

# Elastic Strain-Concentration Factor of Notched Bars under Combined Loading of Static Tension and Pure Bending

Hitham M. Ttilan

**Abstract**—The effect of notch depth on the elastic new strain-concentration factor (SNCF) of rectangular bars with single edge U-notch under combined loading is studied here. The finite element method (FEM) and super position technique are used in the current study. This new SNCF under combined loading of static tension and pure bending has been defined under triaxial stress state. The employed specimens have constant gross thickness of 16.7 mm and net section thickness varied to give net-to-gross thickness ratio  $h_o/H_o$  from 0.2 to 0.95. The results indicated that the elastic SNCF for combined loading increases with increasing notch depth up to  $h_o/H_o = 0.7$  and sharply decreases with increasing notch depth. It is also indicated that the elastic SNCF of combined loading is greater than that of pure bending and less than that of the static tension for  $0.2 \leq h_o/H_o \leq 0.7$ . However, the elastic SNCF of combined loading is the elastic SNCF for static tension and less than that of pure bending for shallow notches (i.e.  $0.8 \leq h_o/H_o \leq 0.95$ ).

**Keywords**—Bar, notch, strain, tension, bending

## I. INTRODUCTION

**M**OST engineering components contain geometrical discontinuities, such as shoulders, keyways, and grooves, generally termed notches. When a notched component is loaded, local stress and strain concentrations are generated in the notch area. The stresses often exceed the yield limit of the material in the small region around the notch root, even at relatively low nominal elastic stresses. When a notched component is subjected to cyclic loading, cyclic inelastic strains in the area of stress and strain concentrations may cause formation of cracks and their subsequent growth could lead to component fracture. For cracks that nucleate from a shallow or blunt notch, the fatigue behavior is often dominated by crack nucleation. Cracks that nucleate from a sharp notch often nucleate rather quickly due to the elevated local stresses. Failure in machine elements takes place mainly at stress concentrations. Methods for predicting failure must take their effect into account. In design, it is common to use textbooks; [1], [2], to obtain the stress-concentration factor (SSCF). Also there have been many studies to tackle this problem, beginning with the work of Hardrath and Ohman and Neuber [3],[4].

A considerable amount of work has been completed with regard to the determination of elastic stress concentration factor for common discontinuities or geometrical irregularities under static loading. Results of these studies have been presented in graphical representation of experimental results, empirical formulae, and in theoretical solutions [5] - [9].

Fewer studies have been done examining the SNCF of discontinuities under static loading. For static tension, it has been predicted by Neuber that the plastic SNCF increases and the plastic SSCF decreases from their elastic values as plastic deformation develops from the notch root [2]. This prediction has been confirmed experimentally or analytically and given in literature [8]-[13]. These results indicate that for any type of loading, the SNCF is more important than the SSCF [14]-[19]. This is because the plastic SNCF maintains a high value much greater than unity, while the plastic SSCF decreases towards unity. The notches employed in the above studies are of intermediate depth, considered to give a strong notch effect.

A new SNCF has been defined under the triaxial stress state at the net section. This new SNCF provides reasonable values consistent with the concave distributions of the axial strain on the net section. Moreover, this new SNCF has removed the contradiction in the conventional SNCF having the values less than unity in spite of the concave distributions of the axial strain under elastic-plastic deformation. On the other hand, the conventional SNCF has been defined under the uniaxial stress state, which is completely different from the stress state at the net section, namely; the triaxial stress state [14]-[20]. This causes the above contradiction of the conventional SNCF. The SNCF for any type of loading must therefore be defined under the triaxial stress state at the net section. This is because the axial strain at the notch root occurs under the triaxial stress state SSCF [14]-[20]. The new SNCF has made it possible to clarify the strain concentration in notches under creep for the first time.

In this work, the finite element analysis and super position technique will be used in this study to predict the strain concentration factor (SNCF) of rectangular with single edge U-notch under combined loading. A newly defined SNCF is introduced here that has been defined under triaxial stress state.

## II. NEW STRAIN-CONCENTRATION FACTOR

Recently, a new strain-concentration factor (SNCF) under combined loading is introduced here. This new SNCF is defined as the ratio of the axial strain at notch root  $(\epsilon_x^{CL})_{max}$  to new average axial or nominal strain at the net section  $(\epsilon_x^{CL})_{av}$ ;

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$$(K_{\varepsilon})_{CL}^{new} = \frac{(\varepsilon_x^{CL})_{max}}{(\varepsilon_x^{CL})_{av}} \quad (1)$$

For combined loading, the maximum axial strain at the notch root  $(\varepsilon_x^{CL})_{max}$  is independent of definition and given as follows,

$$(\varepsilon_x^{CL})_{max} = (\varepsilon_x^P)_{NR} + (\varepsilon_x^M)_{NR} \quad (2)$$

and hence the new SNCF under combined loading depends on the definition of the average axial strain. For static tension, the axial strain is assumed to be uniformly distributed at the net section if the notch effect is negligible. This assumption gives the average axial or nominal strain, as shown in Fig. 1a. For single edged rectangular bar cylindrical bars [17]

$$\begin{aligned} (\varepsilon_x^P)_{av} &= \frac{1}{A} \int_{-h_c}^{h_t} \varepsilon_x(\psi) dA \\ &= \frac{1}{b_o(h_t + h_c)} \int_{-h_c}^{h_t} \varepsilon_x(\psi) b_o d\psi \\ &= \frac{1}{(h_t + h_c)} \int_{-h_c}^{h_t} \varepsilon_x(\psi) d\psi \end{aligned} \quad (3)$$

On the other hand, for pure bending the new nominal strain is therefore given by the maximum tensile longitudinal strain obtained from this assumed linear distribution at the notch root, as shown in Fig. 1b [18].

$$(\varepsilon_x^M)_{av} = \frac{6}{(h_t + h_c)^2} \int_{-h_c}^{h_t} \varepsilon_x(\psi) \psi d\psi \quad (4)$$

As mentioned earlier in this section, the new SNCF under combined loading is introduced by a new definition of the average or nominal axial strain as follows;

$$\begin{aligned} (\varepsilon_x^{CL})_{av}^{new} &= \left[ (\varepsilon_x^P)_{av} \right]_{NR} + \left[ (\varepsilon_x^M)_{av} \right]_{NR} \\ &= \frac{1}{(h_t + h_c)} \int_{-h_c}^{h_t} \varepsilon_x(\psi) d\psi + \frac{6}{(h_t + h_c)^2} \int_{-h_c}^{h_t} \varepsilon_x(\psi) \psi d\psi \end{aligned} \quad (5)$$

For elastic deformation and plane strain condition  $\sigma_z = \nu(\sigma_x + \sigma_y)$ , the new SNCF can be transformed to following form:

$$(\varepsilon_x^{CL})_{av}^{new} = \frac{1-\nu^2}{E} \left[ (\sigma_x^P)_{av} - \frac{6M}{(h_t+h_c)^2} \right] - \frac{\nu(1+\nu)}{E(h_t+h_c)} \left[ \int_{-h_c}^{h_t} \left[ \sigma_y^P - \frac{6\sigma_y^M}{(h_t+h_c)} \psi \right] \psi d\psi \right] \quad (6)$$

where  $E$ ,  $\nu$ ,  $P$ , and  $M$  are Young's modulus, Poisson's ratio, tensile load and bending moment, respectively. For elastic deformation  $h_o \approx h_t + h_c$ ; Equation (6) can be rewritten as

$$(\varepsilon_x^{CL})_{av}^{new} = \frac{1-\nu^2}{E} (\sigma_x^{CL})_{av} - \frac{\nu(1+\nu)}{Eh_o} \left[ \int_{-h_c}^{h_t} \left[ \sigma_y^P - \frac{6\sigma_y^M}{h_o} \psi \right] \psi d\psi \right] \quad (7)$$

This equation indicates that is defined under the triaxial stress state at the net section. The definition under the triaxial stress state gives the reasonable SNCF consistent with the concave distribution of the axial strain [14]-[20].

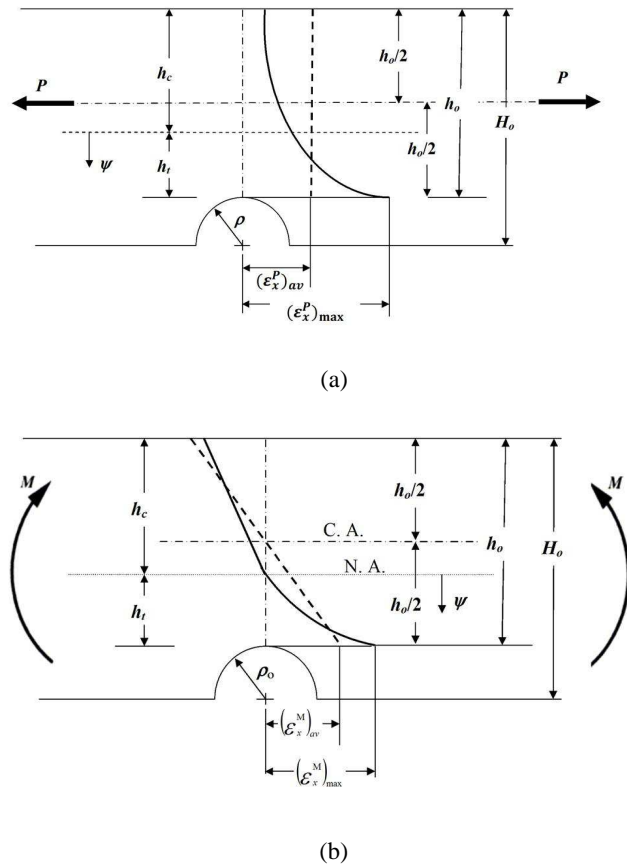


Fig. 1 Schematic diagram of the assumed axial strain average distribution at the net section; (a) Static tension (b) Pure bending

### III. MATERIALS AND GEOMETRIES

The material employed in this study is an Austenitic stainless steel with tensile yield strength of 245.9 (MPa), Young's modulus of 206 (GPa), and Poisson's ratio of 0.3, respectively. Rectangular bars with single edge U-notch are employed here, as shown in Fig. 2. Three different fillet radii 0.5, 1.0 (mm), and 2.0 (mm) are employed. In order to study the effect of notch depth on the elastic SNCF, the gross thickness ( $H_o$ ) is kept constant of 16.7 (mm), while the net-section thickness ( $h_o$ ) has been varied to give net-to-gross thickness ratio ( $h_o/H_o$ ) from 0.2 (i.e. extremely deep notch) to 0.95 (i.e. shallow notch). The half length of the employed bars is 80 (mm), as shown in Fig. 2. The distance from the end of the bar to the point load  $F$  is 35 (mm). The bar within the inner loading span of the four point bending arrangement experiences a pure bending moment.

### IV. FINITE ELEMENT SIMULATIONS

The finite element method analysis shows that stresses and strains are very high on the notched member in the vicinity of the notch root and gradually decrease in the direction of the unnotched part. Taking into account the symmetry of the specimens, only a quarter of the geometry was modeled. FE

models of the employed specimen were constructed in the Marc code using 8-node plain-strain element. Since the super position technique is used here, a biquadratic interpolation and full integration type 27 for static tension and type 28 for pure bending in MARC classifications, is employed in this study. A typical mesh of one-half for simulating the loaded notched bars is shown in Fig. 2.

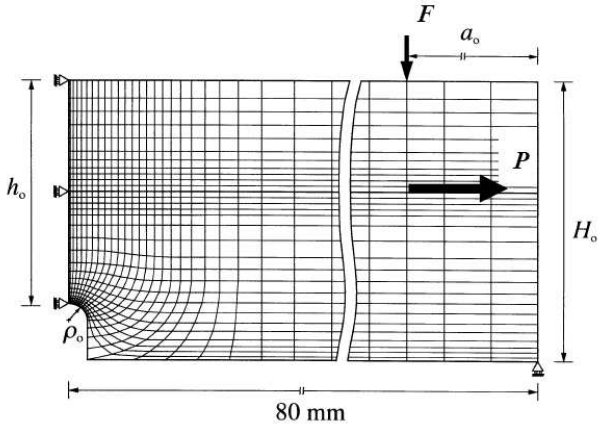


Fig. 2 FEM model of rectangular bar with single edge U-notch

V. RESULTS AND DISCUSSION

A. Effect of notch depth on the new SNCF

For different notch configurations, the elastic axial strain in the notch root was used to obtain the strain concentration factors presented in Figs. 3, 4 and 5. In general, Figs. 3~5 reveal that elastic strain concentration factors increase significantly when the notched bars subjected to axial tension, pure bending and combined loadings.

It is pointed out that even relatively shallow notches introduce large strain concentrations for combined loading of static tension and pure bending. Particularly, the elastic SNCF gradually increases with increasing notch depth and reaches maximum value at  $h_0/H_0 = 0.7$  (i.e. deep notch). After that, the elastic SNCF sharply decreases with increasing notch depth for shallow notches (i.e.  $h_0/H_0 > 0.7$ ).

Results are also compared to the previous results that found the current author for static tension and pure bending [17], [18]. It can be seen from Figs. 3~5, that the elastic SNCF for combined loading is greater than the elastic SNCF for pure bending and less than the elastic SNCF for static tension in the range  $0.2 \leq h_0/H_0 \leq 0.7$ . On the contrary, the elastic SNCF of combined loading is less than that of the static tension and less than that of the pure bending for shallow notches ( $h_0/H_0 > 0.7$ ). The difference between the elastic new SNCF values under combined loading and those under static tension and under pure bending decreases with increasing notch radius.

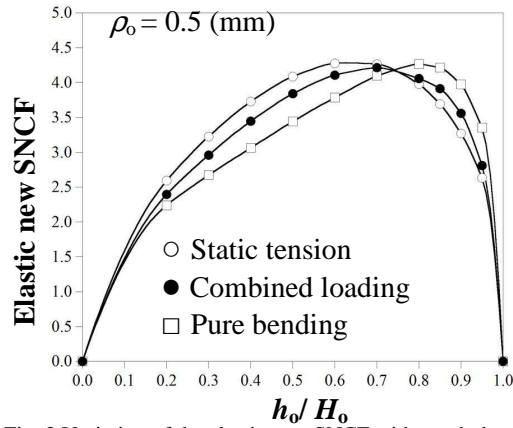


Fig. 3 Variation of the elastic new SNCF with notch depth for  $\rho_0 = 0.5$  (mm)

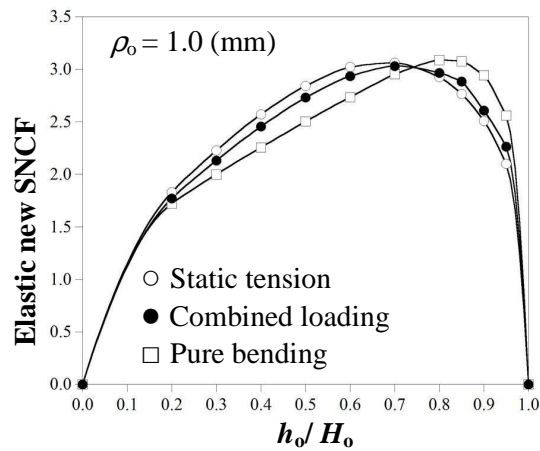


Fig. 4 Variation of the elastic new SNCF with notch depth for  $\rho_0 = 1.0$  (mm)

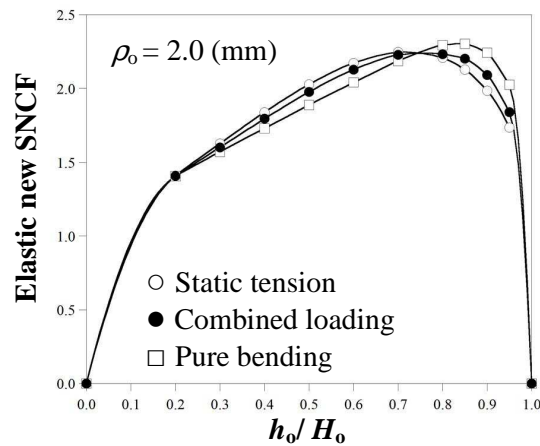


Fig. 5 Variation of the elastic new SNCF with notch depth for  $\rho_0 = 2.0$  (mm)

## VI. CONCLUSION

For different notch configurations, the variations of elastic SNCF with notch depth under combined loading are studied here using FEM. The newly defined strain concentration introduced here is defined under triaxial stress state at the net section. Since an infinitesimal deformation level is used in this study, the super position technique has been applied. It is concluded that the elastic new SNCF under combined loading has the same manner of variation with notch depth as that under static tension and under pure bending. However, the values of the elastic new SNCF under combined loading are less than under static tension for extremely deep and deep notches; i.e.  $0.2 \leq h_0/H_0 \leq 0.7$ . On the other hand, it is greater than the elastic SNCF under pure bending for the same range of notch depths. For shallow notches, i.e.  $h_0/H_0 > 0.7$ , the new SNCF values under combined loading is greater than on static tension and less than that of pure bending. It is prominent that the difference between the elastic new SNCF under combined loading values and that under static tension and under pure bending decreases with increasing notch depth.

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