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# Empirical Mode Decomposition Based Denoising by Customized Thresholding

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***Abstract*****—**This paper presents a denoising method called EMD-Custom that was based on Empirical Mode Decomposition (EMD) and the modified Customized Thresholding Function (Custom) algorithms. EMD was applied to decompose adaptively a noisy signal into intrinsic mode functions (IMFs). Then, all the noisy IMFs got threshold by applying the presented thresholding function to suppress noise and to improve the signal to noise ratio (SNR). The method was tested on simulated data and real ECG signal, and the results were compared to the EMD-Based signal denoising methods using the soft and hard thresholding. The results showed the superior performance of the proposed EMD-Custom denoising over the traditional approach. The performances were evaluated in terms of SNR in dB, and Mean Square Error (MSE).

***Keywords*****—**Customized thresholding, ECG signal, EMD, hard thresholding, Soft-thresholding.

## I. INTRODUCTION

THE EMD method has been widely used for analyzing the nonlinear and non-stationary signals. The aim of the EMD method is to adaptively decompose any signal into oscillatory components called IMFs using a sifting process [1]. The signal reconstruction process is achieved by total sum of the IMFs and the residual. Then, EMD was used for signals denoising in wide range of applications such as biomedical signals and acoustic signals [2]-[9]. The denoising method can be based on the signal estimation using all the IMFs previously thresholded as in wavelet analysis [2], [3]. The noise components of a noisy signal are centered on the first IMFs (high-frequency IMFs), and the useful information of the signal is often concentrated on the last IMFs (low-frequency IMFs) [4]. Thereby, the denoising method can also be based on the partial construction of the signal using only the last relevant IMFs [4], [5]. Several EMD-Based denoising methods using thresholding were proposed in [6]. Indeed, it was shown that the direct application of wavelet thresholding to IMFs can lead to very bad results for the continuity of the reconstructed signal. The main factors affecting the quality of Wavelet Threshold Denoising are threshold and selection of the suitable wavelet threshold function. The hard threshold function does not change the local properties of the signal, but it can lead to some fluctuation in the reconstruction of the original signal. The hard threshold function leads to a loss of some high frequency coefficients above the threshold. In order to overcome the drawbacks of the classical threshold functions, Yoon and Vaidyajnathan proposed a customized thresholding function [10]. In this paper, we proposed a new customized thresholding function named EMD-Custom that can improve the results of soft and hard thresholding significantly. Numerical simulation and real data test were performed to evaluate this method, and the results were compared to EMD soft and hard threshold function in terms of SNR and MSE.

The paper is organized as follows. Section II introduces the EMD algorithm. Section III describes the EMD-Soft, EMD-Hard thresholding and the EMD-Custom thresholding. The simulation results are illustrated in section IV. Finally, Section V presents the conclusion.

## II. EMD ALGORITHM

EMD is an adaptive method to decompose a signal $x(t)$ into a series of IMFs. The IMFs must satisfy the following two conditions:

(i) The number of maximum must equal the number of zeros or differ at most by one.
(ii) In each period, it is necessary that the signal average is zero.

The EMD algorithm consists of the following steps [1]:

1. Find local maxima and minima in x(t) to construct the upper and lower envelopes respectively using cubic spline interpolation.
2. Calculate the mean envelope $m(t)$ by averaging the upper and lower envelopes.
3. Calculate the temporary local oscillation

$$h(t) = x(t) - h(t)$$

4. Calculate the average of $h(t)$, if average $h(t)$ is close to zero, then $h(t)$ is considered as the first IMF, named $c_i(t)$ otherwise, repeat steps (1)–(3) while using $h(t)$ for $x(t)$.
5. Calculate the residue $r(t) = x(t) - c_i(t)$.
6. Repeat steps from (1)-(5) using $r(t)$ for $x(t)$ to obtain the next IMF and residue.

The decomposition process stops when the residue $r(t)$ becomes a monotonic function or a constant that no longer satisfies the conditions of an IMF.

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$$x(t) = \sum_{i=1}^{N} c_i(t) + r_N(t) \quad (1)$$

## III. EMD Based Denoising

### A. EMD Soft Thresholding and EMD Hard Thresholding

Having a noisy signal $y(t)$ given by:

$$y(t) = x(t) + \eta(t) \quad (2)$$

where $x(t)$ is the noiseless signal and $\eta(t)$ is independent noise of finite amplitude. In EMD-Soft thresholding method, the noisy signal $y(t)$ was first decomposed into noisy IMFs $c_{ni}(t)$. These noisy IMFs was thresholded by soft or hard function in order to obtain an estimation of the noiseless IMFs $\hat{c}_i(t)$ of the noiseless signal. In this work, the universal threshold is used proposed in [11] and it identified as follows:

$$\tau_i = C\sqrt{E_i \, 2\ln(n)} \quad (3)$$

where $C$ is a constant depending of the type of signal that was set to 0.5 in this work, $n$ is the length of the signal and $E_i$ is given by [4]:

$$\hat{E}_k = \frac{E_1^2}{0.719} 2.01^{-k}, \; k = 2,3,4....N \quad (4)$$

where $E_1^2$ is the energy of the first IMF defined by:

$$E_1^2 = \left( \frac{median\ (|c_{n1}(t)|}{0.6745} \right)^2 \quad (5)$$

A direct application of wavelet soft thresholding [12] in the EMD case:

$$\hat{c}_i(t) = \begin{cases} c_{ni}(t) - \tau_i & \text{if } c_{ni}(t) \geq \tau_i \\ 0 & \text{if } |c_{ni}(t)| < \tau_i \\ c_{ni}(t) + \tau_i & \text{if } c_{ni}(t) \leq -\tau_i \end{cases} \quad (6)$$

A direct application of wavelet hard thresholding [12] in the EMD case:

$$\hat{c}_i(t) = \begin{cases} c_{ni}(t) & if \quad |c_{ni}(t)| > \tau_i \\ 0 & if \quad |c_{ni}(t)| \leq \tau_i \end{cases} \quad (7)$$

A reconstruction of the denoised signal is given by:

$$\hat{x}(t) = \sum_{i=1}^{N} \hat{c}_i(t) + r_N(t) \quad (8)$$

### B. EMD-Custom Thresholding

Based on [10], we defined a modified custom thresholding functions as follows:

$$\hat{c}_i(t) = \begin{cases} c_{ni}(t) - \text{sgn}(c_{ni}(t))[1-\alpha]\tau_i & \text{if } |c_{ni}(t)| \geq \tau_i \\ 0 & \text{if } |c_{ni}(t)| \leq \gamma \end{cases} \quad (9)$$

where $0 < \gamma < \tau_i$ and $0 \leq \alpha \leq 1$.

A reconstruction of the denoised signal is given by "(8)".

## IV. Simulation Results

In this section, we assess our proposed denoising algorithm compared to EMD-soft and EMD-hard denoising methods. The new EMD-Custom approach was applied to five test signals (Doppler, blocks, bumps, heavy sine, and piece-regular). The size of the signals was equal to 2048. The method was also tested on real ECG signal using the MIT-BIH database [13]. For simulated signals, the SNR before denoising was maintained at 15 dB. The original signals and the corresponding noisy versions are depicted in Figs. 1 and 2. The SNR before denoising of the real ECG signal was 20 dB. Each noisy signal was decomposed into IMFs using EMD process, and all IMFs are thresholded by the soft, hard, and new customized thresholding functions. The performance of the proposed method was affected by the choice of the $\alpha$ value. However, in order to obtain the best results, the parameter $\alpha$ has to be chosen appropriately as shown in Fig. 3 that depicts the SNR after denoising as function of $\alpha$. The $\alpha$ values for which the SNR after denoising are maximum are 0.5, 0.4, 0.1, 0.3, 0.2, 0.2 for ECG, bumps, heavy sine, Piece Regular, blocks, and Doppler signals, respectively. A comparative study with soft and hard thresholding methods considered in this work is presented in Tables I-VI. Clearly, the modified custom thresholding function gives the best estimates in terms of SNR and MSE for all test signals. Therefore, the proposed EMD-Custom outperforms totally the conventional EMD-Soft and EMD-Hard thresholding methods. Fig. 4 shows the denoising results of applying EMD-Soft and EMD-Custom to simulated signals. Fig. 5 displays the denoising results of real ECG signal using EMD-Soft and EMD-Custom. Fig. 6 illustrates the application of EMD to noisy ECG signal. Therefore, we conclude that our algorithm is, in general, able to remove noise from signals and it improves the results obtained by EMD hard thresholding and EMD soft thresholding.

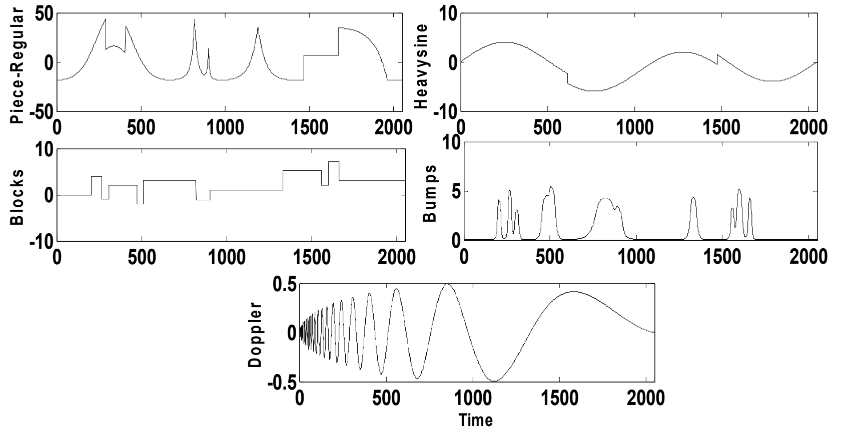


Fig. 1 Test signals with *n=2048*

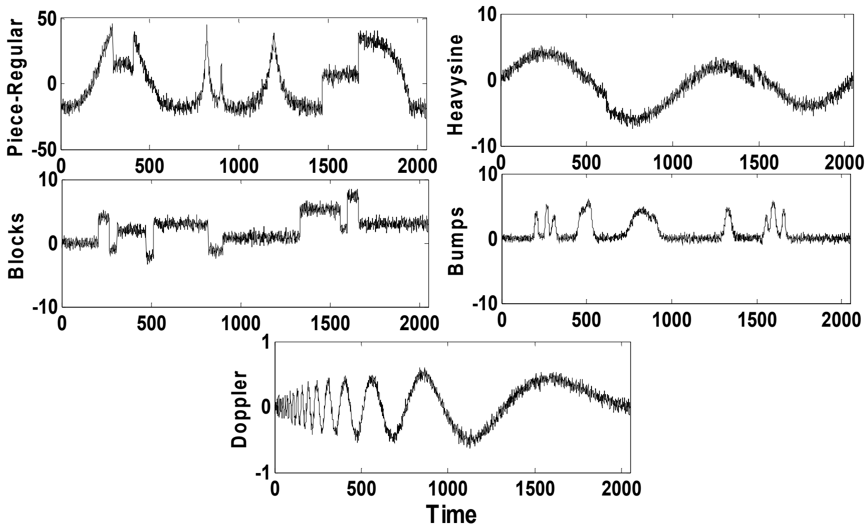


Fig. 2 Noisy test signals *SNR=15* dB

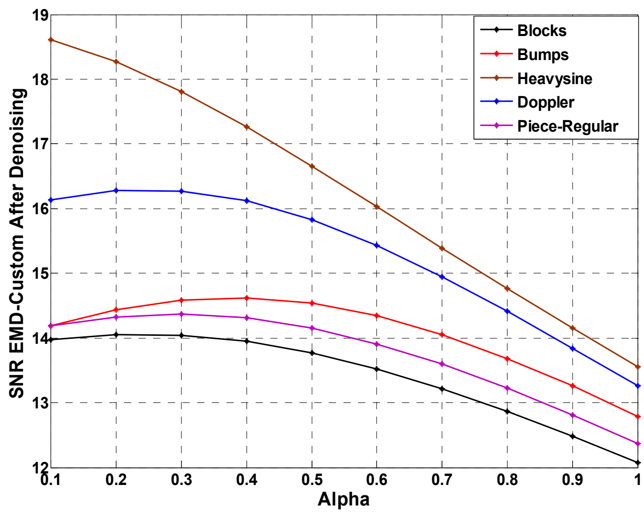


Fig. 3 Performance evaluation of EMD-Custom *SNR=5* dB

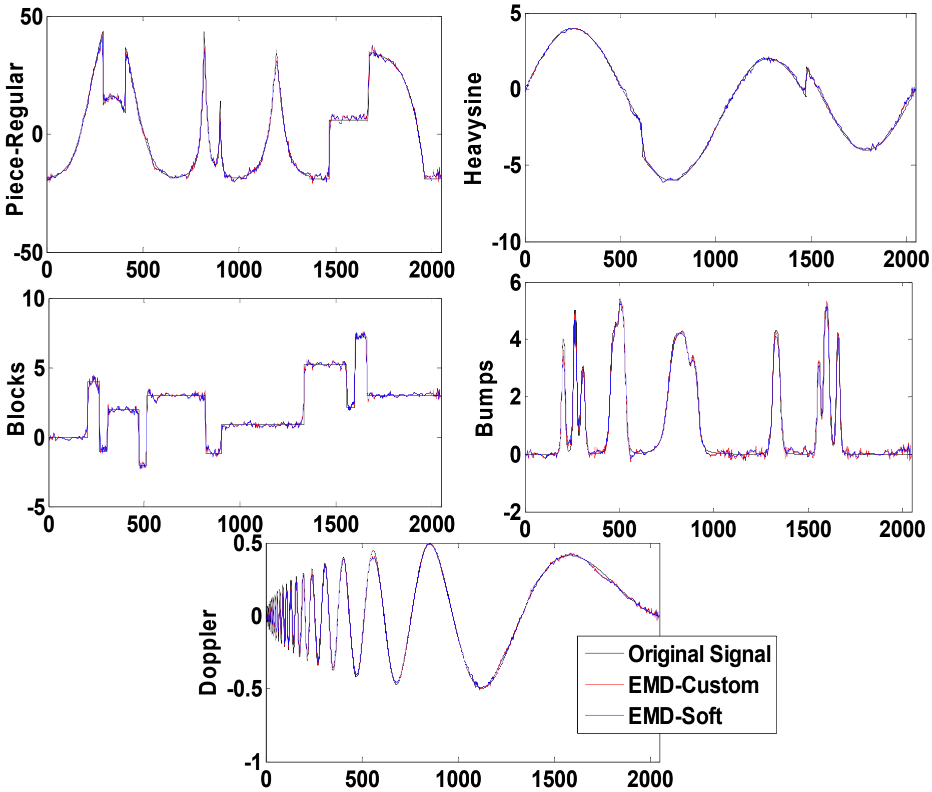


Fig. 4 Denoising results in *SNR =15* dB of test signals corrupted by Gaussian noise

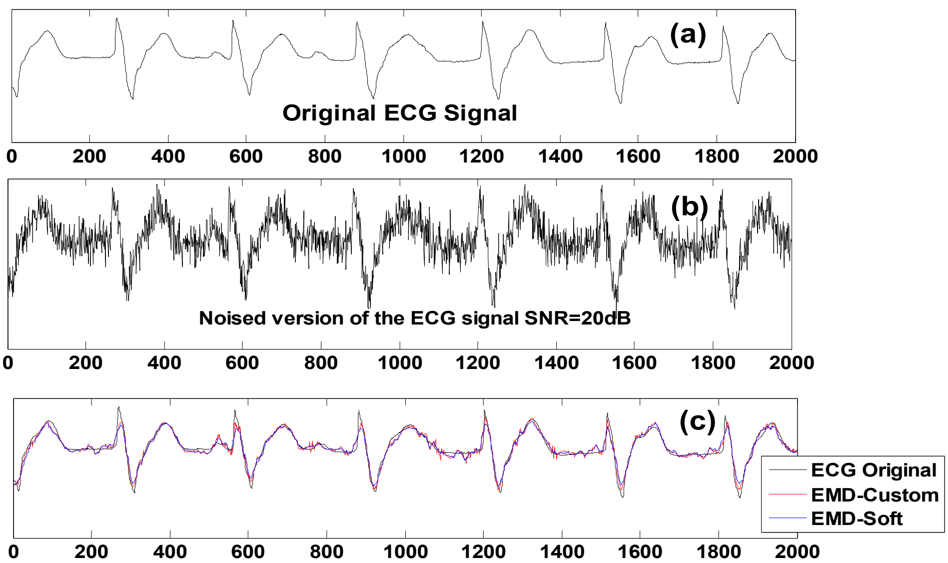


Fig. 5 Denoising results in SNR (20 dB) of real ECG signal

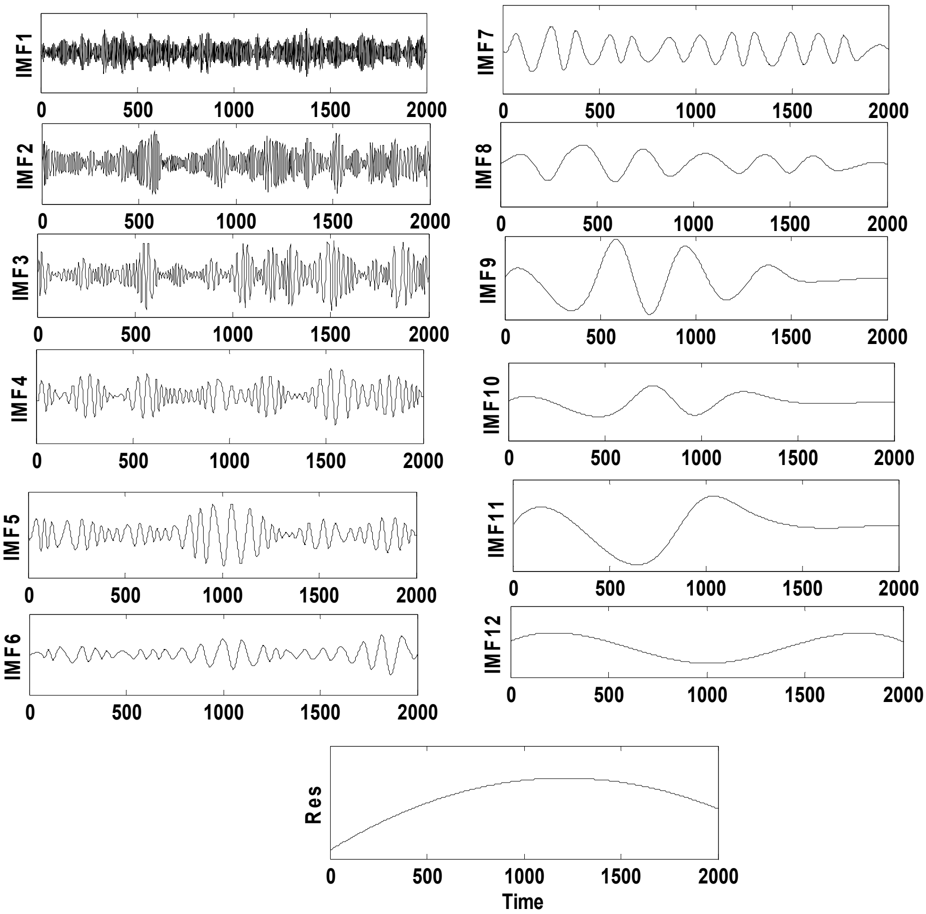


Fig. 6 EMD decomposition of the noisy ECG signal with SNR=20 dB

TABLE I
COMPARISONS OF *SNR* (AFTER DENOISING) VALUES AT $SNR = 5\, dB$

| Methods | SNR (dB) (after denoising) | | | | |
|---|---|---|---|---|---|
| | ***Blocks*** | ***Bumps*** | ***Heavy sine*** | ***Doppler*** | ***Piece-Regular*** |
| EMD-Soft | 13.8133 | 13.8488 | 18.7782 | 15.8444 | 13.9588 |
| EMD-Hard | 12.0748 | 12.7898 | 13.5555 | 13.2650 | 12.3659 |
| EMD-Custom | 14.04899 | 14.62062 | 18.80859 | 16.27758 | 14.36802 |

TABLE II
COMPARISONS OF *MSE* (AFTER DENOISING) VALUES AT $SNR = 5\, dB$

| Methods | ***MSE*** | | | | |
|---|---|---|---|---|---|
| | ***Blocks*** | ***Bumps*** | ***Heavy sine*** | ***Doppler*** | ***Piece-Regular*** |
| EMD-Soft | 0.3664 | 0.1335 | 0.1261 | 0.0022 | 0.1288 |
| EMD-Hard | 0.5468 | 0.1704 | 0.4198 | 0.0040 | 0.1859 |
| EMD-Custom | 0.34709 | 0.11181 | 0.12524 | 0.00202 | 0. 117272 |

TABLE III
COMPARISONS OF *SNR* (AFTER DENOISING) VALUES AT $SNR = 10\, dB$

| Methods | ***SNR (dB)(after denoising)*** | | | | |
|---|---|---|---|---|---|
| | ***Blocks*** | ***Bumps*** | ***Heavy sine*** | ***Doppler*** | ***Piece-Regular*** |
| EMD-Soft | 16.8143 | 17.4557 | 23.0320 | 18.7871 | 17.6147 |
| EMD-Hard | 16.0032 | 17.7700 | 17.9741 | 17.6755 | 16.6911 |
| EMD-Custom | 17.2527 | 18.8086 | 23.4599 | 19.5136 | 18.1645 |

TABLE IV
COMPARISONS OF *MSE* (AFTER DENOISING) VALUES AT $SNR = 10\,dB$

| Methods | MSE | | | | |
|---|---|---|---|---|---|
| | *Blocks* | *Bumps* | *Heavy sine* | *Doppler* | *Piece-Regular* |
| EMD-Soft | 0.1836 | 0.0582 | 0.0473 | 0.0011 | 0.0555 |
| EMD-Hard | 0.2213 | 0.0541 | 0.1517 | 0.0014 | 0.0686 |
| EMD-Custom | 0.1659 | 0.0426 | 0.0429 | 0.0009 | 0.0489 |

TABLE V
COMPARISONS OF *MSE* (AFTER DENOISING) VALUES AT $SNR = 15\,dB$

| Methods | *SNR (dB) (after denoising)* | | | | | |
|---|---|---|---|---|---|---|
| | *Blocks* | *Bumps* | *Heavy sine* | *Doppler* | *Piece-Regular* | *ECG* $SNR = 20dB$ |
| EMD-Soft | 20.1370 | 20.4058 | 26.4670 | 22.2368 | 21.6292 | 26.2561 |
| EMD-Hard | 20.0450 | 22.2445 | 22.8530 | 22.8495 | 21.3182 | 27.2952 |
| EMD-Custom | 20.72509 | 22.29713 | 26.34644 | 23.31444 | 22.41842 | 27.50995 |

TABLE VI
COMPARISONS OF *MSE* (AFTER DENOISING) VALUES AT $SNR = 15\,dB$

| Methods | *MSE* | | | | |
|---|---|---|---|---|---|
| | *Blocks* | *Bumps* | *Heavy sine* | *Doppler* | *Piece-Regular* |
| EMD-Soft | 0.0854 | 0.0295 | 0.0214 | 0.00051 | 0.0220 |
| EMD-Hard | 0.0872 | 0.0193 | 0.0493 | 0.00044 | 0.0236 |
| EMD-Custom | 0.07461 | 0.01909 | 0.02207 | 0.00040 | 0.01837 |

## V. CONCLUSION

In this paper, we proposed a new signal denoising method based on EMD and the modified custom thresholding function to suppress noise in the signal and improve the output SNR. The proposed method was tested on real ECG signal and simulated signals (Doppler, blocks, bumps heavy sine, and piece-regular) corrupted by white Gaussian noise. Based on SNR and MSE, simulation results show the advantages of the proposed EMD-Custom denoising method. We showed that the new approach is useful for removing noise and can improve the denoised results of soft and hard thresholding significantly.

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