

# Effect of Pre-Plasma Potential on Laser Ion Acceleration

Djemai Bara, Mohamed Faouzi Mahboub, Djamila Bennaceur-Doumaz

**Abstract**—In this work, the role of the preformed plasma created on the front face of a target, irradiated by a high intensity short pulse laser, in the framework of ion acceleration process, modeled by Target Normal Sheath Acceleration (TNSA) mechanism, is studied. This plasma is composed of cold ions governed by fluid equations and non-thermal & trapped with densities represented by a "Cairns-Gurevich" equation. The self-similar solution of the equations shows that electronic trapping and the presence of non-thermal electrons in the pre-plasma are both responsible in ion acceleration as long as the proportion of energetic electrons is not too high. In the case where the majority of electrons are energetic, the electrons are accelerated directly by the ponderomotive force of the laser without the intermediate of an accelerating plasma wave.

**Keywords**—Cairns-Gurevich Equation, ion acceleration, plasma expansion, pre-plasma.

## I. INTRODUCTION

IN recent years, the intensity of lasers has increased enormously, reaching  $10^{24}$  W/cm<sup>2</sup> at the focal point, with ultra-short laser pulses reaching the femtosecond. The energy of the laser is so strong that the material is ionized immediately after its irradiation by the pulse of the laser and a plasma is immediately created. The merit of the discovery of high intensity lasers belongs to the Chirped Pulse Amplification (CPA) technique, introduced at the University of Rochester in 1985 by Mourou [1]. The development of this technique was motivated by the need to increase the energy of the input pulses of the lasers in order to explore new regimes of interaction laser material accessible only by theory or numerical simulation. The interest of this type of lasers is justified for the duration of the pulse making possible the study of very brief phenomena which would not be accessible by other methods so several applications are envisaged in the physics of plasmas, chemistry, biology and radiology. Since then, hot and dense plasmas created by these lasers have been found to be sources of accelerated particles at high energies [2].

Great progress has been made in controlling these sources of energy particles and, recently, increasing their performance. Nowadays, due to their compact dimensions, these sources become a complementary alternative to the large instruments that are the plasma-laser power accelerators [3].

Bara, D. and Bennaceur-Doumaz, D. are with Centre de Développement des Technologies Avancées, Cité du 20 août 1956, B.P. 17, Baba-Hassen, Alger, Algérie (e-mail: dbara@cdta.dz, ddoumaz@cdta.dz).

Mahboub, M. F. is with the Theoretical Physics Laboratory, Faculty of Physics USTHB, B.P.32 El Alia, Algiers 16079, Algeria (corresponding author, e-mail: mohamedfaouzimahboub@gmail.com).

The first experimental observations of ion beams emitted on the rear surface of a solid target were published in 2000 by three independent research groups [4]–[6]. These experiments showed the possibility of accelerating protons up to 58 MeV with an intense laser. In 2001, this acceleration mechanism called TNSA was proposed by Wilks et al. [7] to explain the experimental results [8], [9]. The TNSA theory is based on previous works on plasma expansion into vacuum [10], [11], and the acceleration process is based essentially on four steps.

At the same time that laser acceleration experiments developed and advanced at high speed, it was necessary to develop computational models to interpret them and to understand the mechanisms of laser-plasma interaction. The problem in this area of research is the estimation of ionic energy which is still below the expected value for the various applications. Numerical simulations indicate that the proton energy range suitable for cancer therapy is in the order of 70–240 MeV, whereas the energy obtained in the laboratory to date does not exceed 60 MeV [12]. Experimental, analytical and semi analytical models have been developed to deepen the understanding of the physical phenomena that contribute to the control of these ion beams in order to verify the numerical methods used in the simulations and to deduce useful scaling laws and easy to operate [13].

The mechanisms of acceleration and transport of the pre-plasma electrons formed in stage 1 of TNSA are of great importance in understanding the processes of ion acceleration. It is for this reason that throughout this work, great importance has been given to the modeling of these electrons, responsible for the ionic acceleration on the back of the target. Recent experiments [14] have demonstrated that the presence of pre-formed plasma and its scale in front of a solid target play a very important role in establishing the ion energy and then influencing ion acceleration. It was also established that an electrostatic potential well developed in the pre-plasma, accelerated electrons which gained large amounts of energy from laser, after being trapped inside the potential well.

The present work is interested mainly in fast, energetic electrons of the pre-plasma, ejected during laser matter interaction. These electrons are not always Maxwellian but follow distributions far from thermal equilibrium, influencing the acceleration process [15], [16]. Another phenomenon which occurs in the process is electron trapping. The latter is an initially non-negligible nonlinear effect, generated by the principal pulse of the laser in interaction with the front surface of the target, which consists in the creation of sufficiently large potential wells in the pre-plasma. These wells may be significant in the ion acceleration mechanism, thus influencing

the dynamics of the non-thermal electrons and the accelerating electric field of charge separation associated with them.

The contribution through this study will therefore be devoted to the study of the two effects, trapping and non-thermality of electrons in the pre-plasma on the ions acceleration mechanism, the electrons are supposed having densities governed by so-called "Cairns-Gurevich" equation, established earlier by Bara et al. [17], [18].

## II. MODEL EQUATIONS AND SELF-SIMILAR SOLUTION

Historically, the first most well-known analytical study on plasma expansion has been the relaxation in vacuum of a semi-infinite, quasi-neutral and isothermal plasma developed by Gurevich et al. in 1966 [19]. It was subsequently proven by Mora [20] that this model remained valid even for the study of plasma expansion and ion acceleration in intense laser-plasma interaction experiments, which makes it possible to deduce the maximum energy of the ions over time and the shape of the obtained spectrum. These studies used single-dimensional hydrodynamic codes considering either hot ions and electrons as an expanding plasma described with fluid or hybrid models (with a kinetic description for electrons and fluid for ions) [21]. As a result of this study, other authors have attempted to specify this model by taking into account the finite thickness of the target [22], the decrease in the energy of the electron reservoir, the existence of several ionic and electronic populations [23].

For the study of the role of pre-plasma on ion acceleration, the use of numerical simulation such as the Particle-in-Cell (PIC) method allowed identifying all the mechanisms responsible for the emission of protons from the front face or the rear face. All the experimental results are accompanied by numerical simulations in order to highlight the different processes that explain the observed results [24]–[26].

The hydrodynamic model used in this paper considers non-collisional, non-relativistic and non-magnetized, completely ionized pre-plasma created by CPA laser, expanding into space, consisting of one species of ions, which will simultaneously support the effects of electronic non-thermality and trapping phenomena.

The normalized electron density to the initial value is given by the Cairns-Gurevich equation established earlier by Bara et al. [17], [18]

$$\tilde{n}_e = n_e / n_{e0} = (b\Phi^2 - b\Phi + 1) \left\{ e^\Phi \left( 1 - \operatorname{erf} \left( \sqrt{|\Phi|} \right) \right) + 2 \frac{\sqrt{|\Phi|}}{\sqrt{\pi}} \right\} \quad (1)$$

$|\Phi| = |e\phi|/T_e$  is the normalized electrostatic potential of the pre-plasma.  $e$ ,  $T_e$ , and  $n_{e0}$  are the electron charge, the electron temperature and the unperturbed electronic density, respectively.  $b$  ( $0 < b < 1$ ) is a parameter that determines the population of non-thermal electrons in the plasma.

When there is no energetic electrons, *i.e.*,  $b = 0$ , we are in the situation of total electron trapping in the maximum height potential well, described by Gurevich in [27].

Ions with density  $n_i$  and velocity  $v_i$ , in the cold approximation, are described by the following continuity and motion equations, in 1D dimension:

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0 \quad (2)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{1}{m_i n_i} \frac{\partial P_i}{\partial x} + \frac{Ze}{m_i} \frac{\partial \phi}{\partial x} = 0 \quad (3)$$

$m_i$  is the ion mass and  $Z$  is the ion charge number taken equal to 1. The plasma is considered as an ideal gas such as  $P_i = n_i T_i$ , where  $T_i$  is the ion temperature.

The size of the phenomenon of pre-plasma expansion is very big compared with the characteristic Debye length of electron oscillations  $\lambda_D$ , so it is justified to use the quasi-neutrality of charges valid along the expansion.

$$n_e = n_i \quad (4)$$

Equations (2) and (3) with quasi-neutrality assumption (4) describing plasma expansion phenomenon admits a self-similar solution [28], [29]. This latter is constructed by using the ansatz defined with normalized variables such as  $\tilde{n}_i = n_i / n_{i0}$  and  $\tilde{v}_i = v_i / c_s$ ,  $c_s$  is the ion sound velocity given by  $c_s = \sqrt{T_e / m_i}$  and  $n_{i0}$  is the initial density of the plasma. In the self-similar approach adopted in this study, the two variables of space and time are combined into one dimensionless self-similar variable  $\xi = x / c_s t$ .

The transformation of variables leads to the following system of normalized ordinary hydrodynamic equations:

$$(\tilde{v}_i - \xi) \frac{d\tilde{n}_i}{d\xi} + \tilde{n}_i \frac{d\tilde{v}_i}{d\xi} = 0 \quad (5)$$

$$(\tilde{v}_i - \xi) \frac{d\tilde{v}_i}{d\xi} + \frac{\delta}{\tilde{n}_i} \frac{d\tilde{n}_i}{d\xi} + \frac{d\Phi}{d\xi} = 0 \quad (6)$$

$\delta = T_i / T_e$  represents the ratio of ion temperature to electron one. It takes small values when the ions are assumed to be much colder than electrons ( $\delta = 0.01$  in this study).

Differentiating (1) and assuming the plasma quasi neutral (4), we get:

$$\frac{d\tilde{n}_i}{d\xi} = H(\Phi) \frac{d\Phi}{d\xi} \quad (7)$$

where

$$H = 2(2b\Phi - b) \left( \frac{\sqrt{|\Phi|}}{\sqrt{\pi}} \right) + (b\Phi^2 - b\Phi + 1 - b) e^\Phi \operatorname{erfc}(\sqrt{|\Phi|})$$

Then, (6) takes the form:

$$\left(\frac{1}{H}\right)\frac{d\tilde{n}_i}{d\xi} + (\tilde{v}_i - \xi)\frac{d\tilde{v}_i}{d\xi} = 0 \quad (8)$$

Solve the system of (5) and (8), we consider the derivative terms as independent variables and the solution is nontrivial only if the determinant of its coefficients vanishes [30].

The chosen solution is that concerning the expansion of plasma in the positive values of  $\xi$  and considering that ion velocity increases with  $\xi$ :

$$\tilde{v}_i = \xi + \sqrt{\delta + \frac{\tilde{n}_i}{H}} \quad (9)$$

With the new equation obtained by differentiating (9), ion density and electrostatic potential are given by the solution of the following system of equations:

$$\frac{d\tilde{n}_i}{d\xi} = \frac{-\tilde{n}_i\sqrt{\delta + \tilde{n}_i/H}}{(\delta + 1.5\tilde{n}_i/H - 0.5\tilde{n}_i^2L/H^3)} \quad (10)$$

$$\frac{d\Phi}{d\xi} = \frac{-\tilde{n}_i\sqrt{\delta + \tilde{n}_i/H}}{H(\delta + 1.5\tilde{n}_i/H - 0.5\tilde{n}_i^2L/H^3)} \quad (11)$$

where

$$L = e^{\Phi} \operatorname{erfc}\left(\sqrt{|\Phi|}\right)(b\Phi^2 - b\Phi + 1 - b) + 4b\left(\frac{\sqrt{|\Phi|}}{\sqrt{\pi}}\right) - \left(\frac{b\Phi^2 - b\Phi + 1}{\sqrt{\pi}\sqrt{|\Phi|}}\right)$$

At  $t = 0$ , where the plasma is at rest, the beginning of expansion into vacuum corresponds to a point  $\xi_0$  to be determined from (9) and the initial conditions are the following:  $\tilde{v}_i(\xi_0) = 0$ ,  $\tilde{n}_i(\xi_0) = 1$  and  $\Phi(\xi_0) = \Phi_0$ .

Solving numerically the system of (10) and (11) with (9) gives the density, velocity and potential profiles according to the variable  $\xi$ , depending on the initial conditions of the plasma expansion.

#### A. Parametric Investigation

The goal of this study is to analyze the influence of the two coupled phenomena (non-thermality and trapping) present in the pre-plasma, on ion acceleration by varying the two parameters  $b$  and  $\Phi_0$  and deduce what effect dominates and plays the main role in the ion acceleration mechanism. It is in fact an estimation of the influence of these electrons on the dynamics of ions.

We recall that trapping is a non-linear phenomenon which takes place in the pre-plasma created at the front face of the target by exciting plasma waves. This phenomenon increases with the intensity and the pulse duration of the laser beam focused on the front surface of the target.

The numerical value of the initial potential well of the pre-plasma  $\Phi_0$  is chosen arbitrary, but large enough to have non-negligible trapping [31].

The energetic electrons are generated by the interaction of the main pulse of the CPA laser with the electrons of the pre-plasma and are transported in the target, creating a strong electric field of charge separation, responsible of the ion acceleration.

The values of  $b$  are extended from ( $b = 0.01$ ), a situation where only few electrons are non-thermal to a situation where the majority of the electrons are non-thermal ( $b = 0.6$ ).  $b$  is limited to  $4/7 \approx 0.6$  because there is no appreciable effects beyond this value and in order to overcome physical instabilities that could appear in the electronic distribution [32], [33].

#### B. The Potential and the Longitudinal Electrostatic Field in the Pre-Plasma

The self-similar solution allows us to represent in Fig. 1, the electrostatic potential  $\Phi$  decreasing along the plasma expansion, as function of  $\xi$  for different values of  $b$ . The depth  $\Delta\Phi$  is calculated from Fig. 2, for each value of  $b$  as the potential difference from the beginning of the expansion where the initial ion velocity is zero, to the end of the expansion  $\xi_{lim}$ , corresponding to vanishing ion densities, given by the solution of the system of (10), (11).

The figure shows that for an enough large initial potential well, when the proportion of energetic electrons is decreasing, ( $b$  decreasing), the depth of the potential well  $\Delta\Phi$  is increasing through the expansion. For example: for  $b = 0.05$ ,  $\Delta\Phi \approx 0.63$  and for  $b = 0.6$ ,  $\Delta\Phi \approx 0.05$ . It means that the depth of the potential in the pre-plasma depends on the proportion of the thermal of electrons which are trapped and oscillate in the well.

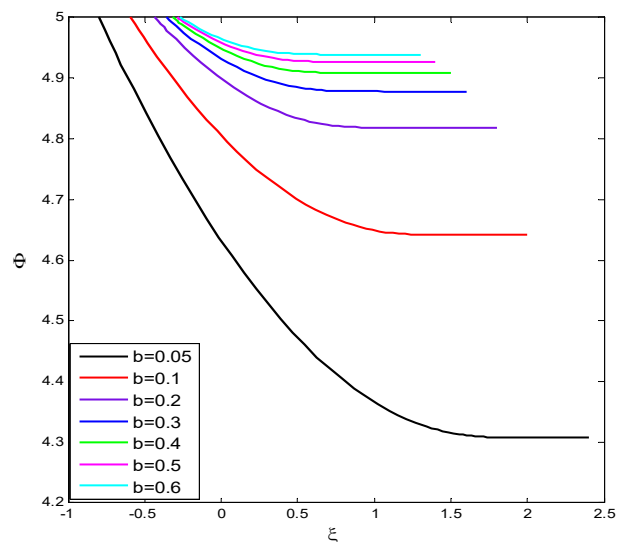


Fig. 1 Normalized electrostatic potential wells  $\Phi$  in the pre-plasma for different values of  $b$  for  $\Phi_0 = 5$

#### C. Ion Velocity and Ion Acceleration

In Figs. 2 (a) and (b), normalized ion velocities are deduced from the self-similar solution and plotted for small and large values of  $b$ , respectively in the case of  $\Phi_0 = 10$ .

Fig. 2 shows that ion velocities are increasing almost linearly (according to (9)) as function of  $\xi$  for different values of  $b$  and the plasma expansion is shortened with increasing  $b$  ( $\xi_{lim}$  decreasing with increasing  $b$ ).

For a given value of  $\xi$ , the ion velocity increases when  $b$  is decreasing, meaning that a greater number of trapped electrons contribute to ion acceleration process then front ion velocity of the plasma is increasing.

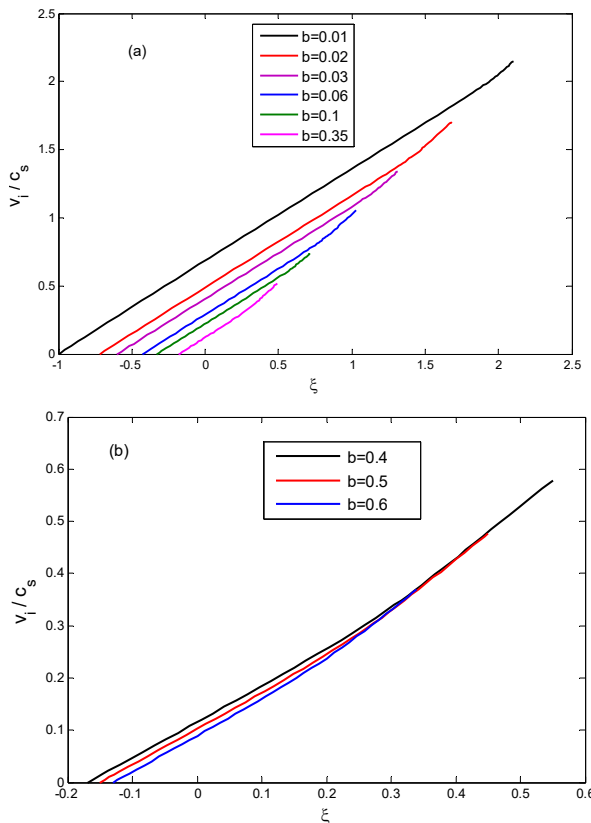


Fig. 2 (a), (b) Normalized ion velocities for small and large values of  $b$  respectively, in the case of  $\Phi_0=10$

The evolution of ion acceleration is represented by the ion velocity gradient and shown in Fig. 3 as a function of the proportion of energetic electrons  $b$  for different values of  $\Phi_0$ .

Here we notice that acceleration  $\Delta v_i / \Delta \xi$  of the ions is increasing with  $b$ , showing the role of energetic electrons, responsible of the increase of ion acceleration in the longitudinal direction (see Fig. 3). In this situation, in the range of small  $b$ ,  $0 \leq b \leq 0.4$ , the trapped electrons accelerated at high velocities, gain kinetic energy effectively from the excited plasma wave by the laser, to leave out the potential well and enter to the target, ionizing new atoms, and then participate to reinforcing the electric field of charge separation, and then in the process of ionic acceleration by ionizing new atoms. It is the same phenomenon that is observed in Laser Wake Field Acceleration mechanism (LWFA) in which electrons have enough energy to travel in resonance with the plasma waves and then will be trapped and

accelerated by the wakefield [34], [35].

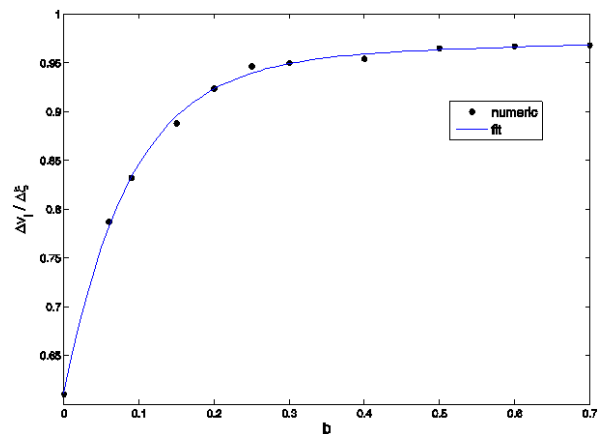


Fig. 3 Ion acceleration fitting for  $\Phi_0=20$  as function of  $b$

The large values of  $b$ ,  $0.4 \leq b \leq 0.6$ , correspond to the case where almost all electrons are energetic and free, resulting in a decrease of the potential in the pre-plasma. It may concern the first electrons ejected from the outer layers of the atoms of the front surface of the target, and are accelerated directly by the ponderomotive main force of the laser which is commonly known as conventional or direct acceleration without intermediate plasma wave [26].

In this situation, the shallow depth of the potential well corresponds to this range of  $b$ , as shown in Fig. 1. These fast electrons enter the target and ionize the atoms of the surface and accelerate the first ions by collision, and thus, there will be generation of new electrons by ionization.

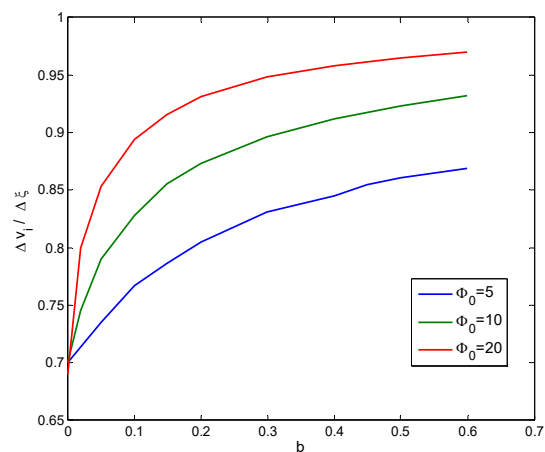


Fig. 4 Ion acceleration as a function of  $b$  for different  $\Phi_0$

We also notice in Fig. 4, a saturation of the ion acceleration showing that, when the proportion of nonthermal energetic electrons is very important, the combined effect of trapping and acceleration by the pre-plasma becomes negligible and the potential well in the pre-plasma has no role in the ion acceleration process. Here, it is the phenomenon of direct acceleration by ponderomotive force of the laser which

dominates [26], meaning that the acceleration of the ions is mainly due to the population of the energetic electrons which constitutes the component for large values of  $b$ .

This phenomenon is interpreted as if there is no effect of pre-plasma anymore on ion acceleration and the situation is similar to the interaction of the main pulse of laser on a “clean” target without pre-plasma formation and the absorption of the laser became ideal, a situation which is not realistic.

The presence of pre-plasma is then important in the ion acceleration process, having the role of increasing laser energy absorption and providing more energy to the electron component which facilitates the ion acceleration [24], [36].

It is also shown from the figure that for a fixed  $b$ , increasing initial potential leads to accelerate more electrons participating in ion acceleration.

In Fig. 5, ion velocity is observed as function of  $\xi$  for different values of the potential in the case where the majority of electrons are energetic ( $b = 0.6$ ). In this case, it is shown that increasing initial potential has nearly no effect on ion acceleration. The energetic electrons remain practically free during the expansion and do not undergo the effects of the plasma waves because of their high energy; they override the wave and go away from the ensemble of the trapped electrons in the potential well of the pre-plasma, creating an electrostatic field of charge separation.

Consequently, the obtained self-similar solution shows that when the population of the energetic electrons is dominant, it is the phenomenon of acceleration due to these electrons which overrides the trapping in the expansion of plasma.

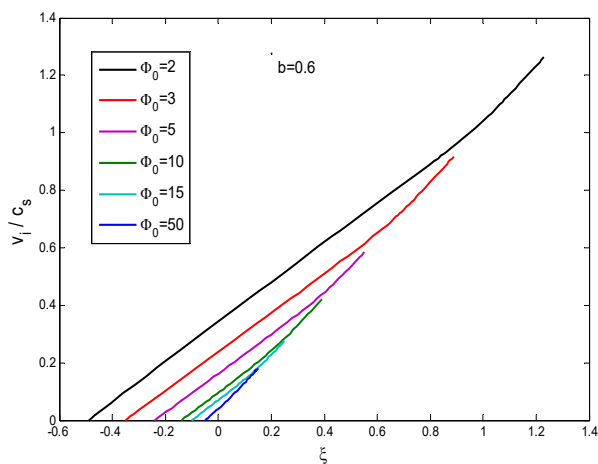


Fig. 5 Normalized ion velocities for different values of  $\Phi_0$  in a case of large number of energetic electrons

The result of this parametric study of the electron acceleration mechanism in the pre-plasma potential well and their effects on the acceleration of ions at the first moments of the ion acceleration process is that the electric field of separation of charge of the ions in the target is reinforced by the longitudinal electric field of the pre-plasma created by the oscillation of the electrons in phase with the potential well of

the pre-plasma excited by the main pulse of the target.

This allowed us to optimize the importance of the presence of the pre-plasma on the proportion of electrons that effectively participate in the strengthening of the electric field of separation of the ionic accelerating in the TNSA mechanism.

These results can be interpreted as if the global potential gradient governing the ion acceleration had two contributions, one of them is represented by the ambipolar electric field of charge separation induced by free energetic electrons and the other which is created by the oscillations of trapped electrons in the potential well of the pre-plasma [36].

### III. CONCLUSION

Ion acceleration process associated with the expansion of expanding pre-plasma in the vacuum is studied, taking into account two physical phenomena that can occur in the pre-plasma created from the first acceleration stage of TNSA mechanism.

There, non-linear effects are generated by the arrival of the main pulse of the laser on the pre-plasma, namely the phenomenon of electronic trapping, as well as the generation of energetic, non-thermal electrons.

The obtained results show that the proportion of the energetic electrons influences the acceleration of the ions in the presence of large potential wells in the pre-plasma.

To summarize, the results found from this work can be pinpointed as follows:

- i) The smaller the parameter  $b$ , the greater the number of trapped thermal electrons and the greater the potential in the pre-plasma.
- ii) Ion acceleration is more efficient with increasing  $b$ .
- iii) When almost all electrons are energetic, there is no influence of trapping on the self-similar expansion and the electron acceleration is mainly due to the ponderomotive force of the main pulse of the laser.

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