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Effect of Progressive Type-I Right Censoring on Bayesian Statistical Inference of Simple Step–Stress Acceleration Life Testing Plan under Weibull Life Distribution

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Abstract**—**This paper discusses the effects of using progressive Type-I right censoring on the design of the Simple Step Accelerated Life testing using Bayesian approach for Weibull life products under the assumption of cumulative exposure model. The optimization criterion used in this paper is to minimize the expected pre-posterior variance of the P^{th} percentile time of failures. The model variables are the stress changing time and the stress value for the first step. A comparison between the conventional and the progressive Type-I right censoring is provided. The results have shown that the progressive Type-I right censoring reduces the cost of testing on the expense of the test precision when the sample size is small. Moreover, the results have shown that using strong priors or large sample size reduces the sensitivity of the test precision to the censoring proportion. Hence, the progressive Type-I right censoring is recommended in these cases as progressive Type-I right censoring reduces the cost of the test and doesn't affect the precision of the test a lot. Moreover, the results have shown that using direct or indirect priors affects the precision of the test.

Keywords**—**Reliability, Accelerated life testing, Cumulative exposure model, Bayesian estimation, Progressive Type-I censoring, Weibull distribution.

I. Introduction

RELIABILITY in engineering can be defined as: the probability that the system or the component will conduct its intended functions satisfactorily at least for a given period of time when used under normal operating conditions. Life tests (LT) are used to predict this probability. As the products become more reliable, Simple Life Tests (SLT), become less useful as the testing time increases exponentially, the matter that renders such tests unpractical in terms of time and cost. Accelerated Life Testing (ALT) comes as a remedy for this problem as it can be used to stimulate the failures in the test by testing the samples under harsh conditions: conditions more severe than normal operating conditions. To reduce the cost of the test and further reduces the testing time, Step-Stress Accelerated Life Testing (SSALT) can be used. Typically, two steps are used in this test. In a typical SSALT a certain number of samples is placed in the test under certain stress level (usually a little bit above the normal operating conditions) for a certain period of time, after which the stress level increases on the surviving samples tell the end of the test.

Two main stopping criteria for ALT are usually adopted: Type-I and Type-II censoring. Under the first stopping criterion, the test stops when a certain amount of time is elapsed, during which the failure times of the samples are recorded. Under the second stopping criterion, the test stops when a certain number of failures are observed, during which the failure times of the samples are recorded. Moreover, a hybrid stopping criterion is also used for SSALT. Progressive Type-I right censoring is an example of such a hybrid criterion used to reduce the cost of the test. Under this stopping criterion, certain proportion of the sample size is removed while it is still working at each test step. Gouno and Balakrishnan [1] discuss this matter extensively.

After the early work of Cohen [2], many efforts are done to design the SSALT based on progressively censored data. Miller and Nelson [3] used exponential lifetimes and complete failure data for SSALT using cumulative exposure model. Bai et al. [4] used the work of Miller and Nelson [3] with time-censored data. The case of general K-level and M-variable case were discussed by Khamis [5]. Ng et al. [6], Wu et al. [7], and Balakrishnan [8] directed their efforts to the point and interval estimation of the test parameters and optimization of the test plans including SSALT.

The vast majority estimation method for life testing in literature is the Maximum Likelihood Estimation method (MLE). Nelson and Kielpinski [9], Bai and Kim [10], Escobar and Meeker [11], Khamis [5] have used MLE to design their tests. Bayesian method also has found its application in this area. The Bayesian method allows the experimenter to reflect his/her expert opinion into the design problem. This other source of information allows for test planning when the MLE performance is under question because of the low number of failures (low information available). Some of the references that have used this method include Van Dorp et al. [12] where the authors developed a Baye's approach to Step-stress accelerated test plans including ramping phenomenon and Ramadan and Ramadan [13] where exponential SSALT under progressive Type-I right censoring was considered.

This paper presents a Bayesian approach for designing an optimal SSALT for Weibull life products and Progressive Type-I right censoring under the assumptions of cumulative

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exposure model and small censoring proportion. The optimization criterion that will be used in this paper is to minimize the expected pre-posterior variance of the P^{th} percentile time of failures. The model variables are the stress changing time and the stress value for the first step. The durations of the steps are not considered equal and the uncertainty in the model parameters will be considered through Bayesian statistics. A comparison between the conventional and progressive Type-I right censoring will be provided.

II. Model Description and Assumptions

The removal of some sample units before their failure in the test reduces the cost of the test as these removed samples can be used somewhere else or in other tests. Hence, the progressive Type-I censoring tests are considered cheaper than traditional Type-I censoring tests.

The SSALT under progressive Type I censoring can be described as follows: n samples are stressed under x_1 stress level for τ time during which n_1 samples fail. At the end of the first step, certain proportion, π, of the samples will be removed from the test and a higher stress, x_2, will be applied until the end of the test time t_c. Hence the amount of samples removed can be calculated as c = $n \times \pi$.

In this model, the format of the Weibull probability distribution will be written such that the scale parameter of the distribution is exp (μ_i). The log-linear relation between μ_i and the stress level x_i is assumed as in (1):

$$\ln(\mu_i) = a + b\mathrm{x_i}, \tag{1}$$

where a and b are the model parameters such that their values depend on the product under test and the test method used. Moreover, cumulative exposure model and small censoring proportion is also assumed.

III. The Proposed Model for SSALT

Fig. 1 shows a schematic diagram for the SSALT under progressive Type-I right censoring.

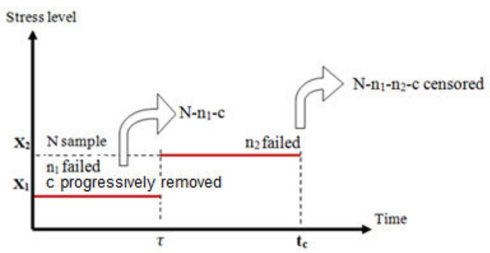

Fig. 1 Schematic diagram for progressive Type-I right censoring for SSALT

The cumulative distribution function considering the cumulative exposure model and the Weibull life distribution is expressed as in (2):

$$F(t|\sigma,\mu_1,\mu_2) = \begin{cases} 1-\exp\left(-(t\times\exp(-\mu_1))^{\frac{1}{\sigma}}\right) & 0<t<\tau \\ 1-\exp\left(-\left(\frac{t-\tau\left(1-\frac{\exp(\mu_1)}{\exp(\mu_2)}\right)}{\exp(\mu_2)}\right)^{\frac{1}{\sigma}}\right) & \tau\le t\le t_c \end{cases}, \tag{2}$$

and the probability density function is:

$$f(t|\sigma,\mu_1,\mu_2) = \begin{cases} \frac{\exp(-\mu_1)}{\sigma}(t\times\exp(-\mu_1))^{\frac{1}{\sigma}-1}\exp\left(-(t\times\exp(-\mu_1)^{\frac{1}{\sigma}}\right) & 0<t<\tau \\ \frac{\exp(-\mu_2)}{\sigma}\left(\frac{t-\tau\left(1-\frac{\exp(\mu_1)}{\exp(\mu_2)}\right)}{\exp(\mu_2)}\right)^{\frac{1}{\sigma}-1}\exp\left(-\left(\frac{t-\tau\left(1-\frac{\exp(\mu_1)}{\exp(\mu_2)}\right)}{\exp(\mu_2)}\right)^{\frac{1}{\sigma}}\right) & \tau\le t\le t_c \end{cases}, \tag{3}$$

where σ is the reciprocal of the shape parameter of Weibull distribution. Moreover, the reliability function for the survived units is as follows:

$$R(t_c|\sigma,\mu_1,\mu_2) = \exp\left(-\left(\frac{t_c-\tau\left(1-\frac{exp(\mu_1)}{exp(\mu_2)}\right)}{exp(\mu_2)}\right)^{\frac{1}{\sigma}}\right), \quad t\ge t_c \tag{4}$$

and the reliability function for the removed units is as follows:

$$R(\tau) = \exp\left(-(\tau\times\exp(-\mu_1)^{\frac{1}{\sigma}}\right) \tag{5}$$

IV. Bayesian Plan

Let μ_2 be the μ at stress level $\mathrm{x_2}$, which can be given by

$$\mu_2 = \exp(a + b\mathrm{x_2}), \tag{6}$$

and let μ_1 be the μ at stress level $\mathrm{x_1}$, which can be given by

$$\mu_1 = \exp(a + b\mathrm{x_1}). \tag{7}$$

According to Baye's theorem, the posterior distribution of a and b given the data $\boldsymbol{t}$ is as follows:

$$f(a,b|t) = \frac{f(t|a,b)f(a,b)}{f(t)}, \tag{8}$$

such that $f(\boldsymbol{t}|a,b)$ is the likelihood of the data and $f(\boldsymbol{t}) = \int\int f(\boldsymbol{t}|a,b)f(a,b)dadb$ is the pre-posterior marginal distribution of $\boldsymbol{t}$. Applying bivariate random variables transformation and the log-linear model , the prior joint distribution of the model parameters a and b can be derived from the priors distributions of μ_2 and μ_1 as follows:

$$f(a,b) = f\big(a(\mu_2,\mu_1), b(\mu_2,\mu_1)\big)\times abs|J|, \tag{9}$$

where

$$a(\mu_2,\mu_1) = ln(\mu_2) - bx_2, \tag{10}$$

$$b(\mu_2,\mu_1) = \frac{ln(\mu_2)-ln(\mu_1)}{(x_2-x_1)}, \tag{11}$$

and J is the Jacoubian matrix given by:

$$J = \begin{bmatrix} \frac{\partial \mu_2}{\partial a} & \frac{\partial \mu_2}{\partial b} \\ \frac{\partial \mu_1}{\partial a} & \frac{\partial \mu_1}{\partial b} \end{bmatrix} = \begin{bmatrix} exp(a+bx_2) & x_2 exp(a+bx_2) \\ exp(a+bx_1) & x_1 exp(a+bx_1) \end{bmatrix}. \quad (12)$$

Substituting (10)-(12) into (9), $f(a,b)$ can be written as:

$$f(a,b) = f\left(ln(\mu_2) - bx_2, \frac{ln(\mu_2)-ln(\mu_1)}{(x_2-x_1)}\right) \times abs\left|\begin{bmatrix} exp(a+bx_2) & x_2 exp(a+bx_2) \\ exp(a+bx_1) & x_1 exp(a+bx_1) \end{bmatrix}\right|. \quad (13)$$

The mean and variance of $t_p(x_0)$ at certain set of data $\boldsymbol{t}$ are

$$E\left(t_p(x_0)|t\right) = \iint t_p(x_0) f(a,b|t)\, dadb, \quad (14)$$

and

$$Var\left[t_p(x_0)|t\right] = \iint \left(t_p(x_0) - E\left(t_p(x_0)|t\right)\right)^2 f(a,b|t) dadb, \quad (15)$$

respectively. Therefore, the expected pre-posterior variance of $t_p(x_0)$ is

$$E_t\left[Var\left[t_p(x_0)|t\right]\right] = \int_0^\infty Var\left[t_p(x_0)|t\right] f(t) dt. \quad (16)$$

Gibbs sampling will be used in the optimization process. In each iteration, the values of the model parameters *a* and *b* will be calculated using (10) and (11) based on the values of μ_1 and μ_2 drawn from their joint prior distribution. Utilizing the calculated values of *a* and *b* in each iteration, a value for $\bar{t_p}(x_o)$ is calculated using the following equation:

$$ln(\bar{t_p}) = -\left(ln\left(ln(1-p)\right) + a + bx_0\right). \quad (17)$$

If a large number of iterations used, the distribution of the calculated points of $\bar{t_p}(x_o)$ approximate the actual distribution of $t_p(x_o)$ and the mean and variance of those points can be used as an approximation for the mean and variance of the actual distribution of $t_p(x_o)$, e.g. $E\left[t_p(x_o)|\boldsymbol{t}\right]$ and $\mathrm{Var}\left[t_p(x_0)|\boldsymbol{t}\right]$ respectively. Moreover, if the whole process repeated many times, the average of the $\mathrm{Var}\left[\bar{t_p}(x_0)|\boldsymbol{t}\right]$, which is $\mathrm{E}_{\mathbf{t}}\left[\mathrm{Var}\left[\bar{t_p}(x_0)|\mathbf{t}\right]\right]$, can be seen as an approximation for the expected variance of $t_p(x_o)$, shown in (16).

Under progressive Type-I right censoring, the likelihood function has four parts to account for the two censoring times and the two steps. The likelihood function for *n* samples is:

$$L = \prod_{i=1}^{n_1}\left(\left[\frac{\exp(-\mu_1)}{\sigma}\left(\frac{t_i}{\exp(\mu_1)}\right)^{\frac{1}{\sigma}-1} \exp\left(-\left(\frac{t_i}{\exp(\mu_1)}\right)^{\frac{1}{\sigma}}\right)\right]\right) \times$$
$$\prod_{i=1}^{n_2}\left(\times\left[\frac{\exp(-\mu_2)}{\sigma}\left(\frac{t_i-\tau\left(1-\frac{\exp(\mu_1)}{\exp(\mu_2)}\right)}{\exp(\mu_2)}\right)^{\frac{1}{\sigma}-1} \exp\left(-\left(\frac{t_i-\tau\left(1-\frac{exp(\mu_2)}{exp(\mu_1)}\right)}{exp(\mu_2)}\right)^{\frac{1}{\sigma}}\right)\right]\right) \times$$
$$(n\pi) \exp\left(-\left(\frac{\tau}{exp(\mu_1)}\right)^{\frac{1}{\sigma}}\right) \times (\mathrm{n}(1-\pi) - \mathrm{n}_1 - \mathrm{n}_2)\left[\exp\left(-\left(\frac{t_c-\tau\left(1-\frac{exp(\mu_2)}{exp(\mu_1)}\right)}{exp(\mu_2)}\right)^{\frac{1}{\sigma}}\right)\right], \quad (18)$$

and the log likelihood function that will be used in the Gibbs sampling is:

$$\mathcal{L} = n_1\mu_1 - n_1 log(\sigma) + \left(\frac{-n_1}{\sigma} + n_1\right)\mu_1 + \left(\frac{1}{\sigma} - 1\right) \times \left(\sum_{i=1}^{n_1} log(t_i)\right) - \sum_{i=1}^{n_1}\left(\frac{t_i}{exp(\mu_1)}\right)^{\frac{1}{\sigma}} +$$
$$n_2\mu_2 - n_2 log(\sigma) + \left(\frac{-n_2}{\sigma} + n_2\right)\mu_2 + \left(\frac{1}{\sigma} - 1\right) \times \left(\sum_{i=1}^{n_2} log\left(t_i - \tau\left(1 - \frac{\exp(\mu_1)}{\exp(\mu_2)}\right)\right)\right) - \sum_{i=1}^{n_2}\left(\frac{t_i-\tau\left(1-\frac{exp(\mu_2)}{exp(\mu_1)}\right)}{exp(\mu_2)}\right)^{\frac{1}{\sigma}} - (n\pi)\left(\frac{\tau-\tau\left(1-\frac{exp(\mu_2)}{exp(\mu_1)}\right)}{exp(\mu_2)}\right)^{\frac{1}{\sigma}} - (\mathrm{n}(1-\pi) - \mathrm{n}_1 -$$
$$\mathrm{n}_2)\left(\frac{t_c-\tau\left(1-\frac{exp(\mu_2)}{exp(\mu_1)}\right)}{exp(\mu_2)}\right)^{\frac{1}{\sigma}}.$$

The complete model that will be used to optimize the SSALT is as follows:

$$min\ E_t\left[Var\left[t_p(x_0)|t\right]\right]$$

s.t.

$$\begin{aligned} t_{LB} &< \tau < t_c, \\ x_o &< x_1 < x_2, \\ E[n_1] &\ll n(1-\pi) \\ x_1, \tau &> 0, \end{aligned} \quad (19)$$

where

$$E[n_1] = n \times \iint \ F_1(\tau|a,b) f(a,b) dadb. \quad (20)$$

V. Numerical Example and Analysis

The following assumptions and values are used in this example:

1. τ = [1000 1500 2000 2500 3000 3500 4000 4500 5000 5500]
2. Total testing time t_c = 6000
3. Normal stress level $\mathrm{x}_0 = 1$ volts
4. High stress level $\mathrm{x}_2 = 4$ volts
5. Low stress level x_1=[1.50 1.67 1.83 2.00 2.17 2.33 2.50 2.67 2.83 3.00]
6. Sample size = 20 samples
7. *P* = 0.1
8. μ_1 and μ_2 are considered independent with normal prior distributions as follows:

$$\mu_1 = N(22026,100) @\ \mathrm{x}_1 = 3 \text{ Volts}$$
$$\mu_2 = N(2980,10) @\ \mathrm{x}_2 = 4 \text{ Volts}$$

The data for this example is generated as follows: a full enumeration for the combinations between τ and x_1 is found. For each combination, the expected pre-posterior variance of the *10th* percentile time of failures is evaluated. The combination with the lowest expected pre-posterior variance of the *10th* percentile time of failures is considered as the optimal design of the SSALT.

Table I, shows the values of the censoring proportion along with the corresponding values of the $E_t\left[Var\left[t_p(x_0)|t,\pi_i\right]\right]$

and the corresponding values of the optimal design parameters τ and x_1.

The table shows the optimal solution under the different values of $\boldsymbol{\pi}$. The zero value for the censoring proportion $\boldsymbol{\pi}$ refers to the case where there is no progressive censoring. From the table, it is clear that as the value of $\boldsymbol{\pi}$ increases, the value of the expected variance for the 10th percentile time of failures also increases and thus the test precision decreases. This makes sense because as we increase $\boldsymbol{\pi}$, the number of samples progressively censored in the first step will increase; consequently, the number of failures on the second step will decrease. The net result will be a deterioration on the test precision. Also the table shows that the optimal stress changing time τ did not change (5500 seconds) with changing the censoring proportion while the optimal stress changed.

TABLE I
THE CENSORING PROPORTION ALONG WITH THE OPTIMAL VALUES OF $E_T[VAR[T_P(X_0)|T,\Pi_I]]$, T, AND X_1 AT SAMPLE SIZE OF 20

Censoring Proportion ($\boldsymbol{\pi}$)	$E_t[Var[t_{0.1}(x_0)\|\boldsymbol{t},\pi_i]]$	τ	x_1 Volts
0	2483018	5500	2.67
0.1	2529339	5500	1.50
0.2	2612043	5500	1.50
0.3	2813307	5500	1.67

To see the effect of the sample size on the optimal solution, the sample size was increased to 40 and the example was solved again. Table II contains the results. It is apparent that there is no clear trend between the values of $E_t[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]]$ and the values of $\boldsymbol{\pi}$. The value of $E_t[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]]$ at no progressive censoring is higher than its corresponding value at π = 0.1. This result was unexpected and may be related to the sample size as follows: as the sample size increases, the amount of information contained in the likelihood function increases and thus progressively removing some units from the test has little effect on the test's precision. Moreover, the information contained in the likelihood comes from two sources, the failed units and the censored units. Increasing the sample size under progressive censoring provides the likelihood function with additional information (censored times) due to the increase in the number of the progressively censoring sample units. This may enhance the quality of information contained in the likelihood function the matter that will reduce $E_t[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]]$.

In addition, it is worth to mention here that the overall precision of the SSALT under higher sample size is always better than the corresponding precision for the test at the same censoring proportion π. This result can also be related to the increase in the sample size.

TABLE II
THE CENSORING PROPORTION ALONG WITH THE OPTIMAL VALUES OF $E_T[VAR[T_P(X_0)|T,\Pi_I]]$, T, AND X_1 AT SAMPLE SIZE OF 40

Censoring Proportion ($\boldsymbol{\pi}$)	$E_t[Var[t_{0.1}(x_0)\|\boldsymbol{t},\pi_i]]$	τ	x_1 Volts
0	1743094	5000	1.50
0.1	1735913	5500	2.17
0.2	1746419	5500	2.17
0.3	1923549	5500	2.00

To assess the effect of prior's strength, the strength of the priors were increased to the following values

$\mu_1 = N(22026,10) @ x_1 = 3$ Volts,

and

$\mu_2 = N(2980,1) @ x_2 = 4$ Volts.

The example was solved again and the results were recorded in Table III. Comparing the results contained in Tables I and III, one can see that the trend found in Table I was not repeated in Table III. One can also see that the values of the $E_t[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]]$ are much lower in Table III (stronger priors) comparing to the values in Table I (weaker priors). These results show the importance of the prior's strength. As the priors become more informative the precision of the test enhances. Also one can see that as the priors become more informative, the importance of the effect of the progressive censoring on the test precision decreases as the difference between the values of $E_t[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]]$ at different censoring proportions decreases. This result may be understood if one remembers that under Bayesian statistics the information about the model parameters *a* and *b* comes from two sources: the likelihood and the model parameters' priors, as the strength of the prior increases, the importance of the information contained in the likelihood decreases and thus most of the information about the model parameters comes from the priors the matter that will reduce the effect of the progressive censoring on the test precision.

To assess the effect of changing the priors from being indirect priors to the model parameters *a* and *b* (through μ_1 and μ_2) to direct priors directly on the model parameters *a* and *b*, the example was solved again with the following priors:

$a = N(16,0.1)$,

and

$b = N(-2,0.5)$.

TABLE III
THE CENSORING PROPORTION ALONG WITH THE OPTIMAL VALUES OF $E_T[VAR[T_P(X_0)|T,\Pi_I]]$, T, TC, AND X_1 AT MORE INFORMATIVE PRIORS AND SAMPLE SIZE 20

Censoring Proportion ($\boldsymbol{\pi}$)	$E_t[Var[t_{0.1}(x_0)\|\boldsymbol{t},\pi_i]]$	τ	x_1 Volts
0	131570	2000	2.33
0.1	131325	1000	1.83
0.2	131436	1500	2.83
0.3	131203	1500	2.67

The results showed that the values of $E_t\left[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]\right]$ under the new priors setup increased a lot compared to the previous setup (even though the priors used were informative), hence, the precision of the test reduced. Also the table showed that the value of $E_t\left[Var[t_{0.1}(x_0)|\boldsymbol{t},\pi_i]\right]$ increased as the censoring proportion π increased, which is the same trend found in Table I under small sample size and indirect priors. This result emphasizes that changing the way of assigning the priors can affect the test precision.

TABLE IV
THE CENSORING PROPORTION ALONG WITH THE OPTIMAL VALUES OF $E_T\left[VAR[T_P(X_0)|T,\Pi_I]\right]$, T, AND X_1 AT SAMPLE SIZE OF 20 WITH DIRECT PRIORS FOR MODEL PARAMETERS

Censoring Proportion ($\boldsymbol{\pi}$)	$E_t\left[Var[t_{0.1}(x_0)\|\boldsymbol{t},\pi_i]\right]$	τ	x_1 Volts
0	1191102081	1000	2.83
0.1	1216429662	1000	1.67
0.2	1238939060	1000	3.00
0.3	1263419716	1000	1.50

VI. CONCLUSION

This paper discusses the effects of using progressive Type-I right censoring on the design of the Simple Step Accelerated Life testing using Bayesian approach for Weibull life products under the assumption of cumulative exposure model. The optimization criterion used in this paper is to minimize the expected pre-posterior variance of the Pth percentile time of failures. The model variables are the stress changing time and the stress value for the first step. A comparison between the conventional and the progressive Type-I right censoring is provided. The results have shown that the progressive Type-I right censoring reduces the cost of testing on the expense of the test precision when the sample size is small. Moreover, the results have shown that using strong priors or large sample size reduces the sensitivity of the test precision to the censoring proportion. Hence, the progressive Type-I right censoring is recommended in these cases as progressive Type-I right censoring reduces the cost of the test and doesn't affect the precision of the test a lot. Moreover, the results have shown that whether using direct or indirect priors affects the precision of the test.

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