

# Emission Constrained Economic Dispatch for Hydrothermal Coordination

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**Abstract**—This paper presents an efficient emission constrained economic dispatch algorithm that deals with nonlinear cost function and constraints. It is then incorporated into the dynamic programming based hydrothermal coordination program. The program has been tested on a practical utility system having 32 thermal and 12 hydro generating units. Test results show that a slight increase in production cost causes a substantial reduction in emission.

**Keywords**—Emission constraint, Hydrothermal coordination, and Economic dispatch algorithm.

## I. INTRODUCTION

THE economic dispatch of an electrical power system is the determination of the generation levels that minimizes the system total generation cost such that the system constraints are satisfied.

In the classical economic dispatch, a set of coordination equations is solved using the Lagrange multiplier. Equal incremental cost method is normally used to solve the coordination equations and the Lagrange multiplier is updated from an initial guess in order to make the total generation equal to the system demand plus losses. The constrained problem is normally solved by fixing any unit violating the maximum or minimum limit during iteration to that limit without dispatching it further. Fahmideh-Vojdani and Galiana [1] showed that this limit fixing technique may not yield an optimal solution and proposed a practical algorithm for quadratic cost function. Their algorithm decides which generation level to be fixed during iterations.

The solution method to dynamic programming based hydrothermal coordination problem includes the economic dispatch problem as a subproblem. Many economic dispatches, normally more than a thousand, need to be performed in the course of a 24 hours study. Therefore, the design of an efficient economic dispatch subroutine is crucial to the performance of a dynamic programming based hydrothermal coordination program.

In recent years, several economic emission dispatch strategies have been proposed [2]-[6]. The paper [7] suggests an efficient economic dispatch algorithm that handles nonlinear cost function and constraints, but the algorithm was

developed for a system which did not consider emission constraint.

In this paper, the formulation [7] has been modified to cope with emission constraint. It is then incorporated into the dynamic programming based hydrothermal coordination program [8]. The program has been tested on a practical utility system. Test results show that a slight increase in production cost reduces the emission substantially.

## II. PROBLEM FORMULATION

### A. Notations

The list of symbols used in this paper is as follows:

- $M$  - number of thermal units
- $T$  - number of periods for dividing the scheduling time horizon
- $i$  - Index of the thermal unit
- $t$  - time index
- $P$  - MW power output of a generating unit
- $\bar{P}$  - maximum MW power of a generating unit
- $\underline{P}$  - minimum MW power of a generating unit
- $D_t$  - demand in period  $t$
- $P_{loss_t}$  - transmission loss in period  $t$
- $C_i(\cdot)$  - production cost function of the  $i$ -th thermal unit
- $E_i(\cdot)$  - emission function of the  $i$ -th thermal unit
- $b_0, b_1, b_2$  - coefficients for production cost function
- $e_0, e_1, e_2$  - coefficients for emission function
- $B_0, B_1, B$  - transmission loss coefficients

### B. Objective function and constraints

Mathematically, the economic dispatch problem can be expressed as follows:

$$\sum_{i=1}^M C_i(P_{i,t}) \quad (1)$$

subject to the energy balance equation

$$\sum_{i=1}^M P_{i,t} = P_{loss_t} + D_t \quad (2)$$

and to the generation limits

$$\underline{P}_i \leq P_{i,t} \leq \bar{P}_i \quad (3)$$

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The new constraint function is the emission function. The total emission from the system is

$$\sum_{i=1}^M E_i(P_{i,t}) \quad (4)$$

### III. PROPOSED APPROACH

The cost objective function in (1) is then augmented by constraint (4) using Lagrange multiplier  $\omega$ , called the emission weighting factor, as follows

$$\min F = \sum_{i=1}^M [C_i(P_{i,t}) + \omega_i E_i(P_{i,t})] \quad (5)$$

For the optimization problem (5), (2) the necessary conditions for optimality are [9]

$$\frac{\partial F_i}{\partial P_{i,t}} = \lambda [1 - \frac{\partial Ploss_i}{\partial P_{i,t}}] \quad (6)$$

$$\sum_{i=1}^M P_{i,t} = Ploss_t + D_t \quad (7)$$

where  $\lambda$ , the Lagrange multiplier is an unknown value to be determined and  $\frac{\partial F_i}{\partial P_{i,t}}$  is known as the incremental cost function.

Thermal unit cost is represented as quadratic function of its generation. Sulfur oxide emissions are assumed proportional to each unit's cost curve. This is a reasonable assumption since such emissions are proportional to fuel consumption [10]. This leads to the following cost and emission functions

$$C_i(P_{i,t}) = b0_i + b1_i P_{i,t} + b2_i P_{i,t}^2$$

$$E_i(P_{i,t}) = e0_i + e1_i P_{i,t} + e2_i P_{i,t}^2$$

If we neglect the transmission loss ( $Ploss_t = 0$ ), (6) and (7) can be written as

$$b1_i + 2b2_i P_{i,t} + \omega_i e1_i + 2\omega_i e2_i P_{i,t} = \lambda \quad (8)$$

$$\sum_{i=1}^M P_{i,t} = D_t \quad (9)$$

From (8),

$$P_{i,t} = \frac{\lambda - b1_i - \omega_i e1_i}{2(b2_i + \omega_i e2_i)} \quad (10)$$

Putting the value of  $P_{i,t}$  from (10) in (9):

$$\sum_{i=1}^M \frac{\lambda - b1_i - \omega_i e1_i}{2(b2_i + \omega_i e2_i)} = D_t$$

or

$$\lambda = \frac{D_t + \sum_{i=1}^M \frac{b1_i + \omega_i e1_i}{2(b2_i + \omega_i e2_i)}}{\sum_{i=1}^M \frac{1}{2(b2_i + \omega_i e2_i)}} \quad (11)$$

Omitting generation limit constraints, (10) and (11) determine the analytical solution for the lossless case. The availability of an analytical solution to (8) and (9) substantially increases the efficiency of the classical algorithm [11].

If we consider transmission loss, the equations to be solved are as follows,

$$b1_i + 2b2_i P_{i,t} + \omega_i e1_i + 2\omega_i e2_i P_{i,t} = \lambda [1 - \frac{\partial Ploss_t}{\partial P_{i,t}}] \quad (12)$$

$$\sum_{i=1}^M P_{i,t} = Ploss_t + D_t \quad (13)$$

These are nonlinear equations of unknown variables  $P_{i,t}$  and  $\lambda$  and they can only be solved iteratively. Let  $\lambda^{old}$  and  $P_{i,t}^{old}$  be approximate solutions to (12) and (13). It is necessary to find new approximations

$$\lambda^{new} = \lambda^{old} + \delta\lambda$$

$$P_{i,t}^{new} = P_{i,t}^{old} + \delta P_{i,t}$$

Now, the transmission loss is represented by the following expression

$$Ploss_t = \sum B_{i,j} P_{i,t} P_{j,t} + \sum B1_i P_{i,t} + B0 \quad (14)$$

Using (14),  $\lambda^{new}$  and  $P_{i,t}^{new}$  are, to the first order, given by:

$$2(b2_i + \omega_i e2_i) P_{i,t}^{new} + b1_i + \omega_i e1_i = \lambda^{new} (1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i) - 2\lambda^{old} \sum B_{i,j} \delta P_{j,t} \quad (15)$$

$$\sum (1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i) P_{i,t}^{new} = Ploss_t^{old} + D_t - \sum (2 \sum B_{i,j} P_{j,t}^{old} + B1_i) P_{i,t}^{old} \quad (16)$$

In the classical solution algorithm, the last term in (15) as well the term  $(2 \sum B_{i,j} P_{j,t}^{old} + B1_i) (P_{i,t}^{new} - P_{i,t}^{old})$  i.e.  $(2 \sum B_{i,j} P_{j,t}^{old} + B1_i) \delta P_{i,t}$  from (16) are neglected. To retain the classical form and to improve convergence, it is possible to include only the  $i$ -th term of the summation, i.e.  $2\lambda^{old} B_{i,i} \delta P_{i,t}$ , or rather  $2\lambda^{old} B_{i,i} (P_{i,t}^{new} - P_{i,t}^{old})$ . We now have

$$2(b2_i + \omega_i e2_i + \lambda^{old} B_{i,i}) P_{i,t}^{new} + b1_i + \omega_i e1_i - 2\lambda^{old} B_{i,i} P_{i,t}^{old} = \lambda^{new} (1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i)$$

$$\sum (1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i) P_{i,t}^{new} = P_{loss_t}^{old} + D_t - \sum (2 \sum B_{i,j} P_{j,t}^{old} + B1_i) P_{i,t}^{old} \quad (17)$$

If we denote,

$$P_{i,t}^* = P_{i,t}^{new} (1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i)$$

$$b2_i^* = \frac{b2_i + \omega_i e2_i + \lambda^{old} B_{i,i}}{(1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i)^2}$$

$$b1_i^* = \frac{b1_i + \omega_i e1_i - 2 \lambda^{old} B_{i,i} P_{i,t}^{old}}{1 - 2 \sum B_{i,j} P_{j,t}^{old} - B1_i}$$

$$b^* = P_{loss_t}^{old} + D_t - \sum (2 \sum B_{i,j} P_{j,t}^{old} + B1_i) P_{i,t}^{old}$$

(17) can be written in the equivalent lossless form [1] as

$$2b2_i^* P_{i,t}^* + b1_i^* = \lambda^{new}$$

$$\sum P_{i,t}^* = b^*$$

We can now apply the technique [1] to fix the generation constraints (3).

IV. TEST RESULTS

This emission constrained economic dispatch algorithm has been coded in ‘C’ language for use in the dynamic programming based hydrothermal coordination program [8] developed in ‘C’ language too. The program has been tested on a practical utility system using the data of the generating units and system demands. The test system consists of 32 thermal and 12 hydro generating units, of which seven are gas turbine units. The total thermal capacity of the system is 3,640 Megawatt (MW) and the total hydro capacity is 848 MW. Numerical results presented here are based on three data sets: Case 1, Wednesday; Case 2, Saturday; Case 3, Sunday. The scheduling horizon is 24 hours in all cases. A summary of the system characteristics and parameters for these data sets is shown in Table 1.

TABLE I  
SUMMARY OF POWER SYSTEM

System characteristics	Number of units	Total capacity or requirements (MW)
Steam units	25	3360
Gas turbine units	7	280
Hydro units	12	848
All units	44	4488
Peak demand		2198
Minimum demand		1563
Maximum reserve		166

To avoid the voluminous amount of results, the complete solution process will be presented for Case 2 only. However summary of results of all cases will be presented at the end. For Case 2, using the hydrothermal coordination program without the emission constraint ( $\omega=0$  for all units), a schedule shown in Table 2 was suggested whose cost of operation is

\$2,024,321. The total emission from this schedule was 766.54 tons. The emissions from each unit are shown in Table 3. Emissions from units 1, 2, 9 and 10 were found to be quite high.

TABLE II  
SCHEDULE

(\* 1-unit is on; 0-unit is off. \*\* 1-nonzero generation; 0-zero generation)

Hour	Thermal Units *			Hydro Units **		
	000000000	111111111	222222222	333	00000001111	
	123456789	0123456789	0123456789	012	123456789012	
1	111111111	100000000	000000000	000	11111111111	
2	111111111	100000000	000000000	000	000011110000	
3	111111111	100000000	000000000	000	000011110000	
4	111111111	100000000	000000000	000	000011110000	
5	111111111	100000000	000000000	000	111111110000	
6	111111111	100000000	000000000	000	111111110000	
7	111111111	100000000	000000000	000	111111110000	
8	111111111	100000000	000000000	000	111111111111	
9	111111111	1001110111	000000000	000	111111111111	
10	111111111	1001111111	000000000	000	111111111111	
11	111111111	1111111111	000000000	000	111111111111	
12	111111111	1111111111	000000000	000	111111111111	
13	111111111	1111111111	000000000	000	111111111111	
14	111111111	1111111111	000000000	000	111111111111	
15	111111111	1111101111	000000000	000	111111111111	
16	111111111	1111100111	000000000	000	111111111111	
17	111111111	1001100111	000000000	000	111111111111	
18	111111111	1001100111	000000000	000	111111111111	
19	111111111	1001100111	000000000	000	111111111111	
20	111111111	1001100111	000000000	000	111111111111	
21	111111111	1001100111	000000000	000	111111111111	
22	111111111	1000000111	000000000	000	111111111111	
23	111111111	1000000000	000000000	000	111111111111	
24	111111101	1000000000	000000000	000	111111111111	

TABLE III  
EMISSIONS FROM THERMAL UNITS

Unit	Emission (tons)	Unit	Emission (tons)	Unit	Emission (tons)
1	115.23	12	6.54	23	0.00
2	116.85	13	11.33	24	0.00
3	34.42	14	11.45	25	0.00
4	34.25	15	6.77	26	0.00
5	33.67	16	6.84	27	0.00
6	32.34	17	12.67	28	0.00
7	32.55	18	12.7	29	0.00
8	32.81	19	13.78	30	0.00
9	123.55	20	0.00	31	0.00
10	122.43	21	0.00	32	0.00
11	6.36	22	0.00		

In order to reduce emissions from the above mentioned four units, hydrothermal coordination program was rerun using  $\omega=200$  for these particular units. The schedule obtained corresponds to an operating cost of \$2,034,143. The deviation of this schedule from the earlier one was as follows:

- Thermal unit 10 was not committed during hours 1 to 4
- Thermal units 17 and 18 were committed during hours 1 to 8
- All hydro units had nonzero generation during hours 2 to 4

The total emission from this schedule was 744.35 tons. The increase of \$9822 i.e. 0.49% in total cost in this run over the

previous one was due to inclusion of emission constraint. But this caused a reduction of 22.19 tons i.e. 2.89% in total emission. Cost and emission summary of all three cases are shown in Table 4.

TABLE IV  
COST AND EMISSION SUMMARY

Data Set	$\omega=0$ for all units		$\omega \neq 0$ for all units	
	Cost (\$)	Emission (tons)	Cost (\$)	Emission (tons)
Case 1	2,150,712	802.22	2,157,342	784.68
Case 2	2,024,321	766.54	2,034,143	744.35
Case 3	1,641,014	644.48	1,652,546	614.82

Emission constraint was enforced through a set of weighting factors. Higher emitters were given higher weighting factors to limit the emission to a greater extent. In the study system, only four units were found to be larger emitters of sulphur oxide. The emissions from them contribute to more than 60% of the total emissions. Hence, it is quite logical to use higher weighting factors for those units only while keeping the weighting factor equal to zero for other units as the emissions from them are quite low. The reduction of emission is done by shifting some loadings of these units to more expensive units thus resulting in higher operating cost. However, the percentage reduction in emission obtained is much higher than the percentage increase in operating cost.

#### V. CONCLUSIONS

An efficient economic dispatch algorithm for dealing with nonlinear functions such as the thermal cost, transmission loss and emission constraint is developed. It is then incorporated into the dynamic programming based hydrothermal coordination program. The program has been tested on a practical utility system using the data of the generating units and system demands. Emission weighting factor is varied to mitigate the impact of the emission of the corresponding unit. Higher emitters of sulphur oxide are given a higher weighting factor in order to limit the emissions to a greater extent. The reduction of emissions is accomplished by shifting some loading of these higher emitter units to more expensive units. This results in higher operating cost. However, it has been observed that a slight increase in production cost causes a substantial reduction in emission.

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