

# Error Factors in Vertical Positioning System

Hyun-Gwang Cho, Wan-Seok Yang, Su-Jin Kim, Jeong-Seok Oh, Chun-Hong Park

**Abstract**—Machine tools are improved capacity remarkably during the 20th century. Improving the precision of machine tools are related with precision of products and accurate processing is always associated with the subject of interest.

There are a lot of the elements that determine the precision of the machine, as guides, motors, structure, control, etc. In this paper we focused on the phenomenon that vertical movement system has worse precision than horizontal movement system even they were made up with same components. The vertical movement system needs to be studied differently from the horizontal movement system to develop its precision. The vertical movement system has load on its transfer direction and it makes the movement system weak in precision than the horizontal one. Some machines have mechanical counter balance, hydraulic or pneumatic counter balance to compensate the weight of the machine head. And there is several type of compensating the weight. It can push the machine head and also can use chain or wire rope to transfer the compensating force from counter balance to machine head. According to the type of compensating, there could be error from friction, pressure error of hydraulic or pressure control error. Also according to what to use for transferring the compensating force, transfer error of compensating force could be occur..

**Keywords**—Chain chordal action, Counter balance, Setup error, Vertical positioning system..

## I. VERTICAL MOVEMENT SYSTEM

THE vertical movement system has load on its transfer direction than the horizontal one.

The gravity force which effects on the horizontal movement system does not affect at thrust force directly, but load at ball screw of vertical movement system are different when it moves upper side and down side [1],[2].

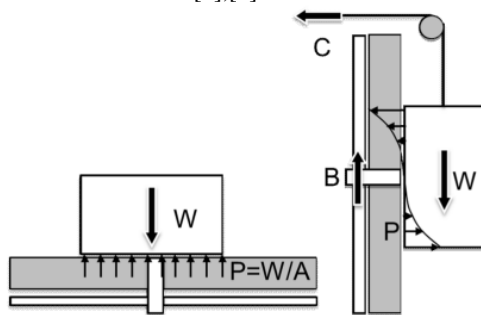


Fig. 1 Horizontal and vertical movement system

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The counter balance to compensate the weight of vertical table is used in heavy machine tools. There are mechanical counter weight and hydraulic counter weight. The advantage of counter weight is reducing the weight and moment of vertical table and the disadvantage of it is the poor dynamic characteristic. And the chain connecting vertical table and counter weight causes counter force transmission error [3].

Vertical table is supported by guide such as slide guide, rolling guide and hydro static guide. The traction force moving vertical table up and down is made by ball screw or linear motor. If the table is too heavy to be sustained by motor, counter balance is used to compensate the weight of the table. The mechanical counter weight is indirectly connected by chain that transmits counter force [4].

## II. FORCE AND MOMENT IN VERTICAL GUIDE

Fig. 3 shows guide, ball screw, machine head, counter balance and force, moment that works on it.

Machine tool has setup error of it and also has assembly errors in it. So the gravity force direction of vertical table is not always same to counter force direction.

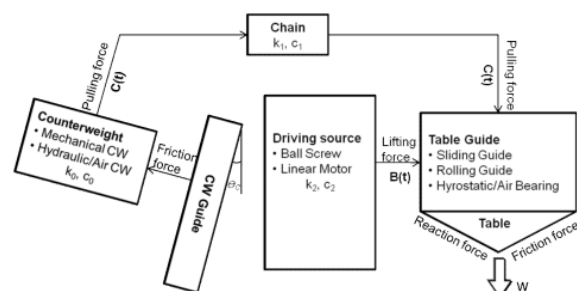


Fig. 2 Compensation system in vertical table

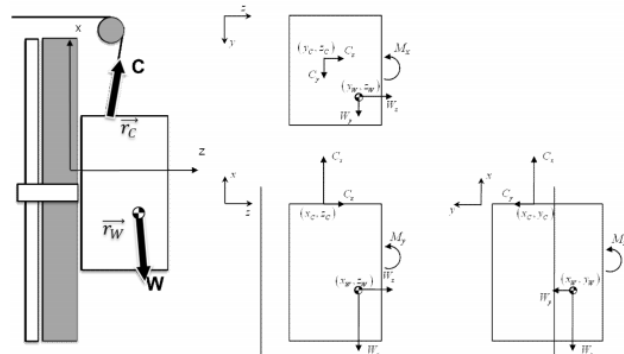


Fig. 3 Force and moment in vertical guide

If we define the moving direction  $x$  and normal direction  $y$  and  $z$ , the gravity force and counter weight force vector has not only  $x$  value but also  $y$  and  $z$  values. The acting point of gravity force is at the center of weight of vertical table and acting point of counter weight is at the connected point of head. In ideal case the acting line of counter force is parallel with gravity force passing the center of vertical table. But in general two force vectors are not parallel which cause the moment force. The net force and moment acting the vertical table of machine tool are sustained by guide and ball screw. The net force along  $x$  axis is sustained by ball screw and  $y$  and  $z$  axis are sustained by guide.

### III. COUNTER BALANCE

The table weight in vertical motion causes large axial load on ball screw. The counterweight is used to compensate the gravity force on vertical table of machine tools. Although vertical movement system is assembled with counterweight to increase their capacity and reliability, become less precise due to vibration [5],[6]. After the vertical table decelerates and stop at the desired point the remained vibration causes wavy marks on the work piece that reduce the surface quality. There are three kinds of counterweight: mechanical, hydraulic and pneumatic counterweight, which can be divided into two vibration models.

#### A. Vibration model

Counterweight is used to compensate the gravity force of vertical table. Although it is effective at static valance, the vibration is increased at dynamic case.

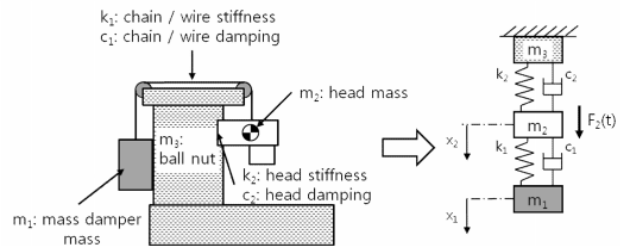
Adding one more component to the machine structure means dynamic problem needs to be reconsidered. And vibration analysis is coming up. By simplifying machine center as Lin's work [1], vibration model can be carried out. Ball nut is considered as a source getting acceleration  $a(t)$  and connected to machine head  $m_2$ . Machine head is connected to counterweight system  $m_1$  by chain or steel wire with stiffness  $k_1$  and damping  $c_1$ . Connection between the spindle and ball nut have stiffness  $k_2$  and damping  $c_2$ . Fig. 4 (a) shows the mechanical counterweight model and vibration equation is (2). Fig. 4 (b) presents the hydraulic or pneumatic counterweight model. Vibration equation in this case can be expressed as (2) with  $m_1 \ll m_2$ .

#### B. Vibration simulation by Matlab

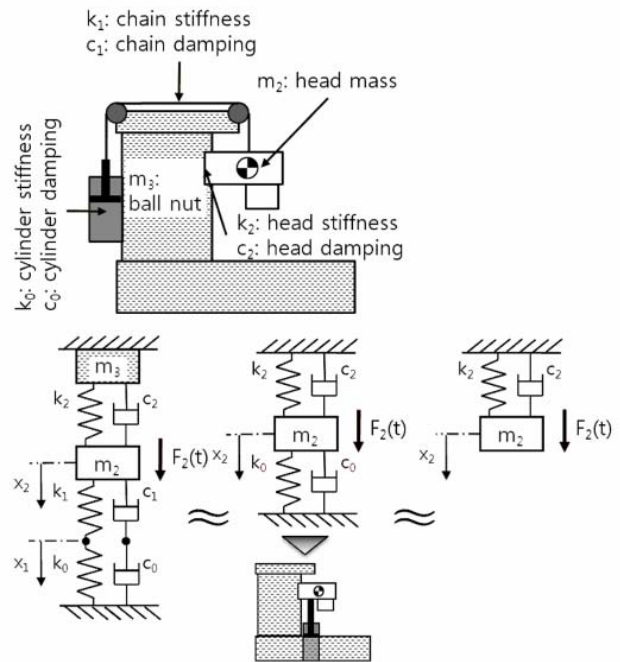
Fig. 5 introduces a schematic model built in matlab simulink. Initial parameters of this study case are Table I. One complete move of machine head to the target is normally divided in 3 stages: acceleration, constant velocity and deceleration. Fig. 6 expresses velocity and acceleration through time during one move of the ball nut, meanwhile  $a(t)$  in (3) is input as:

$$\begin{aligned} F &= W + C \\ M &= r_w \times W + r_w \times C \end{aligned} \quad (1)$$

$$\begin{aligned} M \ddot{x} + C \dot{x} + K x &= F(t) \\ \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1 + c_2 & c_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} \\ + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} &= \begin{bmatrix} 0 \\ 0 \\ a(t) \end{bmatrix} \end{aligned} \quad (2)$$



(a) Mechanical counterweight



(b) Hydraulic and Pneumatic counterweight

$$a(t) = \begin{cases} 2/3 & \text{if } 0 \leq t < 0.5 \\ 0 & \text{if } 0.5 \leq t < 1.5 \\ -2/3 & \text{if } 1.5 \leq t < 2.0 \end{cases} \quad (3)$$

TABLE I  
INITIAL PARAMETERS OF MODEL

Symbol	QUANTITY	Symbol	QUANTITY
m1	28kg	m2	38kg
k1	$1.15 \times 10^4$ (N/m)	k2	$9.86 \times 10^6$ (N.m)
c1	121 (Ns/m)	c2	0.1m

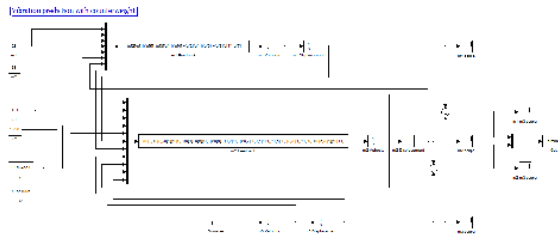


Fig. 5 Block diagram

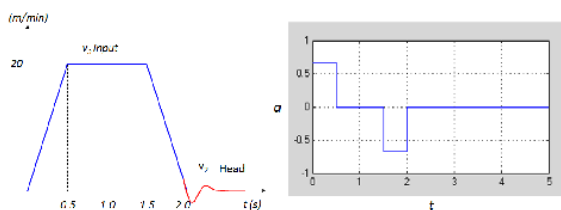
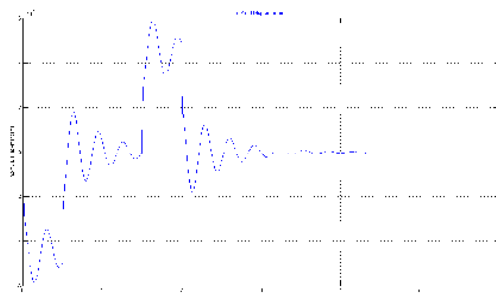
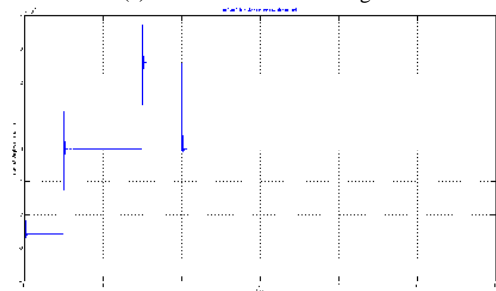


Fig. 6 Input velocity and acceleration



(a) Mechanical counterweight

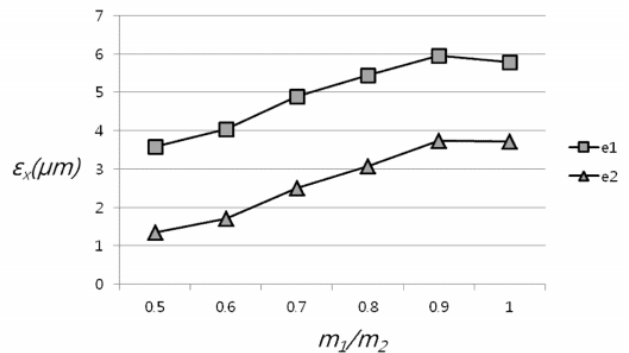


(b) Hydraulic and pneumatic counterweight, or without counterweight  
Fig. 7 Displacement of vertical table due to input of ball nut

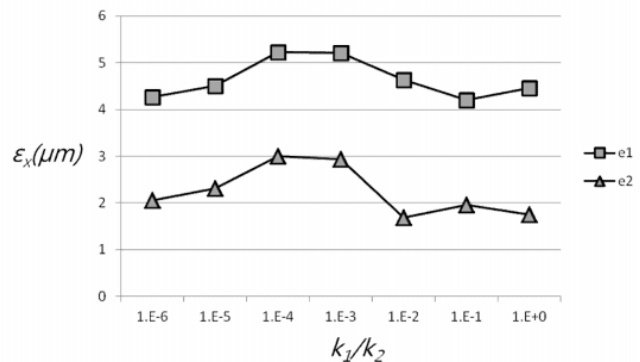
From that input condition displacement or position error between spindle head and ball nut position are plotted in Fig. 7. Fig. 7 (b) gives a better result compares to Fig. 7 (a), which mean hydraulic and pneumatic counterweight and non-counterweight are more reliable than mechanical counterweight in dynamic.

As mentioned above, after the head moves to the target point, it keeps moving up and down for a very short period before considered as permanent stop. Initial two upward errors are

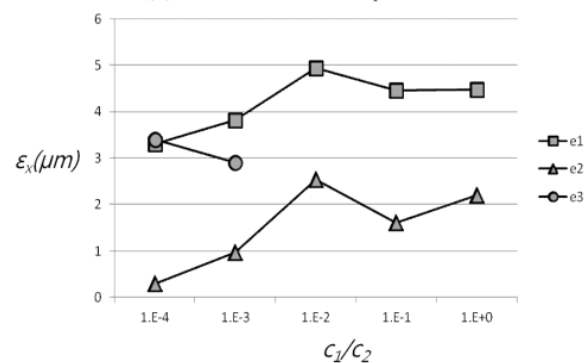
expressed in Fig. 8. Figs. 8(a), (b) and (c) gives the relation between position errors and weight ratio of  $m_1$  and  $m_2$ , stiffness ratio of  $k_1$  and  $k_2$ , damping ratio of  $c_1$  and  $c_2$ , respectively. The vertical dash lines are marked for the study case above. Fig. 8 (a) shows that the remained vibration of vertical table is enlarged as the weight of mechanical balance is increased. Vibration of mechanical counterweight and hydraulic or pneumatic counterweight system are predicted and evaluated. The mechanical balance makes the vertical table weak in vibration compared to hydraulic or without counterweight. The remained vibration of vertical table is enlarged as the weight of mechanical balance is increased.



(a) Weight ratio and amplitude



(b) Stiffness ratio and amplitude



(c) Damping ratio and amplitude

Fig. 8 Position errors after the vertical table reach the target

## IV. SETUP ERROR

In ideal case the acting line of counter force is parallel with gravity force passing the center of vertical table. But in general two force vectors are not parallel because of setting error of counter balance and it causes compensating force transfer error. So the gravity force and counter weight force vector has not only x value but also y and z values.

Fig. 9 shows the x, y, z elements of counter force when y, z vector are defined as  $\varepsilon_y$  and  $\varepsilon_z$ . The moments are simulated as the setting error changes when the tables are at different two positions and Table II show the initial value of it. Fig. 10 shows the variation of the counter force that works on vertical direction while the table is moving under each setting error. And the counter force error become large when the table moves to near the sprocket as shown in Fig. 11.

TABLE II  
INITIAL VALUE OF THE SIMULATION

Symbol	QUANTITY	Symbol	QUANTITY
W	(-1000, 0, 0)	$x_c$	0.3m
C	( $C_x, 0, C_z$ )	$x_w$	-0.2m
C	800N	$y_c$	0.1m
h	0.4m	$y_w$	-0.1m
L	1m	$z_c$	0.2m
		$z_w$	0.4m

W= weight , C=counter balance, h=initial height, L=travelling height, N=Newton, m = meter

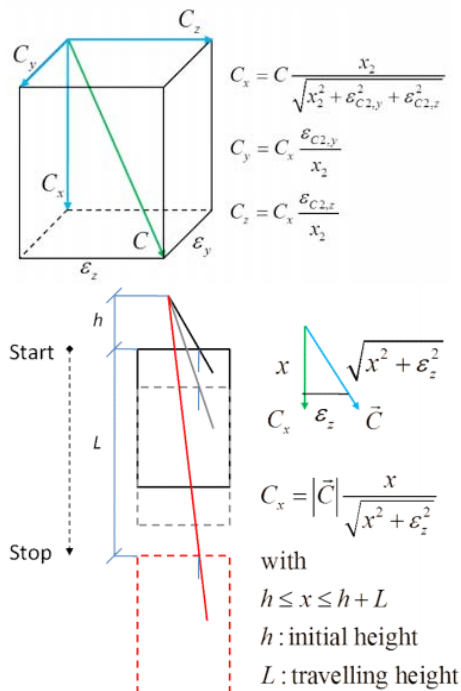


Fig. 9 Setup error

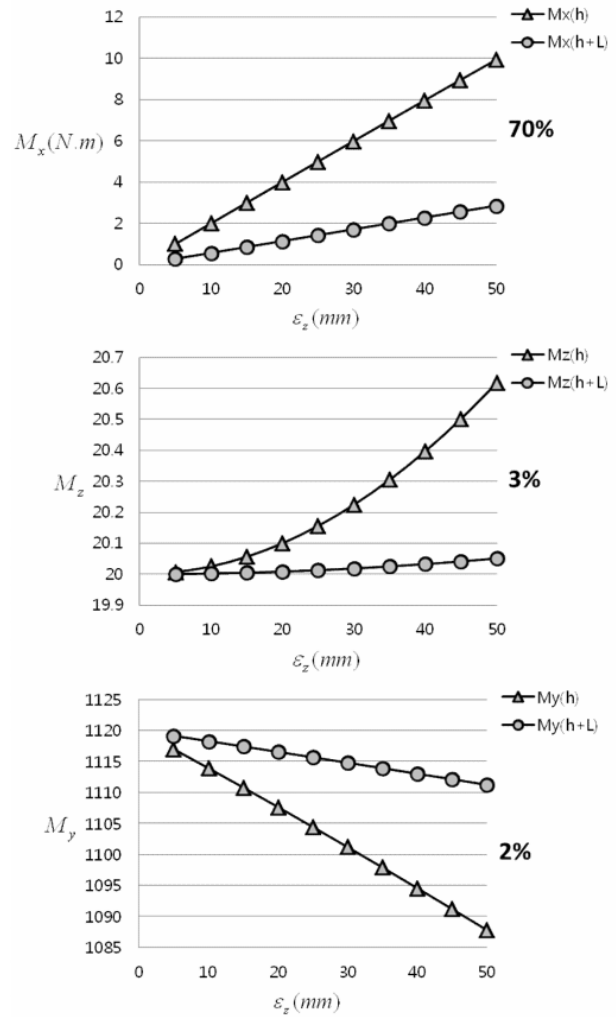


Fig. 10 Moment on the vertical table

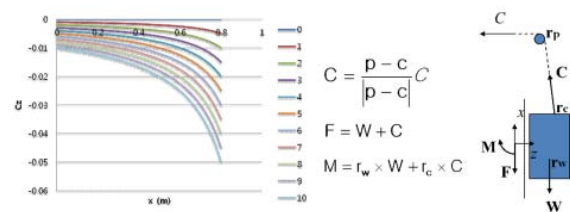


Fig. 11 Counter force errors because of setting error

## V.CHAIN

Mechanical, hydraulic or pneumatic counter balance is connected directly to the vertical system or indirectly connected by chain or wire loop that transmits counter force.

## A. The variation of pitching line

The pitching line of chain changes when the chain link with sprocket and when the chain departs from the sprocket as the sprocket rotates [7]. Fig. 12 shows the variation of the pitching line when the sprocket rotates. If we define the distance from the center to the knot of sprocket to  $r_P$ , the pitch radius of chain is

changed as  $r_p \cos \theta$  where the  $\theta$  is the rotation angle of the sprocket. The  $c$  function was defined instead of  $\cos$  function in (4) where the  $n$  is the count of sprocket teeth and  $\theta$  is changed from  $-\pi/n$  to  $\pi/n$  which is repeated if out of limit. The pitch radius is computed by defined  $c(\theta)$  function and the number of sprocket teeth.

#### B. The variation of transmission force

The chain passing a sprocket is shown in Fig. 14. The force ratio of left and right side of chain is changed because of the pitch radius changed by the number of teeth and the angle  $\delta$ . The pitch radius of chain passing both side of sprocket is changed and affects the transmission force. The setting angle is defined to  $\delta$  as shown in Fig. 14 and force ratio is computed by  $r_p c(\theta)$  and  $\delta$  in (5). The force ratio is shown in Fig. 15 when the number of teeth is 5 and setting angle is from 0 to  $(2\pi/n)$ .

The force transmission ratio is 1 when the setting angle is  $(2\pi/n)m$ . The force transmission error is maximized when the setting angle is  $(2\pi/n) \times (m+0.5)$ . The maximum error of force transmission ratio when the number of teeth is from 5 to 30 is shown in Fig. 16. It is decrease as the number of teeth is increase.

$$\begin{cases} c(\theta) = \cos(\theta) & \text{if } -\frac{\pi}{n} \leq \theta \leq \frac{\pi}{n} \\ c(\theta) = c\left(\theta - \frac{2\pi}{n}\right) & \text{others} \end{cases} \quad (4)$$

$$\frac{F_1}{F_2} = \frac{r_p c(\theta_2)}{r_p c(\theta_1)} = \frac{r_2}{r_1} = \frac{r_p c(\theta_1 - \delta)}{r_p c(\theta_1)} \quad (5)$$

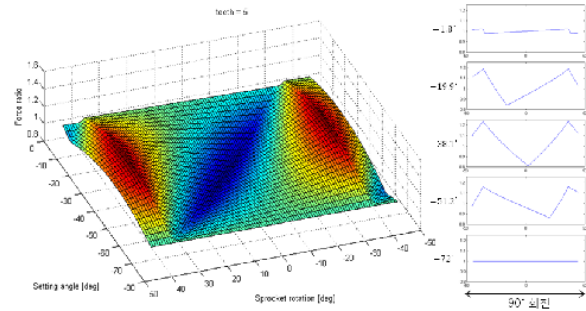


Fig. 15 Force ratio on a sprocket

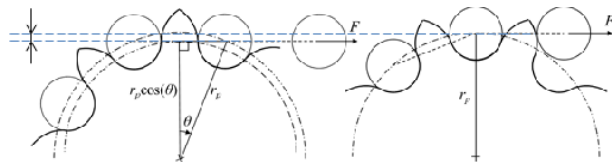


Fig. 12 Pitch line rise and fall

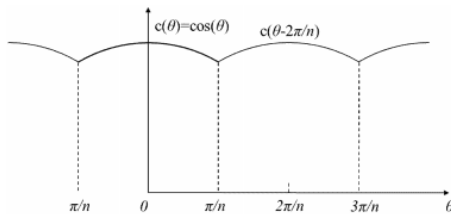


Fig. 13 Function  $c(\theta)$

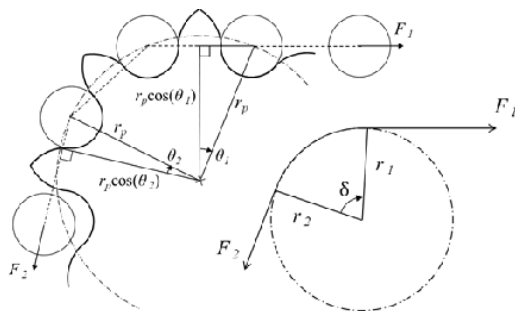


Fig. 14 Sprocket and chain

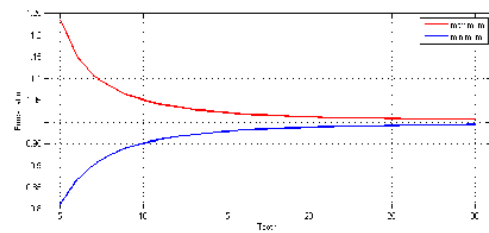


Fig. 16 Force ratio with teeth number

#### C. Chain with two sprockets

The variation of transmission force in case of single chain passing a sprocket has been studied. But machine tool uses two sprockets commonly to compensate the weight of vertical system as shown in Fig. 17 and the distance between two sprockets affects transmission error of counter force. Fig. 18 shows the force transmission ratio changed by the distance between the sprockets when the setting angle is  $90^\circ$  and two sprockets with 17 teeth are used.

The force transmission ratio is changed by the distance between two sprockets and it is repeated every pitches.

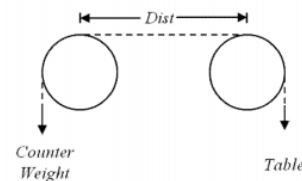


Fig. 17 Setting condition of a chain and two sprockets

TABLE III  
INITIAL VALUE FOR FORCE RATIO SIMULATION

Symbol	QUANTITY
Table weight	300kg
Counter weight	210kg
Center of weight	(0,0,0)mm
Counter pulling point	(300,10,20)mm
Measure point	(0,0,0)mm
Relative pulling position and setting error	(1000,10,-10)mm
Chain pitch	20mm
Sprocket teeth	17
Sprocket distance	500mm

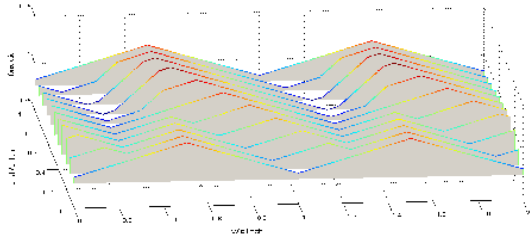


Fig. 18 Force ratio on two sprockets

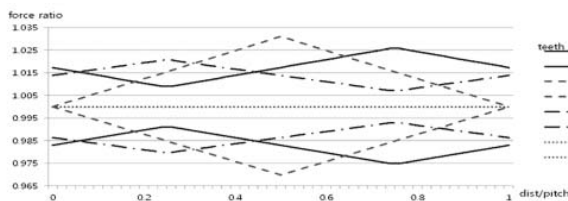


Fig. 19 Effect of teeth number and distance on Force ratio

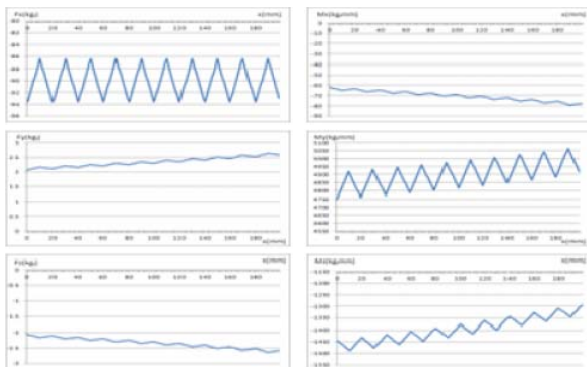


Fig. 20 Force & moment of vertical table

Fig. 19 shows the force ratio change when the teeth of sprocket are from 17 to 20 and other conditions are same with Fig. 17. The software that simulates counter force transmission error affected by setting error and chordal action of chain was developed. Table III is initial value used at the simulation and Fig. 20 shows the net force and moment at the vertical head of machine tools. The reason of counter force transmission error is the number of teeth of sprocket, the setting angle of chain and the distance between two sprockets in case of machine tools with counter balance connected by chain. The counter force transmission error could be eliminated by control upper three elements.

## VI. CONCLUSION

The vertical movement system has load on its transfer direction and it makes the movement system weak in precision than the horizontal one. The force and moment equation of vertical movement system was proposed to evaluate the reason that decreases precision. Counter balance used to compensate the weight of vertical system decries the dynamic characteristic of system. And the mechanical counter weight is weak to vibration compared to other counter weight. Positioning error increase as when weight is increased. Positioning error is effected by the stiffness and damping less than weight

The setting error of counter balance distorts the counter force. And counter force distortion that is changed while vertical system moves is predicted. The compensation force error occurs because of the chordal action of chain and sprocket when they are chosen to force transmission media. The force transmission error caused by the setting error of counter balance and the chordal action of chain and sprocket was simulated. And the error caused by the chordal action can be eliminated with specific design.

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