Electrical and Magnetic Modelling of a Power Transformer: A Bond Graph Approach

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Abstract— **Bond graph models of an electrical transformer including the nonlinear saturation are presented. The transformer using electrical and magnetic circuits are modelled. These models determine the relation between self and mutual inductances, and the leakage and magnetizing inductances of power transformers with two windings using the properties of a bond graph. The equivalence between electrical and magnetic variables is given. The modelling and analysis using this methodology to three phase power transformers can be extended.**

Keywords— Bond graph, electrical transformer, magnetic circuits, nonlinear saturation.

I. INTRODUCTION

TRANSFORMERS make large power systems possible. To transmit hundreds of megawatts of power efficiently over long distances. The main uses of electrical transformers are for changing the magnitude of an AC voltage providing electrical isolation, and matching the load impedance to the source [1].

Bond graphs were devised by H. Paynter at MIT in April 1959 and subsequently developed into a methodology together with Karnopp and Rosenberg. Early prominent promoters of bond graph modeling techniques among others were J. Thoma, van Dixhoorn and P. Dransfield.

On the other hand, a bond graph is a model of a dynamic system where a collection of components interact with each other through energy ports. A bond graph consist of subsystems linked by lines to show the energetic connections. It can represent a variety of energy types and can describe how the power flows through the system [2], [3].

In [4] a magnetic circuit model of power transformer which takes into account the nonlinear hysteresis phenomenon is analyzed. However, this paper uses a special nonlinear function to introduce the hysteresis.

In [6] a bond graph model of a transformer based on a nonlinear conductive magnetic circuit is described. Here, the state space nonlinear magnetic model has to be known.

Therefore, in this paper bond graph models of a transformer with two windings using an I-field and TF element are proposed. The relationship between these models allow to determine the self and mutual inductances equations in terms of leakage and magnetizing inductances of each winding. Moreover, bond graph models with I-field and TF elements of a transformer with three windings in order to obtain the relation between both models are proposed. Also, a basic electromagnetic model for the magnetizing branch of a transformer with two or three windings in the physical domain is described. This magnetizing branch consists of a resistor

and inductance. However, in order to introduce the magnetic saturation a nonlinear function is used.

The outline of the paper is as follows: Section II gives some basic elements of the modelling in bond graph. The magnetic circuits modelling in a bond graph approach is described in section III. Section IV summarizes the model of a two winding transformer including the flux linkage and voltage equations. A bond graph model of a transformer with two windings is proposed in section V. The magnetic modelling of the transformer in the physical domain in sectionVI. The two winding transformer considering the linear and nonlinear core are presented in section VII. Finally, section VIII gives the conclusions.

II. MODELLING IN BOND GRAPH

As it is common in other modelling methodologies first of all complex physical systems are partitioned into subsystems which in turn are decomposed further hierarchically top down to components of which the dynamic behavior is known or down to elements that represent physical processes. In bond graph modelling that decomposition is guided by the view that subsystems, components elements interact by exchanging power which is intuitive and essential in the bond graph modelling approach.

Consider the following scheme of a multiport LTI system which includes the key vectors of Fig. 1 [2], [7].
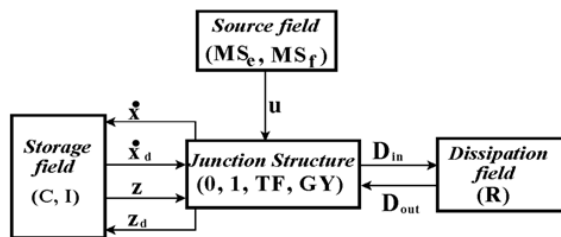

Fig. 1. Key vectors of a bond graph.

In Fig. 1, (MS_e, MS_f), (C, I) and (R) denote the source, the energy storage and the energy dissipation fields, and $(0, 1, TF, GY)$ the junction structure with transformers, TF, and gyrators, GY.

The state $x \in \Re^n$ and $x_d \in \Re^m$ are composed of energy variables p and q associated with I and C elements in integral and derivative causality, respectively, $u \in \Re^p$ denotes the plant input, $z \in \Re^n$ the co-energy vector, $z_d \in \Re^m$ the derivative co-energy and $D_{in} \in \Re^r$ and $D_{out} \in \Re^r$ are a mixture of e and f showing the energy exchanges between the dissipation field and the junction structure.

The Table1 gives the effort and flow variables for the direct formulation in some physical domains.

Table 1. Power variables in some domains.

Systems	Effort (e)	Flow (f)
Mechanical	Force (F)	Velocity (v)
	Torque (τ)	Angular velocity (ω)
Electrical	Voltage (v)	Current (i)
Hydraulic	Pressure (P)	Volume flow rate (Q)

The linear relations of the storage and dissipation field are,

$$z(t) = Fx(t) \tag{1}$$

$$z_d(t) = F_d x_d(t) \tag{2}$$

$$D_{out}(t) = LD_{in}(t) \tag{3}$$

The relations of the junction structure are [2], [7],

$$\begin{bmatrix} \dot{x}(t) \\ D_{in}(t) \\ z_d(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & 0 \\ S_{31} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ D_{out}(t) \\ u(t) \\ \dot{x}_d(t) \end{bmatrix} \tag{4}$$

The entries of S take values inside the set $\{0, \pm 1, \pm k_t, \pm k_g\}$ where k_t and k_g are transformer and gyrator modules; S_{11} and S_{22} are square skew-symmetric matrices and S_{12} and S_{21} are matrices each other negative transpose. The state equation is,

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{5}$$

where

$$A = (S_{11} + S_{12}MS_{21})F \tag{6}$$

$$B = S_{13} + S_{12}MS_{23} \tag{7}$$

$$E = I - S_{14}F_d^{-1}S_{31}F \tag{8}$$

being

$$M = (I - LS_{22})^{-1}L \tag{9}$$

It is very common in electrical power systems to use the electrical current as state variable of this manner taking the derivative of (1) and (5), we have

$$\dot{z}(t) = \overline{A}z(t) + \overline{B}u(t) \tag{10}$$

where

$$\overline{A} = FE^{-1}AF^{-1} \tag{11}$$

$$\overline{B} = FE^{-1}B \tag{12}$$

Next section summarizes the basic elements of an electrical transformer.

III. BOND GRAPH MODEL OF MAGNETIC SYSTEMS

Many useful electrical and electromechanical devices contain magnetic circuits. Multiport models for some of these devices have already been studied in previous section, but here the magnetic flux paths will be modelled in detail.

When considering magnetic circuits from the viewpoint of electronics, it is natural to regard magnetomotive force (mmf) as analogous to voltage and magnetic flux as analogous to current. This traditional pairing results in the reluctance-resistance analogy for modeling magnetic components. The mmf produced by an N-turn winding carrying a current of $i\{A\}$ is $F = Ni\{A\}$. Since N is dimensionless, the units of mmf are properly amperes, but ampere-turns are in common use. mmf is visualized as a forcelike quantity that pushes a magnetic flux Φ around the magnetic circuit. The unit of flux is the webber $\{Wb\}$ or volt-seconds $\{V \cdot s\}$. In a linear, lossless magnetic material, mmf and flux are proportional. Reluctance $\Re \{H^{-1}\}$ is then the couterpart of resistance $R\{\Omega\}$ in an electrical circuit: in a magnetic circuit, $\Phi = F/R$, corresponding directly to Ohm's law, $i = v/R$ [8].

Considering the relationships between electrical and magnetic variables is possible to convert a magnetic system to electrical system. However, this is not universally successful: if the magnetic circuit is nonplanar, the conversion procedure fails because, as proved by elementary graph theory, nonplanar networks have no dual.

A recently introduced technique retains the resistance model, linking it to the electrical circuit via a magnetic interface. The interface implements the pair of equations governing a winding:

$$v = N\frac{d\Phi}{dt}$$
$$F = Ni$$

Although it has served for many years, confidence in the traditional resistance model is undetermined by a simple question: magnetic reluctances store energy, so why are they made analogous to electrical resistances, which dissipate energy? The objection is particularly worrying in power electronics, where energy relations are of prime importance. At the root of the problem is the initial choice of mmf and flux as the natural magnetic circuit variables.

A generalized energy-based network, such as an electrical, hydraulic, or mechanical system, is characterized by effort variables and flow variables. The system variables are usually chosen so that when an effort and its corresponding flow are multiplied together, the result has the dimensions of power.

The product of voltage and current is power, in hydraulic and mechanical systems, the product of effort and flow variables is again power. However, with the conventional choice of mmf and flux as the magnetic system variables, the product of effort and flow is energy. For consistency with an electrical equivalent circuit, the product should be power, not its integral. The energy stored in a magnetic component should equal that stored in its equivalent circuit, but there is no energy stored in the resistance model.

In the late 1960's, Buntenbach proposed an alternative analogy in which mmf is retained as the magnetic effort variable, but the rate-of-change of flux $\left(d\Phi/dt = \dot{\Phi}\right)$ is chosen as the flow variable. It is convenient to call Φ the flux rate; its units are webbers/second or volts (per turn) $\{V\}$. The product of effort and flow variables is then power.

In the new system, magnetic flux is analogous to electric charge, not electric current. Just as voltage pushes charge around the electrical circuit causing a flow of current $(i = dq/dt)$, so mmf pushes flux around the magnetic circuit, causing a flow of flux rate $\left(\dot{\Phi} = d\Phi/dt\right)$. With the new variables, magnetic permeance becomes analogous to electrical

capacitance. This may be seen as follows: $\Phi = F/\Re = PF$, where $P\{H\} = 1/\Re$ is permeance; hence $\dot{\Phi} = PdF/dt$. This differential equation corresponds to that governing capacitance in an electrical circuit, $i = Cdv/dt$. The formula for calculating permeance, $P = \mu A/l$, also corresponds to that for capacitance, $C = \epsilon A/l$. A magnetic structure may be represented by a topologically similar capacitance model, each lumped permeance of $P\{H\}$ corresponding to a capacitance of $C\{F\}$. Thus, the alternative analogous are summarized in Table 2 [2], [8].

Table 2. Alternative analogs.

Magnetic		Electrical	
mmf	F	Voltage	v
Flux rate	$\dot{\Phi}$	Current	i
Permeance	P	Capacitance	C
Flux	$\Phi = \int \dot{\Phi} dt$	Charge	$q = \int i dt$
Power	$P = F\dot{\Phi}$	Power	$P = vi$

A distinctive feature of Buntenbach's approach is the way in which windings are treated. A winding may be through of as a two-port element that links the electrical and magnetic circuits. An N-turn winding relates variables v and i at the electrical port to variables F and $\dot{\Phi}$ at the magnetic port:

$$v = N\dot{\Phi} \tag{13}$$
$$i = F/N \tag{14}$$

Thus, the electrical effort variable v is proportional to the magnetic flow variable $\dot{\Phi}$, while the electrical flow variable i is proportional to the magnetic effort variable F. A relationship where effort and flow are exchanged is characteristic of a gyrator, which is shown in Fig. 2.

IV. MODEL OF A TWO-WINDING TRANSFORMER

Charles P. Steinmetz (1865-1923) developed the circuit model that is universally used for the analysis of iron core transformers at power frequencies. His model has many advantages over those resulting from straightforward application of linear circuit theory, primarily because the iron core exhibits saturation and hysteresis and is thus definitely nonlinear [1]. However it is good idea to consider transformers first from the point of view of basic linear circuit theory to better appreciate the Steinmetz model.

A. Flux Linkage Equations

Consider the magnetic coupling between the primary and secondary windings of a transformer shown in Fig. 2 [5].
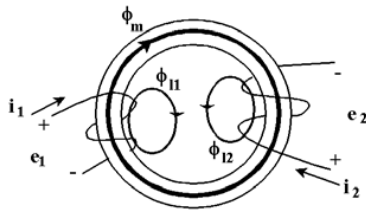

Fig. 2. Magnetic coupling of a two-winding transformer.

The total flux linked by each winding may be divided into two components: a mutual component, ϕ_m, that is common to both windings, and a leakage flux components that links only the winding itself. In terms of these flux components, the total flux by each of the windings can be expressed as,

$$\phi_1 = \phi_{l1} + \phi_m \tag{15}$$
$$\phi_2 = \phi_{l2} + \phi_m \tag{16}$$

where ϕ_{l1} and ϕ_{l2} are the leakage flux components of windings 1 and 2, respectively. Assuming that N_1 turns of winding 1 effectively link ϕ_m and ϕ_{l1}, the flux linkage of winding 1 is defined by,

$$\lambda_1 = N_1\phi_1 = N_1(\phi_{l1} + \phi_m) \tag{17}$$

the leakage and mutual fluxes can be expressed in terms of the winding currents using the magneto-motive forces (mmfs) and permeances. So, the flux linkage of winding 1 is,

$$\lambda_1 = N_1[N_1i_1P_{l1} + (N_1i_1 + N_2i_2)P_m] \tag{18}$$

where $P_{l1} = \dfrac{\phi_{l1}}{N_1i_1}$ and $P_m = \dfrac{\phi_m}{N_1i_1 + N_2i_2}$.

Similarly, the flux linkage of winding 2 can be expressed as,

$$\lambda_2 = N_2(\phi_{l2} + \phi_m) \tag{19}$$

and using mmfs and permeances for this winding,

$$\lambda_2 = N_2[N_2i_2P_{l2} + (N_1i_1 + N_2i_2)P_m] \tag{20}$$

The resulting flux linkage equations for the two magnetically coupled windings, expressed in terms of the winding inductances are,

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{21}$$

where L_{11} and L_{22} are the self-inductances of the windings, and L_{12} and L_{21} are the mutual inductances between them.

Note that the self-inductance of the primary can be divided into two components, the primary leakage inductance, L_{l1} and the primary magnetizing inductance, L_{m1}, which are defined by,

$$L_{11} = L_{l1} + L_{m1} \tag{22}$$

where $L_{l1} = N_1^2P_{l1}$ and $L_{m1} = N_1^2i_1P_m$.

Likewise, for winding 2

$$L_{22} = L_{l2} + L_{m2} \tag{23}$$

where $L_{l2} = N_2^2P_{l2}$ and $L_{m2} = N_2^2i_2P_m$.

Finally, the mutual inductance is given by,

$$L_{12} = N_1N_2i_2P_m \tag{24}$$
$$L_{21} = N_1N_2i_1P_m \tag{25}$$

Taking the ratio of L_{m2} a L_{m1},

$$L_{m2} = \frac{N_2\phi_m}{i_2} = \frac{N_2L_{12}}{N_1} = N_2^2P_m = \left(\frac{N_2}{N_1}\right)^2 L_{m1} \tag{26}$$

B. Voltage Equations

The induced voltage in winding 1 is given by,

$$e_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} \qquad (27)$$

replacing L_{11} by $L_{l1} + L_{m1}$ and $L_{12}i_2$ by $N_2 L_{m1} i_2/N_1$, we obtain

$$e_1 = L_{l1}\frac{di_1}{dt} + L_{m1}\frac{d\left(i_1 + (N_2/N_1)\,i_2\right)}{dt} \qquad (28)$$

Similarly, the induced voltage of winding 2 is written by,

$$e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{d\left(i_2 + (N_1/N_2)\,i_1\right)}{dt} \qquad (29)$$

Finally, the terminal voltage of a winding is the sum of the induced voltage and the resistive drop in the winding, the complete equations of the two windings are,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 i_1 \\ r_2 i_2 \end{bmatrix} + \begin{bmatrix} L_{l1} + L_{m1} & a^{-1}L_{m1} \\ aL_{m2} & L_{l2} + L_{m2} \end{bmatrix} \begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix} \qquad (30)$$

where $a = N_1/N_2$.

Next section a bond graph model of a transformer with two windings is proposed.

V. BOND GRAPH MODEL OF A TRANSFORMER WITH TWO WINDINGS

The bond graph methodology allows to model a system in a simple and direct manner. Using fields and junction structures, one may conveniently study systems containing complex multiport components using bond graphs. In fact, bond graphs with fields prove to be a most effective way to handle the modeling of complex multiport systems [2]. In Fig 3 shows a equivalent circuit for a two winding transformer using an I-field that represents a transformer with two windings and taking account the self and mutual inductances.
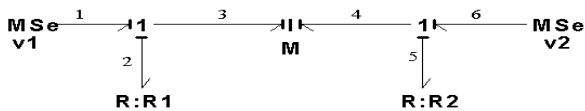
Fig. 3. Bond Graph of a two windings transformer using I-field.

The key vectors of the bond graph are,

$$x = \begin{bmatrix} p_3 \\ p_4 \end{bmatrix}; \dot{x} = \begin{bmatrix} e_3 \\ e_4 \end{bmatrix}; z = \begin{bmatrix} f_3 \\ f_4 \end{bmatrix} \qquad (31)$$

$$D_{in} = \begin{bmatrix} f_2 \\ f_5 \end{bmatrix}; D_{out} = \begin{bmatrix} e_2 \\ e_5 \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_6 \end{bmatrix}$$

the constitutive relations of the fields are,

$$L = diag\{R_1, R_2\} \qquad (32)$$

$$F^{-1} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \qquad (33)$$

and the junction structure is,

$$S_{21} = S_{13} = -S_{12} = I_2; S_{11} = S_{22} = S_{23} = 0 \qquad (34)$$

From (11), (12), (32), (33) and (34) the state space representation is,

$$\dot{z} = \frac{1}{\Delta}\begin{bmatrix} L_{22} & -L_{12} \\ -L_{12} & L_{11} \end{bmatrix}\left\{\begin{bmatrix} -R_1 & 0 \\ 0 & -R_2 \end{bmatrix}z + \begin{bmatrix} e_1 \\ e_6 \end{bmatrix}\right\} \qquad (35)$$

Now, a bond graph model of a transformer with two windings using leakage inductance, L_l and the magnetizing inductance, L_m in each winding is proposed in Fig 4.
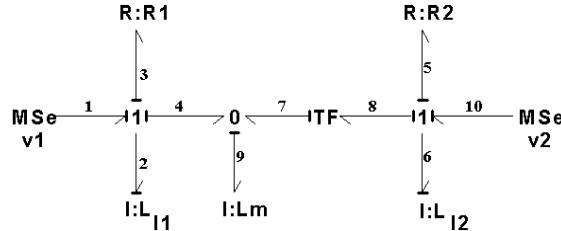
Fig. 4. Bond graph of a two windings transformer using leakage and magnetizing inductances.

The key vectors of the bond graph are,

$$x = \begin{bmatrix} p_2 \\ p_6 \end{bmatrix}; \dot{x} = \begin{bmatrix} e_2 \\ e_6 \end{bmatrix}; z = \begin{bmatrix} f_2 \\ f_6 \end{bmatrix} \qquad (36)$$

$$D_{in} = \begin{bmatrix} f_3 \\ f_5 \end{bmatrix}; D_{out} = \begin{bmatrix} e_3 \\ e_5 \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_{10} \end{bmatrix}$$

$$x_d = p_9; \dot{x}_d = p_9; z_d = f_9$$

the constitutive relations of the fields are,

$$L = diag\{R_1, R_2\} \qquad (37)$$

$$F^{-1} = diag\{L_{l1}, L_{l2}\} \qquad (38)$$

$$F_d^{-1} = L_m \qquad (39)$$

and the junction structure,

$$S_{21} = -S_{12} = S_{13} = I_2; S_{14}^T = \begin{bmatrix} -1 & -a^{-1} \end{bmatrix}$$

$$S_{11} = S_{22} = S_{23} = 0 \qquad (40)$$

From (8), (38), (39) and (40) the relationship between the storage field in integral causality and the storage element in derivative causality is,

$$E = \begin{bmatrix} 1 + \dfrac{L_m}{L_{l1}} & \dfrac{L_m}{aL_{l2}} \\ \dfrac{L_m}{aL_{l1}} & 1 + \dfrac{L_m}{a^2 L_{l2}} \end{bmatrix} \qquad (41)$$

The state matrix of this system is given by,

$$\overline{A} = \frac{1}{\Delta_1}\begin{bmatrix} L_{l2} + \dfrac{L_m}{a^2} & \dfrac{-L_m}{a} \\ \dfrac{-L_m}{a} & L_{l1} + L_m \end{bmatrix}\begin{bmatrix} -R_1 & 0 \\ 0 & -R_2 \end{bmatrix} \qquad (42)$$

where $\Delta_1 = L_{l1}L_{l2} + L_{l1}L_m a^{-2} + L_m L_{l2}$ and

$$\overline{B} = E^{-1}F I_2 \qquad (43)$$

Note that (42) is the same result considering the self and mutual inductances.

If we use the following numerical values of the parameters of the transformer, the simulation of the two bond graph models can be compared.

The numerical values of the parameters of the bond graph of Fig. 4 are $L_{l1} = 1.59mH$, $L_{l2} = 6.34mH$, $L_m = 31.9mH$, $R_1 = 4\Omega$, $R_2 = 16\Omega$, $a = 10$, and comparing (35) with (42) and (43) yields $L_1 = 33.49mH$, $L_2 = 6.659mH$ and $L_{12} = 3.19mH$. The voltage sources are: $V_1 = 20\sin(377t)$ and $V_2 = 60\sin(377t)$. Also, the simulation of both bond graph is shown in Fig. 5.
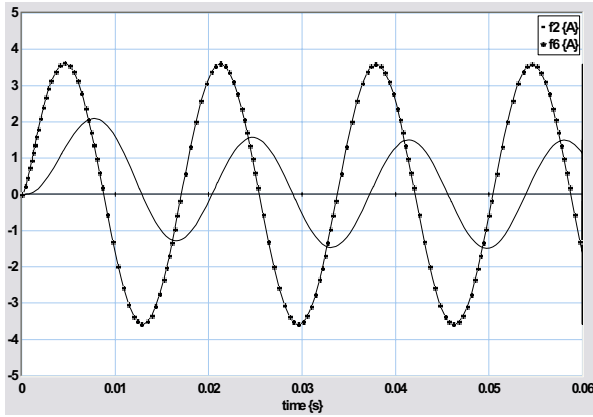

Fig. 5. Simulation of a transformer with two windings.

The electrical currents of the primary and secondary windings are the same by simulating the bond graph with I-field of Fig. 3 or the bond graph using leakage and magnetizing inductances of Fig. 4. Also, the bond graph with I-field is easier than the bond graph with individual inductances. However, the bond graph of Fig. 4 gives the information of the magnetizing section. Fig. 6 shows the linkage flux, voltage and electrical current of the linear magnetizing inductance.
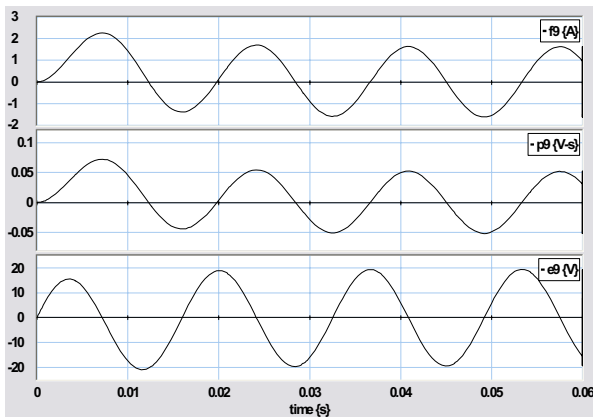

Fig. 6. Magnetizing electrical current, linkage flux and voltage.

Next section proposes a two windings transformer including the linear and nonlinear core in the physical domain.

VI. THE TRANSFORMER WITH A MAGNETIC BOND GRAPH

The description of the single phase two winding transformer presented in the previous section is considered. The bond graph of the transformer using magnetic circuits is shown in Fig. 7.
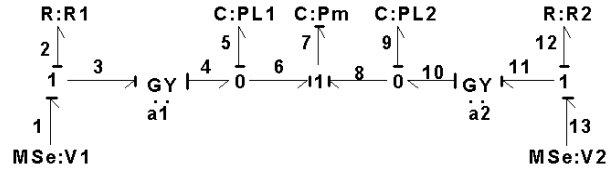

Fig. 7. Magnetic bond graph of the transformer.

The key vectors of the bond graph are

$$x = \begin{bmatrix} q_5 \\ q_9 \end{bmatrix}; \dot{x} = \begin{bmatrix} f_5 \\ f_9 \end{bmatrix}; z = \begin{bmatrix} e_5 \\ e_9 \end{bmatrix}$$

$$D_{in} = \begin{bmatrix} f_2 \\ f_{12} \end{bmatrix}; D_{out} = \begin{bmatrix} e_2 \\ e_{12} \end{bmatrix}; u = \begin{bmatrix} e_1 \\ e_{13} \end{bmatrix}$$

$$x_d = q_7; \dot{x}_d = f_7; z_d = e_7$$

the constitutive relations are

$$L = diag\{R_1, R_2\} \tag{44}$$

$$F = diag\left\{\frac{1}{P_{l_1}}, \frac{1}{P_{l_2}}\right\} \tag{45}$$

$$F_d = \frac{1}{P_m} \tag{46}$$

and the junction structure is

$$S_{21} = -S_{12}^T = S_{13} = diag\left\{\frac{1}{a_1}, \frac{1}{a_2}\right\}$$

$$S_{31} = -S_{14}^T = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$S_{11} = S_{22} = S_{23} = 0 \tag{47}$$

The state space realization by using (6), (7), (8), (9), (44), (45), (46) and (47) is given by

$$A = \begin{bmatrix} \frac{-R_1}{a_1^2 P_{l_1}} & 0 \\ 0 & \frac{-R_2}{a_2^2 P_{l_2}} \end{bmatrix}; E = \begin{bmatrix} 1 + \frac{P_m}{P_{l_1}} & \frac{P_m}{P_{l_2}} \\ \frac{P_m}{P_{l_1}} & 1 + \frac{P_m}{P_{l_2}} \end{bmatrix}$$

$$B = diag\left\{\frac{1}{a_1}, \frac{1}{a_2}\right\}$$

The magnetic bond graph gives interesting information of the transformer performance in a magnetic meaning, this is, the leakage fluxes and mutual flux can be determined.

Fig. 8 shows the magnetomotive force, flux and flux rate of the mutual permeance of the electrical transformer.
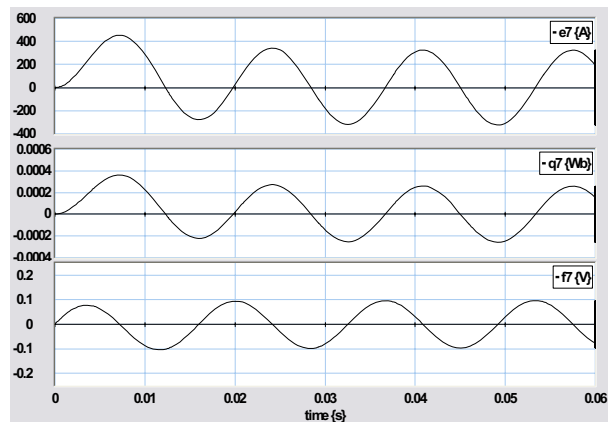

Fig. 8. Magnetic information of the mutual permeance.

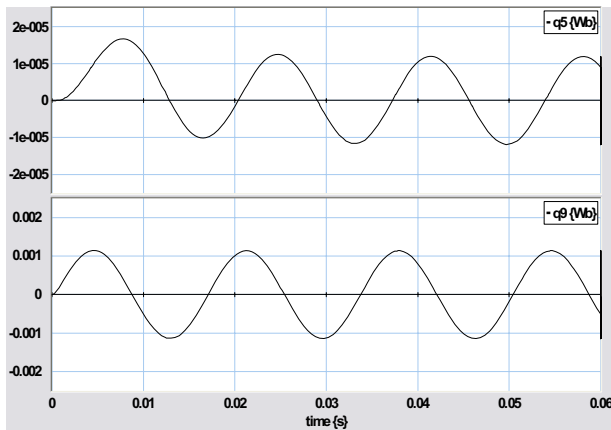The leakage fluxes of the windings are shown in Fig. 9.

Fig. 9. Leakage fluxes of the transformer.

A two windings transformer including a nonlinear core in a bond graph approach is proposed in the next section.

VII. Bond Graph of a Two Windings Transformer with Core

The final concept involved in the Steinmetz transformer model is a scheme for handling the nonlinearity of the core. The Steinmetz model approaches the problem of representing core excitation including the magnetization.

The incorporation of nonlinear effects such as magnetic saturation is achieved in the transformer model with the appropriate modification of the inductance L_m in the bond graph of Fig. 8.

In Fig. 8 the saturation curve is illustrated and this curve is approximated with the equation [9],

$$i_{Lm} = \tan\left(\frac{\lambda_{Lm}}{L_m}\right) \qquad (48)$$
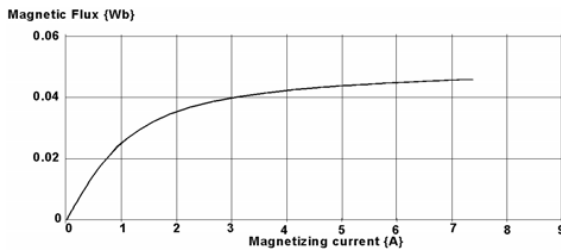
where $L_m = 0.0319H$.

Fig. 8. Saturation curve of equation (48).

The mutual inductance $I : L_m$ of the bond graph of Fig. 4 has a derivative causality assignment, then the state equation is

$$\dot{x}(t) = Ax(t) + Bu(t) + S_{14}\dot{x}_d(t) \qquad (49)$$

from (48), the third line of (4) and (1) we have

$$\dot{x}_d(t) = \frac{L_m}{1 + z_d^2} S_{31} F \dot{x}(t) \qquad (50)$$

considering (49) and (50) a nonlinear state equation of the transformer with saturation can be obtained.

However, if we introduce (48) to the bond graph model of Fig. 4 the nonlinear phenomena is incorporated. Fig. 9 shows the saturation performance in the bond graph model of the transformer.
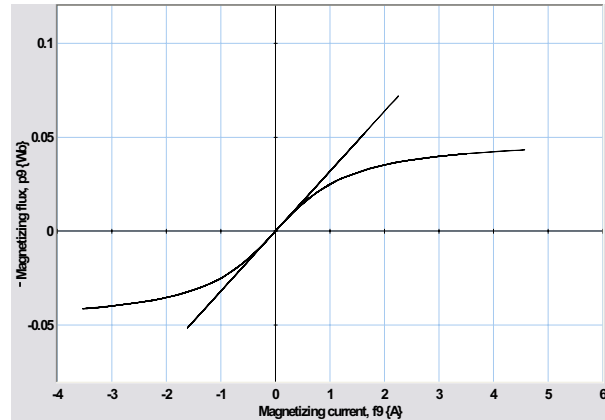

Fig. 9. Nonlinear performance of the transformer of Fig. 4.

The linear and nonlinear relationships of flux and electrical current of the mutual inductance considering the electrical modelling of Fig. 4 is shown in Fig. 9.

In order to include the magnetic saturation to the bond graph of Fig. 7, (48) can be written by

$$\frac{e_7}{a_1} = \tan\left(\frac{a_1 \cdot q_7}{a_1^2 \cdot P_m}\right)$$

reducing

$$e_7 = a_1 \cdot \tan\left(\frac{q_7}{a_1}\right) \qquad (51)$$

The nonlinear saturation characteristic of the mutual permeance of the bond graph of Fig. 7 is shown in Fig. 10.
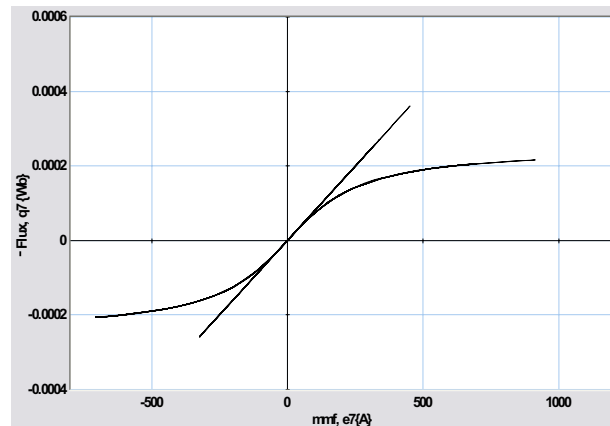

Fig. 10. Nonlinear performance of the transformer of Fig. 7.

The primary and secondary currents of the bond graph model of the transformer are shown in Fig. 11. Note that the primary current has the magnetic saturation effect. The simulation results by using bond graphs of the electrical and magnetic modelling are the same.
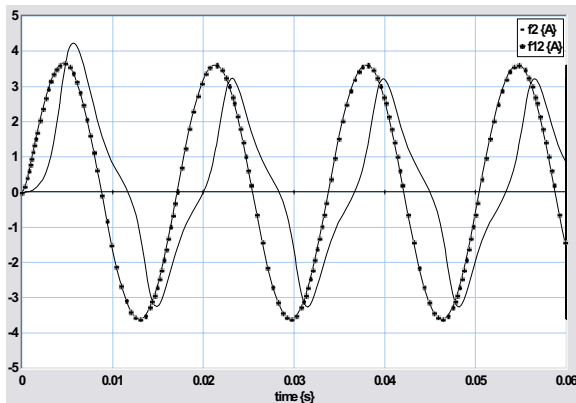
Fig. 11. Primary and secondary currents of a transformer with nonlinear core.

The analysis of a transformer with two windings can be generalized. Hence, a three-phase transformer model with nonlinear core by using magnetic bond graphs can be obtained.

VIII. Conclusions

Bond graph models of a power transformer incorporating the nonlinear saturation are presented. These models allow to obtain relations between the self and mutual inductances, and the leakage and magnetizing inductances in a simple and direct way using the derivative causality assignment of a bond graph. A bond graph model of the transformer using a magnetic system is proposed. The relation between electrical and magnetic modelling in the physical domain is established. Also, the nonlinear magnetic saturation of the electrical modelling to magnetic modelling is obtained. In order to verify the results the graphical simulation are shown. These models can be extended to three phase power transformers.

References

[1] George McPerson and Robert D. Laramore, *An Introduction to Electrical Machines and Transformers*, John, Wiley & Sons, 1990.
[2] Dean C. Karnopp, Donald L. Margolis and Ronald C. Rosenberg, *System Dynamics Modeling and Simulation of Mechatronic Systems*, Wiley, John & Sons, 2000.
[3] P. E. Wellstead, *Physical System Modelling*, Academic Press, London, 1979.
[4] Z. Q. Wu, G. H. Shirkoohi, J. Z. Cao, "Simple dynamic hysteresis modelling of three phase power transformer", *Journal of Magnetism and Magnetic Materials* 160(1996) pp.79-80.
[5] Chee-Mun Ong, *Dynamic Simulation of Electric Machinery Using Mat-Lab/Simulink*, Prentice-Hall, 1998.
[6] H. Fraisse, J. P. Masson, F. Marthouret and H. Morel, "Modeling od a Non-Linear Conductive Magnetic Circuit. Part 2: Bond Graph Formulation", *IEEE Transactions on Magnetics* Vol. 31, No. 6, November 1995.
[7] C. Sueur and G. Dauphin-Tanguy, "Bond graph approach for structural analysis of MIMO linear systems", *Journal of the Franklin Institute*, Vol. 328, No. 1, pp. 55-70, 1991.
[8] David C. Hamill, "Lumped Equivalent Circuits of Magnetic Components: The Gyrator-Capacitor Approach", *IEEE Transactions on Power Electronics*, Vol. 8, No. 2, April 1993.
[9] S. Garcia, A. Medina and C. Perez, "A state space single-phase transformer model incorporating nonlinear phenomena of magnetic saturation and hysteresis for transient and period steady-state analysis", *IEEE Power Engineering Society Summer Meeting*, Vol. 4, pp. 2417-2421, July 2000.