

# Error Rate Performance Comparisons of Precoding Schemes over Fading Channels for Multiuser MIMO

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**Abstract**—In Multiuser MIMO communication systems, interuser interference has a strong impact on the transmitted signals. Precoding technique schemes are employed for multiuser broadcast channels to suppress an interuser interference. Different Linear and nonlinear precoding schemes are there. For the massive system dimension, it is difficult to design an appropriate precoding algorithm with low computational complexity and good error rate performance at the same time over fading channels. This paper describes the error rate performance of precoding schemes over fading channels with the assumption of perfect channel state information at the transmitter. To estimate the bit error rate performance, different propagation environments namely, Rayleigh, Rician and Nakagami fading channels have been offered. This paper presents the error rate performance comparison of these fading channels based on precoding methods like Channel Inversion and Dirty paper coding for multiuser broadcasting system. MATLAB simulation has been used. It is observed that multiuser system achieves better error rate performance by Dirty paper coding over Rayleigh fading channel.

**Keywords**—Multiuser MIMO, channel inversion precoding, dirty paper coding, fading channels, BER.

## I. INTRODUCTION

THE recent wireless technology has been shifted from MIMO to Multiuser MIMO (MU-MIMO), where different users can simultaneously functioned by a single base station (BS) with multiple antennas. In precise, MU-MIMO achieves spatial multiplexing gain benefits, with single antenna in the mobile users [1], [2]. The importance of a MU-MIMO downlink is to permit multiple antenna BS to communicate with numerous mobile users at the same time through orthogonal channels [3]. In downlink transmitter section, the information has been transmitted through the multiple antennas; it experiences interuser interference (IUI) which makes the receiver to detect the signal incorrectly, leads to decreases the system performance. In order to suppress the interference, an appropriate precoding technique can be used in MU-MIMO downlink [4], [5].

Precoding is the important technique which is used in the BS before signal transmission. It needs channel state information (CSI) at the BS [6], [7]. The precoding techniques are classified as linear and nonlinear techniques. Linear precoding techniques are sub-optimal transmission technique which provides simple signal processing, low complexity and reasonable performance [8]. However, nonlinear precoding techniques assure high implementation complexity compared to linear precoding techniques.

An attractive research for MU-MIMO downlink precoding techniques to be organized. The linear channel inversion precoding structure such as a zero forcing (ZF) precoding [8], [10], [11], Minimum mean square error (MMSE) precoding [9]-[11] schemes are organized. Researchers compared maximum-ratio transmission (MRT) precoding, ZF and MMSE precoding techniques with respect to sum rate and SNR with the assumption of perfect CSI in the transmitter section under independent Rayleigh fading channel. It is found that MMSE gives superior performance compared to ZF and MRT over the range of SNRs [8]. An interference-free transmission can be recognized by nonlinear precoding like dirty paper coding (DPC) with the assumption of perfect CSI under independent Rayleigh fading channel [11]-[13].

To evaluate the real world MU-MIMO system performance, it is essential to recognize about the signal path between transmitter and receiver which is called as channel fading. Hence, different channel fading models have been explored to estimate the channel response between the BS and mobile users. Some of the channel fading models are well known Rayleigh, Rician, and Nakagami-m distributions [14]-[16]. Linear precoding techniques are analyzed with respect to spectrum efficiency and number of antennas under Rayleigh, Rician, and Nakagami-m channel models. In this paper, certain precoding techniques like ZF, MMSE and DPC have been analyzed over Rayleigh, Rician and Nakagami-m fading channels. To evaluate the MU-MIMO performance, bit error rate (BER) according to a certain signal to noise ratio (SNR) can be determined.

This paper is planned as follows: In Section II, MU-MIMO downlink channel model is introduced. Section III describes channel inversion and DPC precoding schemes. Then the fading channel models are described in Section IV. Finally, simulation results and conclusions are presented in Sections V and VI.

## II. MU-MIMO DOWNLINK CHANNEL MODEL

Consider MU-MIMO single cell system which consists of BS with  $N_b$  transmit antennas connect with  $M$  single antenna mobile users. Assume that the transmitter is having perfect CSI. MU-MIMO downlink Channel model in Fig. 1 denotes that an information vector  $\mathbf{s} \in N_b \times 1$  is precoded before transmission by precoding matrix  $\mathbf{Q} \in N_b \times N_b$  in the BS. The transmitted vector in the BS can be represented as  $\bar{\mathbf{s}} = \mathbf{Q}\mathbf{s}$  with  $\bar{\mathbf{s}} \in N_b \times 1$ . The received vector for  $M$  mobile users is specified by

$$\mathbf{y} = \mathbf{H} \bar{\mathbf{s}} + \mathbf{n} = \mathbf{H}\mathbf{Q}\mathbf{s} + \mathbf{n} \quad (1)$$

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where  $\mathbf{H} \in M \times N_b$  is the channel matrix between BS and M users, and  $\mathbf{n} \in M \times 1$  Gaussian noise with zero mean and unit variance. Note that the BS shares same time and frequency resource to mobile users. Notations: Matrix-vector quantities are denoted by Boldface letters. The operation  $\text{tr}(\cdot)$  and  $(\cdot)^H$  represents the trace and the Hermitian transpose of a matrix, respectively.

### III. DIFFERENT PRECODING TECHNIQUES

This section describes the linear channel inversion and DPC precoding techniques. These techniques are used to synchronize the transmitted information at the receiver.

#### A. Channel Inversion Precoding

Channel Inversion is a significant Precoding technique used to mitigate the interference occurring in the transmitted signal due to the multipath channel fading. In multi user MIMO ZF and MMSE channel inversion, precoding techniques are commonly used. ZF precoding is applied to suppress the interference among multiple users. This technique quashes the interference by executing pseudo-inverse of the channel gain matrix [17], [18]. It can be written as

$$\mathbf{Q}_{ZF} = \frac{1}{\sqrt{\alpha}} \mathbf{H}^H (\mathbf{H}\mathbf{H}^H)^{-1} \quad (2)$$

where  $\alpha$  is a constant of Wiener filter to achieve the transmitted power constraint after precoding and it is given in (3)

$$\alpha = \frac{N_b}{\text{tr}(\mathbf{Q}\mathbf{Q}^H)} \quad (3)$$

As in (2), this channel inversion precoding technique provides deprived performance for noise enhancement. In this situation, noise enhancement is reduced by MMSE criterion. MMSE precoding is created for minimizing mean square error (MSE) on the transmitted symbols which considers noise and interference process [18], [19]. Using the channel information at transmitter, the  $\mathbf{Q}_{MMSE}$  is given as

$$\mathbf{Q}_{MMSE} = \frac{1}{\sqrt{\alpha}} \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \sigma_n^2 \mathbf{I})^{-1} \quad (4)$$

Note that statistical information of noise is needed for the MMSE precoder. It performs better than the ZF precoder at the low SNR.

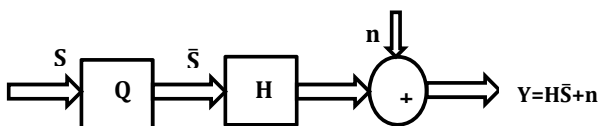


Fig. 1 MU-MIMO downlink channel model

#### B. DPC

One of the nonlinear precoding techniques is DPC, which can be realized when the transmitter is having complete CSI. Here m-th user received interference-free signal by cancelling the interference initiated by the (m-1) users before

transmitting the m-th user signal. LQ decomposition is used if the CSI is known at BS [11]. The channel gain matrix by LQ decomposition can be described as,

$$\mathbf{H} = \mathbf{L} * \mathbf{Q} \quad (5)$$

where  $\mathbf{L}$  is the unitary precoding matrix and  $\mathbf{Q}$  is the lower triangular matrix. Hence  $\mathbf{Q}_{DPC}$  (precoding matrix in DPC) is a scaled inverse matrix of  $\mathbf{Q}$  which is attained from the  $\mathbf{H}$ .

### IV. CHANNEL MODELLING

In cellular systems, the receiver receives the transmitted signal via multi path with distributed amplitudes and phases. The multipath signal will exhibit various fading effects based on the density of the scatter signal. Hence these multipath signals are statistically modeled namely Rayleigh distributions, Rician distributions, and Nakagami distributions. To model the dense scattered signals, Rayleigh and Nakagami distributions are employed, but for stronger line-of-sight signals are modeled by Rician distributions.

#### A. Rayleigh Fading Channel

When the channel contains many scatterers, Rayleigh fading can be a useful model in this situation. Here the received signal includes a number of copies of the original transmitted signal due to multipath transmission and there is no LOS communication between the transmitter and the receiver [14], [15]. This model assumes that summation of large number of statistically independent scattered paths with random amplitudes is an independent and identically distributed (i.i.d) complex Gaussian random variable.

The single element of channel gain matrix can be written as [19]

$$h_{ij} = a + jb = r \exp(j\phi) \quad (6)$$

where  $a, b \in \mathcal{N}\left(0, \frac{1}{\sqrt{2}}\right)$  are random variables. The probability density function (PDF) for Rayleigh fading channel is given in (7):

$$P_{RA}(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right), \quad r \geq 0 \quad (7)$$

#### B. Rician Fading Channel

When the strong direct signal component reaches at the receiver with small attenuation, Rayleigh fading model is not applicable. In such case, Rician fading channel can be used with the assumption of a LOS path between the transmitter and the receiver [15]. The PDF of Rician fading channel is expressed in (8):

$$P_{RC}(r) = \frac{r}{\sigma^2} \exp\left(\frac{-r^2 + A^2}{2\sigma^2}\right), \quad r \geq 0 \quad (8)$$

where the constant A is the strength of the direct component.

#### C. Nakagami-m Channel Fading

Nakagami distribution is regularly used to model the fading

distribution in communication channel where the Rayleigh distribution is failed to evaluate the channel activities over high frequencies and extended distances. Hence, there is no LOS condition between the transmitter and the receiver. Then, Nakagami-m fading channel PDF is described using parametric gamma distribution and it is given as

$$P_N(r) = \frac{r}{\sigma^2} \exp\left(\frac{-mr^2}{\sigma^2}\right) J_0\left(\frac{rA}{\sigma^2}\right) \quad r \geq 0, m \geq 1/2 \quad (9)$$

where  $r \geq 0$  is the amplitude of channel,  $J_0()$  is the 0<sup>th</sup> order modified Bessel function of the first kind and  $m$  is the Nakagami-m shape factor which includes both severe and weak fading. The parameter  $m$  describes degree of the fading propagation due to multipath interference. Nakagami-m distribution becomes Rayleigh and Rician distributions for  $m=0$  and  $m=1$  respectively.

V. SIMULATION RESULTS

This section presents the BER performance analysis of ZF, MMSE and DPC precoding techniques over different fading channels such as Rayleigh, Rician and Nakagami fading channels for MU-MIMO using MATLAB simulation. For simulation, Single cell MU-MIMO system is considered. Furthermore, we have considered BS consisting of 4 ( $N_b$ ) transmitting antennas, and assumed that total single antenna active users are 4 which are selected based on the highest norm values from 15 mobile users. Here 100 packets of data have been transmitted and each packet contains 20 frames. It is essential to note that Monte-Carlo simulation method has been applied to obtain the results. The key parameters BER and SNR are plotted to analyze the performance of the MU-MIMO system.

Fig. 2 shows the BER performance of ZF, MMSE and DPC precoders in Rayleigh fading channel. It shows that when BER decreases SNR will be increasing because signal is stronger than noise. Under the constraint of all the interference to be zero, ZF precoder is used to remove the interference. The noise term has to be considered in MMSE system. The DPC precoding technique gives better performance than the channel inversion techniques across SNR values.

In Fig. 3, error rate performance of precoders in Rician fading channel is depicted. Due to the assumption of LOS signal in Rician fading channel, the multiuser system performance is better than Nakagami fading channel.

Simulation results show that the ZF provides better performance at high SNR. MMSE provides the finest performance across SNR range.

TABLE I  
NUMERICAL BER VALUES AT 5DB

SNR at 5dB	BER values		
	ZF	MMSE	DPC
Rayleigh fading channel	0.248	0.141	0.1
Rician fading channel	0.294	0.178	0.118
Nakagami fading channel	0.356	0.257	0.069

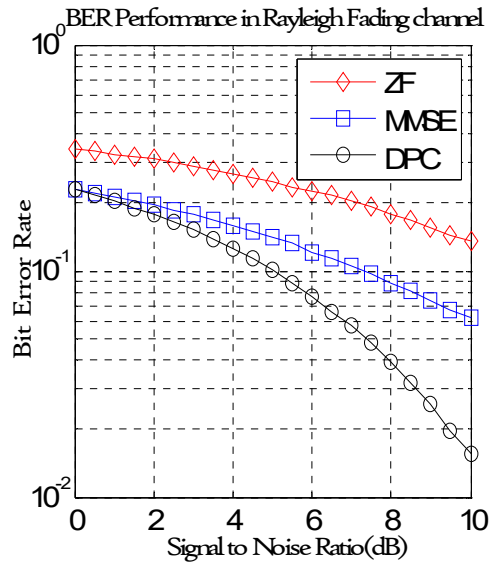


Fig. 2 Performance of precoding schemes in Rayleigh fading channel

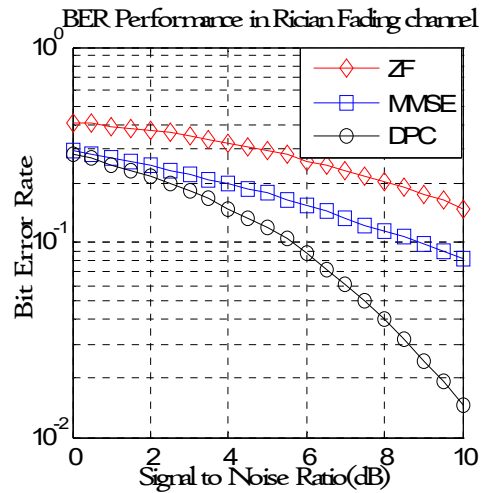


Fig. 3 Performance of precoding schemes in Rician fading channel

In Fig. 4, BER versus SNR is obtained for channel inversion and DPC precoders in Nakagami fading channel with  $m = 0.5$  was simulated. Here the performance of multiuser system is diminished compared to other fading channels. DPC provides better performance compared to channel inversion precoding schemes, but the implementation of DPC requires high complexity. However, channel inversion precoders have low complexity.

Fig. 5 yields BER comparison of ZF, MMSE and DPC precoders in different fading channels. It is observed that system performance is acceptable in presence of Rayleigh compared to Nakagami fading. Table I shows the numerical values of BER at 5 dB. It is perceived that the DPC non-linear precoder attained better performance than the channel inversion linear precoders in all the three fading channels. Among the linear precoding systems MMSE outperforms the ZF precoder.

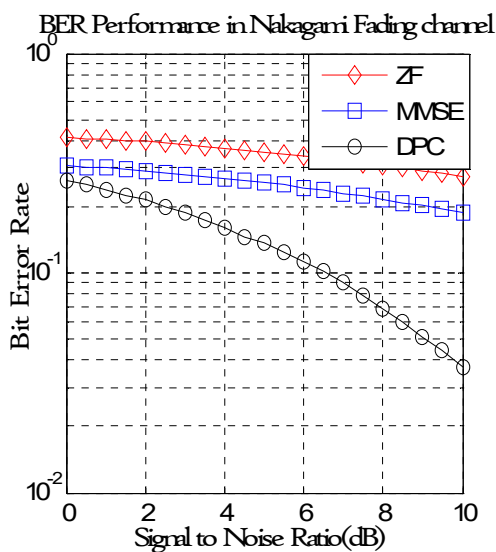


Fig. 4 Performance of precoding schemes in Nakagami fading channel

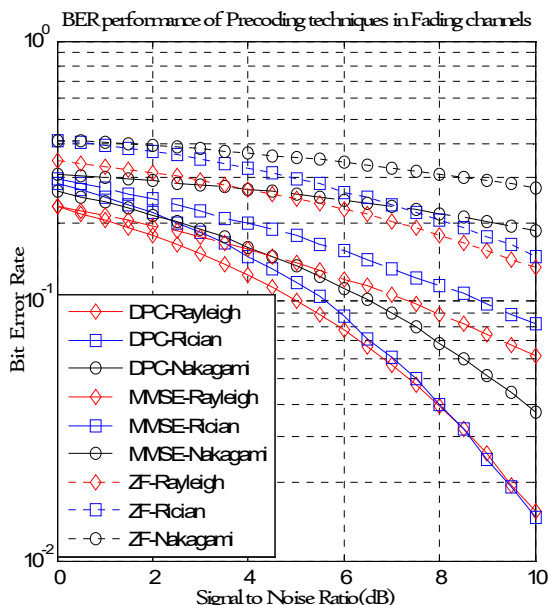


Fig.5 Performance of precoding schemes in different fading channel

VI. CONCLUSION

In this paper, the performance of precoding techniques were studied over Rayleigh, Rician and Nakagami fading channels in terms of BER versus SNR. It has been found that error rate is decreased by using different precoding schemes. The simulation results verify that the DPC scheme outperforms the channel inversion scheme over various channels. However, DPC is optimal and their high complexities make them challenging to implement in practical MU-MIMO. Channel inversions are one of the suboptimal methods with acceptable performance degradation and it is

easy to implement in practical systems because of its lower complexity. Moreover, compared to MMSE, ZF is a simple scheme. Also it is observed that the system performance reduces more in Nakagami channels than Rayleigh and Rician channels. Furthermore, MU-MIMO downlink performance in Rayleigh fading has better results than others.

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