulent quantities are non-dimensionalized by the channel half-width δ, and the wall shear velocity $u_\tau = \sqrt{\tau_w/\rho}$.

The statistical steady state of a fully developed channel flow is identified by a linear profile of the total shear stress, $-\overline{u'v'} + (1/Re_\tau)(\partial\bar{u}/\partial y)$, where $\overline{u'v'}$ is the Reynolds stress and $(1/Re_\tau)(\partial\bar{u}/\partial y)$ expresses the wall shear stress, as shown in the plots for different cases in Fig. 1. Once the velocity field reached the statistically steady state, the equations have been averaged in time to obtain a running time average of the various statistical correlations. From Fig. 1, the behavior of the total shear stress appeared to be a straight line which goes from the highest shear at the wall, where the viscous force is dominant, to the lowest value at the center of the channel, where the inertia force is larger.

The mean velocity distribution has been compared with the experiment from [11] and the DNS results of turbulent channel flow from [3]-[5] in Fig. 2. The superBee flux limiter simiulation results show good agreement with the DNS results of [3]-[5] for the same $Re_\tau = 180$ till wall units $y^+ = 35$ while having a slight deviation in the logarithmic region from the experimental result [11], as shown in Fig. 2 (a), this may be due to a characteristic difference between the channel and the boundary layer flows. This difference has been also observed by [4] in their DNS study. The Gamma, vanAlbada and vanLeer limiters have produced over prediction in the velocity profiles in the outer region of the boundary layer while they were identical in the inner region.

The near wall behavior of the mean velocity profile for all cases has been drawn in Fig. 2 (b), all limiters predict the same behavior in the sub-viscous region while Gamma, vanAlbada and vanLeer start to deviate around $y^+ = 10$ with increase in the over estimation for the velocity profile.

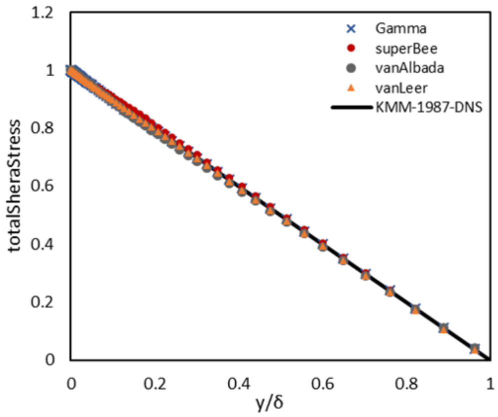
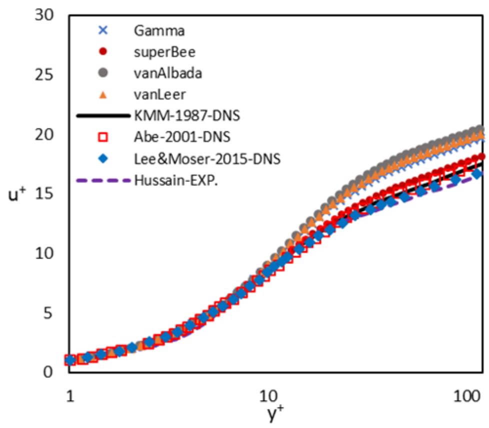
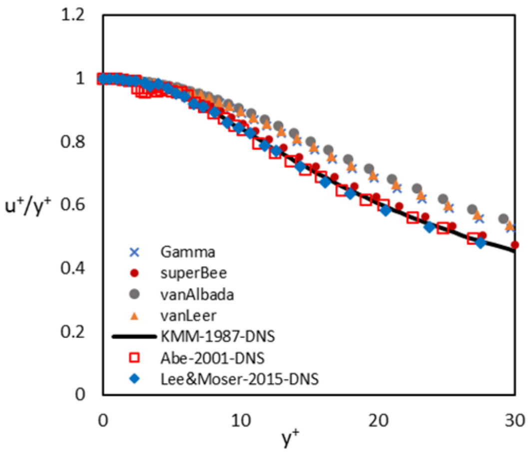


Fig. 1 Total shear stress for fully developed channel normalized by the wall shear velocity



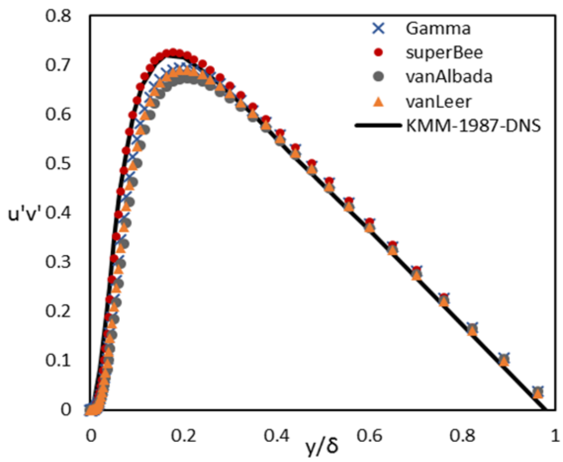
(a)



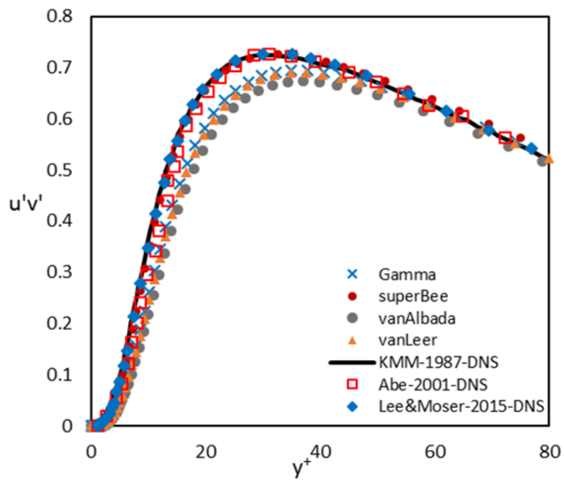
(b)

Fig. 2 (a) Mean-velocity profiles; (b) Near-wall behavior of the mean velocity

The plots of the Reynolds stresses are presented in Fig. 3 (a). It is clear that the superBee flux limiter values prediction are identical with those of the DNS results. The prediction for Gamma, vanAlbada and vanLeer flux limiters is nearly identical. The Reynolds stresses have reached their maximum values at $y^+ = 32$ for the DNS and superBee while being at $y^+ = 37$ for vanLeer and vanAlbada with a value percentage of 95% between both maximum values and $y^+ = 34$ for Gamma with a percentage of 96%. The difference has significantly appeared in Fig. 3 (b) with the wall coordinates where superBee limiter has approved its values with the DNS results.



(a)

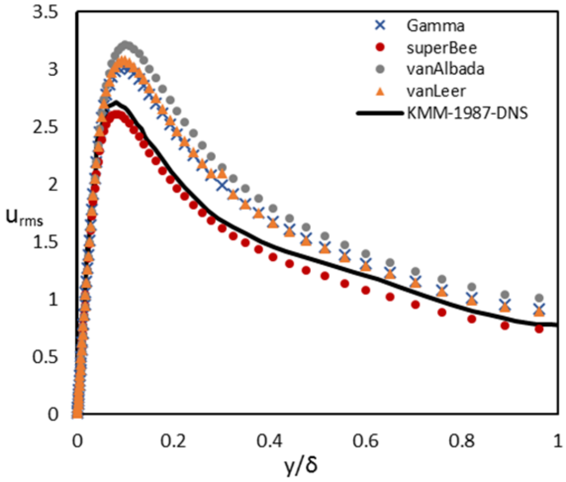


(b)

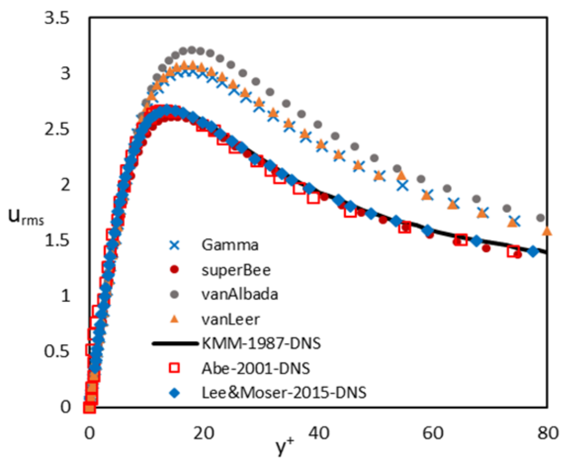
Fig. 3 Reynolds shear stress for fully developed channel normalized by the wall shear velocity: (a) in global coordinates (b) in wall coordinates

The RMS velocity of the fluctuations, normalized by the wall shear velocity from the current study, is compared with those of the DNS studies. In Fig. 4 (a), the streamwise RMS velocity of the fluctuations from superBee limiter has shown very close match with the results from DNS. On the other hand, overestimated values are predicted by Gamma, vanAlbada and vanLeer limiters even in the near wall region with ratios of 89%, 84% and 88%, (as a ratio of the DNS maximum value to their values) respectively, shown in Fig. 4 (b), with the wall coordinates.

The RMS fluctuations in the wall normal direction have been plotted as a function of the normalized y-coordinates in Fig. 5 (a). The superBee limiter predictions are comparable with the results of DNS in the near wall region while there is a noticeable discrepancy between the two results with an acceptable ratio in the near wall region of 92% at the position of maximum fluctuation, $y^+ = 60$. The other flux limiters have given under predictions consistently with a maximum value ratio of 84% at the same y^+, and these deviations are very clear when replots are drawn as a function of y^+ in Fig. 5 (b).

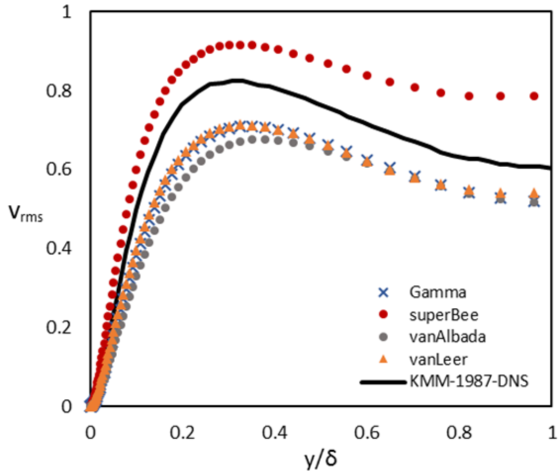


(a)

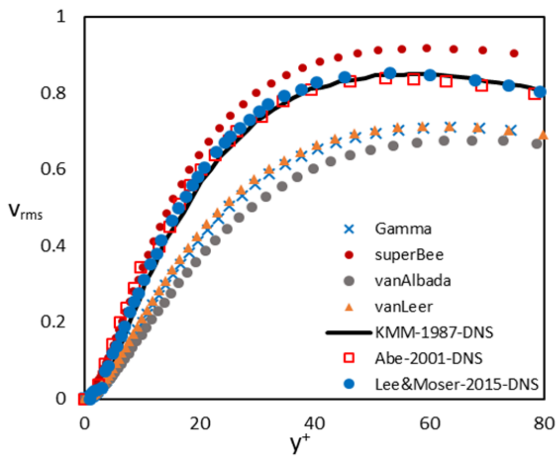


(b)

Fig. 4 RMS velocity fluctuations in streamwise dir. normalized by the wall shear velocity (a) In global coordinates; (b) in wall coordinates



(a)

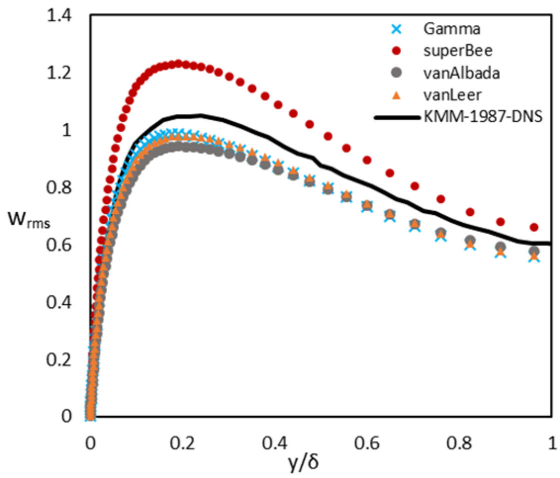


(b)

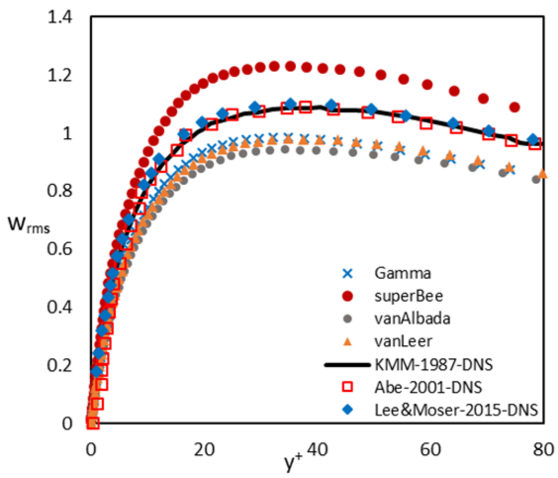
Fig. 5 RMS velocity fluctuations in y-dir. normalized by the wall shear velocity (a) In global coordinates; (b) in wall coordinates

Similar behavior for the spanwise velocity fluctuation is observed as shown in Fig. 6. Gamma, vanAlbada and vanLeer have produced under predictions for the spanwise fluctuation relative to the reference DNS data with a maximum value ratio of 90% for vanLeer and Gamma, while vanAlbada has given the maximum under estimation with a ratio of 86% relative to the maximum values of the fluctuations of the DNS results. On the other hand, superBee has given consistent over estimation for the fluctuation with a maximum value ratio of nearly 89% at $y^+ = 40$.

From the results shown in Figs. 1-6, it is clear that the superBee flux limiter predictions matched very well to the DNS studies in the streamwise directions, while having over predictions in turbulence quantities in the other two channel directions for capturing the turbulence structures in a fully developed channel. But, the discrepancy with other flux limiters considered in this study consistently either over/underpredicted the turbulence quantities. The conclusion can be that superBee limiter is the least diffusive limiter between all limiters. As from the physical meaning, it allows the linear discontinuities, change in the velocity gradient, to propagate for an indefinitely long time without numerical diffusion [10], while it gives the least non-physical oscillations of the values comparing with other flux limiters [12]. On the other hand, Gamma, vanAlbada and vanLeer limiters are more diffusive than superBee limiter. This can be proved with referring to the behavior of the TVD limiters compared with the behavior of linear non-TVD limiters, as illustrated in Fig. 7, in which the diffusivity decreases as the flux-limiters approach that of the superBee limiter.



(a)



(b)

Fig. 6 RMS velocity fluctuations in z-dir. normalized by the wall shear velocity (a) In global coordinates; (b) in wall coordinate

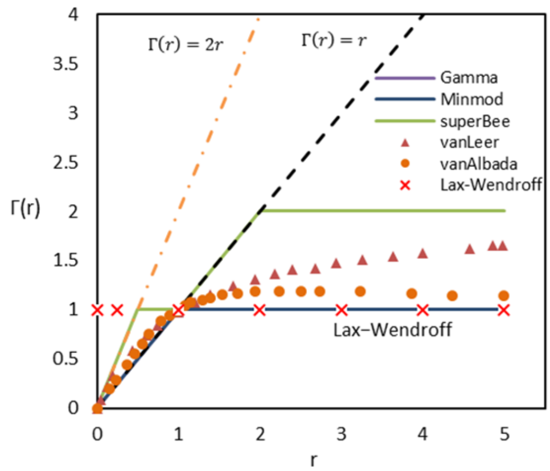


Fig. 7 TVD regions for the first and second order accurate TVD schemes together for the selected limiters (Lax-Wendroff scheme, the simplest version of a second order scheme)

IV. Conclusion

MILES approach has been used in simulating the fully developed turbulent channel flow at friction Reynolds of 180. The open source CFD OpenFOAM libraries are used to develop the MILES solver. The ability of different flux limiters flux schemes namely, Gamma, superBee, van-Albada, van-Leer are assessed by evaluating the statistical turbulent quantities for the channel flow. The simulated results are compared with the well-established DNS results. The results have shown that Gamma and vanLeer have been identical in their output along with vanAlbada limiter as diffusive limiters. While superBee limiter has produced acceptable behavior, which has been very close to the reference DNS data in the streamwise direction and has deviated slightly in the spanwise and normal to the wall directions which can be improved by increasing the grid resolution near the center of the channel to give good estimation turbulence quantities in both wall normal and span wise direction. Overall the superBee flux limiter is looking promising for the developed MILES solver in OpenFOAM context.

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