Effect of Non Uniformity Factors and Assignment Factors on Errors in Charge Simulation Method with Point Charge Model

Gururaj S Punekar, N K Kishore *Senior Member IEEE,* H S Y Shastry

Abstract—Charge Simulation Method (CSM) is one of the very widely used numerical field computation technique in High Voltage (HV) engineering. The high voltage fields of varying non uniformities are encountered in practice. CSM programs being case specific, the simulation accuracies heavily depend on the user (programmers) experience. Here is an effort to understand CSM errors and evolve some guidelines to setup accurate CSM models, relating non uniformities with assignment factors. The results are for the six-point-charge model of sphere-plane gap geometry. Using genetic algorithm (GA) as tool, optimum assignment factors at different non uniformity factors for this model have been evaluated and analyzed.

It is shown that the symmetrically placed six-point-charge models can be good enough to set up CSM programs with potential errors less than 0.1% when the field non uniformity factor is greater than 2.64 (field utilization factor less than 52.76%).

Keywords—Assignment factor, Charge Simulation Method, High Voltage, Numerical field computation, Non uniformity factor, Simulation errors.

I. INTRODUCTION

THE charge simulation method (CSM) is one of the widely used numeric field computation technique ideally suited for simulating open boundary problems [1-3]. The high voltage engineering makes use of this technique extensively [4]. The method in its simplest form (conventional CSM) computes the charge magnitudes by satisfying the boundary conditions at the selected number of contour points. The locations of the charges and the boundary conditions are predetermined and supplied based on the experience [1] of the researcher. The unknown charges are computed from the relation (1) by setting up simultaneous equations

Gururaj. S. Punekar is with the with the Department of Electrical & Electronics Engineering, National Institute of Technology, Karnataka 575025 INDIA (phone:+91-(0)824-2474000Ext3495; fax: +91-(0)824-2474033; e-mail: gururaj.punekar@ gmail.com).

N. K. Kishore, is with the Department of Electrical Engineering, Indian Institute of Technology, Kharagpur 721302 INDIA (e-mail: kishor@ee.iitkgp.ernet.in).

H. S. Y. Shastry is with the Department of Electrical & Electronics Engineering, National Institute of Technology, Karnataka 575025 INDIA (e-mail: hsys@nitk.ac.in).
.

$$[P] [Q] = [V] \qquad (1)$$

Where

[P] is the potential coefficient matrix

[Q] is the column vector of unknown charges

[V] is the column vector of known potentials at the contour points.

Resulting simulation accuracy strongly depends on the type and number of charges, locations of contour points and complexity of electrode geometry [1-2]. As CSM accuracy depends on the choice of type of simulating charge, their number, location of these charges and the contour points, the CSM programs for particular application become case specific and depend on the programmer (or developer). Hence, to set up an accurate CSM model, familiarity and understanding of the programmer (or developer) with the CSM plays a key role. In order to help the user, the empirical relations relating location of charges with those of the contour points become useful. One such widely used parameter is the assignment factor 'f_a' [1]. The attempts have also been made to locate the charges using optimized charge simulation methods [5-9] instead of assignment factor as the guiding parameter. With GA as a tool with number of charges pre-decided (by the programmer or developer) using point charges (also pre-decided by the user) automatic allocation of these charges and contour points is attempted, relatively recently [10]. All these efforts are to reduce the need of users experience in setting up accurate CSM models. Even with all these efforts the CSM programs have remained user and case specific as regards to the choice of type and number of charges. The CSM being a semi analytical technique makes use of potential and field coefficients of simulating charge configurations [1-2], user interference and knowledge can be an advantage. Based on the symmetries of the simulating charges and those of the geometry simulated, user with his/her experience can guide the charge arrangement in relation with the contour point locations. Hence, it is felt that empirical guiding parameter like assignment factor and its impact on the simulation errors needs further understanding. The present work is such an effort, with symmetrically arranged six point charges in simulating electric fields associated with the sphere-plane geometry.

The earlier investigations [1, 11] of CSM errors involving assignment factors are not relating to electric field non

uniformities. Also, the efforts give guidelines with ring charges as the fictitious charges and indicate that assignment factors in the region of 1 to 2 can achieve acceptable accuracies.

The effort in this work has been to relate the assignment factor with the field non uniformity factor [12] which quantifies the degree of non uniformity associated with geometry. This has been attempted with six-point-charge model of sphere-plane gap. Using GA as tool best assignment factors at which errors are minimum have been computed at selected set of non uniformity factors, covering the wide range (near uniform to highly non uniform) of non uniformities. These results are believed to be unique and are being reported, perhaps, for the first time. They should help the CSM programmers, in furthering the understanding of CSM errors and aid in setting up accurate CSM programs.

II. PARAMETERS OF STUDY AND MODEL

A. Assignment Factor 'f_a'

It is defined as the ratio of the distance between a contour point and the corresponding charge 'a2' to the distance between two successive contour points 'a1', as given in relation (2). The schematic showing the charges and the contour points given in figure 1 depicts distances 'a1' and 'a2'.
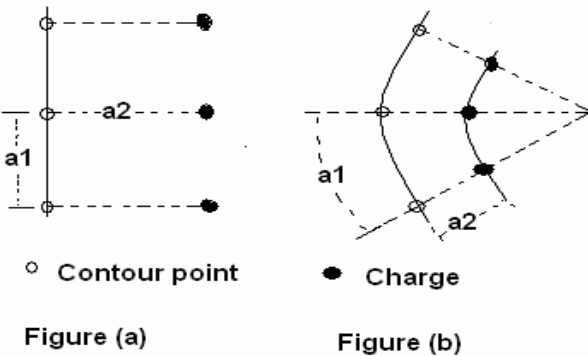

Fig. 1 Schematics (a) and (b) to explain assignment factor.

$$f_a = \frac{a2}{a1} \qquad (2)$$

In setting up a CSM model, in simulating particular gap geometry, user decides on the type of simulating charges based on the profile of the geometry. S/he also decides the number and general arrangement taking in to account the accuracy requirements and symmetry. Further, it is the exact location of these charges in relation with the contour points that needs to be decided, to maximize the accuracy. This aspect is quantified by the parameter, 'assignment factor'. Literature, states that it should be in the region of 1 to 2 for low error [1, 11]. This is true, with ring charges, particularly with large number of charges (number of charges 10 or more). In actuality, the range over which this parameter can vary is

specific to a model based on the type of charges and their number. For sphere-plane model this can vary in the range of 0 to '2/π' (=0.63) as explained below, in the model details.

B. Model details

1) Geometric details of sphere-plane model

The geometric model of the sphere-plane gap with the image sphere is as shown in figure 2. Image sphere is used to simulate the infinite ground plane.
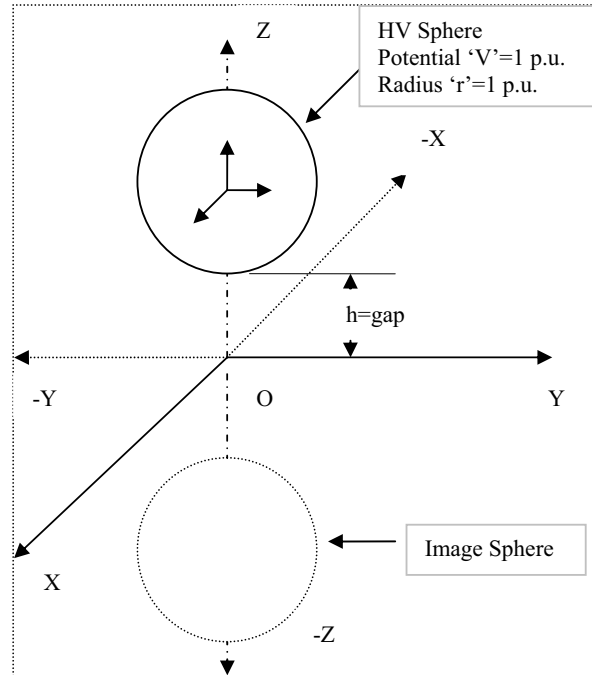

Fig. 2 The sphere-plane gap simulated (shown along with the Cartesian frame of reference and image electrode).

The sphere electrode radius, 'r', is considered as 1 per unit. And with respect to this, the gap separation is, 'h' per unit. It is the dimension of 'h' in relation with 'r' that decides the electric field non uniformity (and hence the electric field non uniformity factor) of the geometry [12].

2) CSM Model details

The sphere-plane geometry (figure 2) is simulated using the six point charges arranged symmetrically as shown in figure 3. The charges are placed inside the sphere with corresponding image charges inside the image sphere. These charges are on a concentric sphere of radius 'rc'. This radius 'rc' can assume values form 0 to 'r'(=1); where 'r' is the radius of the sphere electrode. This forms 'a2' of the assignment factor (equation 2). The contour points in the model are chosen such that, they are on the electrode surface, along the line joining the corresponding charge and the center of the sphere. Then the distance between the two successive contour points is '$\pi r/2$' units. This distance forms 'a1' of the assignment factor (equation 2). Hence, with these ranges for 'a1' and 'a2', the assignment factor can assume value between 0 to '2/π' (i.e. 0
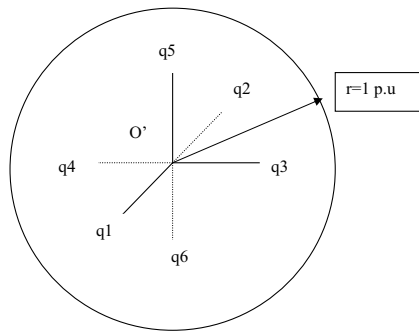
to 0.63).

Fig. 3 General charge arrangement within the HV sphere shown in figure 1. (O' is the center of the sphere. Charges q1, q2, q3, q4, q5 and q6 are the six charges placed on the coordinate axes within the sphere)

C. Field non uniformity factor 'f'

The electric field non uniformity is defined as the maximum electric field intensity in the gap (which occurs near the tip of the high voltage electrode to the average electric field intensity (V/h; figure 2). The reciprocal of this, field non uniformity factor, is called the electric field utilization factor. The field utilization factor can assume value from 0 to 100% and can be used to interpret how best the insulation in the gap is being utilized. These factors have been extensively used in the literature in interpreting the electric field dependency, breakdown and corona inception behaviors of electrical insulation. For this purpose these factors are also computed and reported in the literature for the simple geometric forms [12, 13]. These parameters are used to understand the error variation in simulating the CSM model along with charge and contour point arrangement. The field utilization factors (also the field non uniformity factor) which depend on the gap spacing 'h' and the sphere radius 'r' are reported for the sphere-plane gap in reference [12]. This data is used to compute the corresponding CSM errors by successive simulations.

III. RESULTS AND DISCUSSION

The numerical experiments are conducted on the six-point-charge CSM model of the sphere plane gap with the assignment factor and the field utilization factors as the parameters. The results for maximum potential error on the surface of spherical electrode taking 100 test points, computed for selected values of field utilization factor, chosen from the entire range are reported. The useful range of the assignment factors varying from 0.2 to 0.63 are considered in analyzing the CSM simulation errors.

The plots of percentage potential error on the sphere electrode surface for the field non uniformity factor of 9.55 (electric field utilization factor of 10.5%), with typical assignment factor (with f_a=0.6) is as given in the figure 4. The x and y axes in these plot are the angle 'phi' and 'theta' coordinates of the points on the sphere (polar co-ordinates).

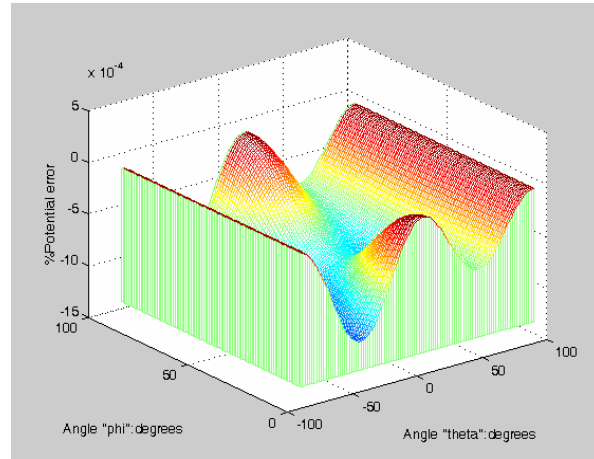The maximum error in potential in this case is seen to be of the

Fig.. 4 Plot of percentage potential error on the sphere surface for sphere-plane model. (Assignment factor fa=0.6, field non uniformity factor f=9.55).

order of 1.3e-3 percent. The field deviation angle error is the angular difference between the normal component of electric field intensity at the electrode surface and surface normal drawn at that point. The field deviation angle error for this particular case is observed to be less than 0.14°. These errors are acceptable, as generally the CSM errors less than 0.1% in the potential is considered reasonable [2]. The similar test simulation runs indicated that the errors less than 0.1% are achievable with the field utilization factors being less than 52.76% (non uniformity factor greater than 2.64).

The variation of maximum potential error as a function of assignment factor obtained by numerical experimentation in the range 0.56 to 0.63 is given in figure 5. As seen from these simulation results the maximum potential error varied in the range of 113% to 1.14e-006%. Due to this wide variation, in this error plot, z-axis is with logarithmic (base 10) values.
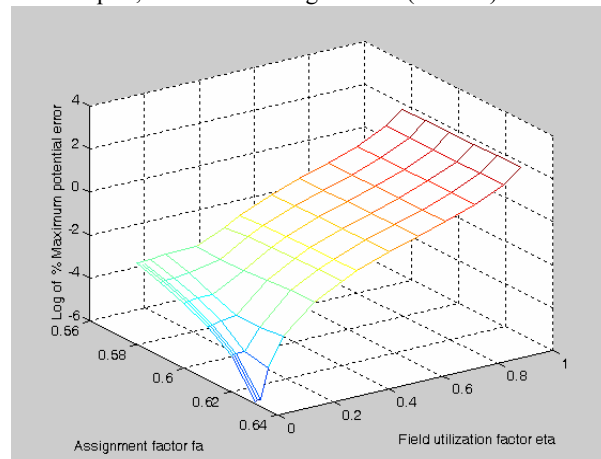

Fig. 5 Percentage maximum potential error on the sphere surface as a function of assignment factor 'f_a' for selected values of field utilization factor (over its entire range).

The plot shows dependency of CSM maximum potential error on both assignment factor and the field utilization factor. At higher field utilization factors the influence of the assignment factors is relatively low with errors being higher. But in the lower range of field utilization factors (<40%), the assignment factors show a higher influence. There is definite unique value of the assignment factor (specific to non uniformity factor) at which the simulation errors are a minimum. The observation made is, as the utilization factor decreases the assignment factor at which best results are obtained shifts towards the higher value.

The maximum field deviation angle error, also termed as Beta error (βmax), varied in the range of 65.3˚ to 1.47e-005˚ (over the entire range of field utilization factor and useful range of assignment factor studied).

At lower values of field utilization factor the assignment factor shows a great degree of influence. On the contrary as the field utilization factor decreases (from 100%), the simulation accuracies increase (for any chosen assignment factor). It is to say that, high accuracy simulations are possible, when the fields are non uniform.

The summation of the simulating charges in CSM should remain constant irrespective of their locations (also of contour point) for a fixed non uniformity. The variation in this can be a possible error criterion, which is investigated. Figure 6 shows the variation in summation of simulating charges as a function of assignment factor and the field utilization factor. As expected, system capacitance decreases with decrease in field utilization factor. But the summation of simulating charges shows little variation with the changes in the assignment factor.
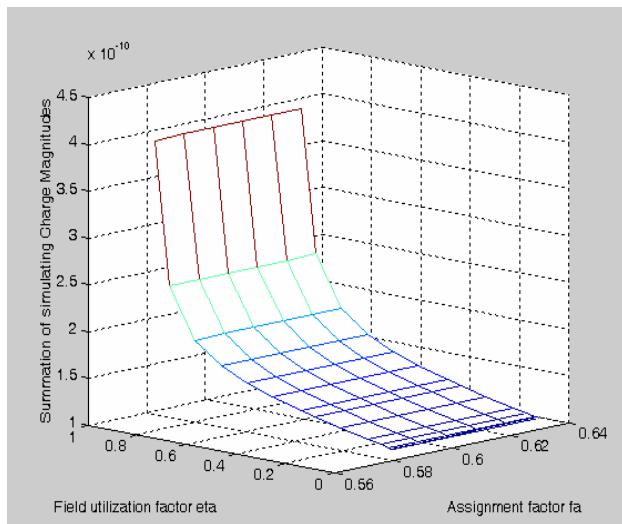

Fig. 6 Plot of summation of simulating charges of the sphere as a function of assignment factor 'fa' for selected values of field utilization factor 'η' over its entire range (six-point-charge model).

Optimal assignment factors using Genetic Algorithm (GA)

The optimal value of the assignment factor is obtained for few typical values of the utilization factors chosen from its entire range (listed in [12]), using GA as the optimization tool [14]. The root mean square (rms) potential error on the surface formed the objective function in error minimization. The rms value is obtained by evaluating errors at 100 regularly spaced points on the surface of sphere. The GA as the optimization tool used randomly generated initial population of size 40. The algorithm used 25 number of generation as the termination criteria. The bound for assignment factor used is 0.2 to 0.63.

The evaluated optimal values of the assignment factors for the corresponding field non uniformity factors are given in table-I. Using these optimal values of assignment factors (obtained using GA-CSM program), the maximum potential error, maximum deviation angle error, rms potential error and rms deviation angle error are calculated for different field non uniformity factors listed in table-I. These results are given in figure 7 and 8. Plots in figure 7 indicates that, at higher values of assignment factor the simulation errors are the lowest; but higher optimal values are possible only at higher non uniform field factors (as indicated by figure 8 and table-I).
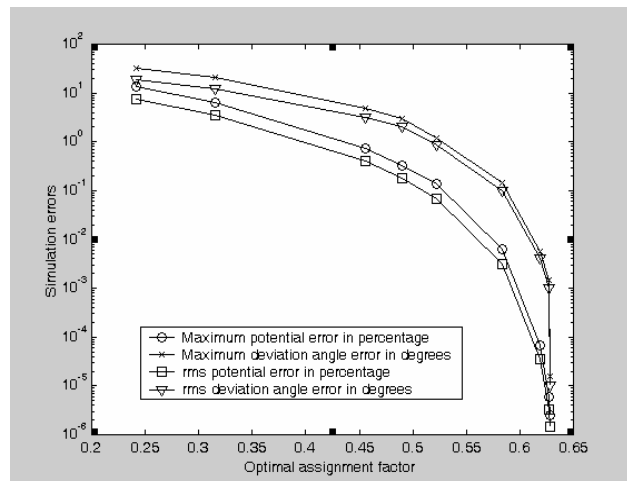

Fig. 7 CSM simulation errors as a function of optimal assignment factor (corresponding to the each non uniformity factor) for sphere plane gap (with six-point-charge model).

These results indicate that even with the optimal assignment factor there is a definite limit to achieving minimum error for a particular non uniformity. Further reduction in errors, is possible only with the different set (shape) of charges and/or increased number of charges. In this sense, CSM models have remained case specific and user specific as regards to the choice of type and number of charges. It is felt, that under such circumstances this effort to analyze the error variations with few (six in this case) symmetrically arranged point charges (being the most basic and simplest charge configuration) and the results reported for different non

TABLE I
FIELD NON UNIFORMITY FACTOR, OPTIMAL ASSIGNMENT FACTORS AND
CSM ERRORS

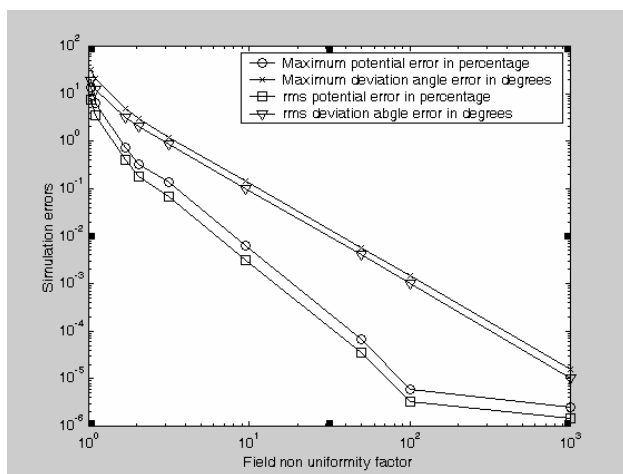| Non Uniformity Factor	Optimal Assignment Factor	Root Mean Square Potential Error in Percentage
1.0336	0.2417	7.5446
1.1020	0.3154	3.4553
1.6846	0.4561	0.4087
2.0342	0.4896	0.1752
3.1516	0.5222	0.0690
9.5511	0.5830	0.0031
49.5049	0.6188	3.5794e-005
100.0000	0.6269	3.2093e-006
1000.0000	0.6286	1.4768e-006

uniformity factors will be useful.

Fig. 8 CSM simulation errors as a function of non uniformity factor (with optimal assignment factor based charge locations) for sphere plane gap (with six-point-charge model).

IV. CONCLUSION

The CSM error dependency on charge-contour point arrangements and field non uniformity is reported for sphere-plane model using most elementary charges, namely, point charges.

Generalized interpretation is, it's simpler to setup highly accurate CSM models when the electric field non uniformity of the geometry being simulated is high. In such situation the charges are clustered far away from the electrode surface (contour points).

REFERENCES

[1] H Singer, H. Steinbigler P. Weiss, "A change simulation method for the calculation of high voltage fields", *IEEE trans, PAS* vol.93, 1974, pp1660-68.
[2] Nazar H Malik, "A review of the charge simulation method and its application", *IEEE Transaction on electrical insulation*, Vol.24, No.1, 1989, pp. 1-20.
[3] S. Chakravorty, "Charge Simulation Method: a critical overview", *Ie (I) journal-el,* Vol. 78, March 1998, pp. 210-214.
[4] E. Kuffel, W.S.Zeangle & J.Kuffel, "High Voltage Engineering fundamentals*"*, Newnes, An imprint of Butterworth-Heinemann, A division of Reed Educational and Publishing Ltd, Woburn, MA. 2000, pp.254-269.
[5] H Anis, A Zeitoun, M El-Ragheb m and El-Desouki, "Field calculations Around Non-Standard Electrodes Using Regression and Their Spherical Equivalence", IEEE Trans on PAS, Vol.PAS-96, no.6, November-December 1977, pp.1721-1730.
[6] A Yializis, E.Kuffel, P H Alexander, "An Optimized Charge Simulation Method for the Calculation of High Voltage fields", IEEE Transaction on PAS-97, 1978, pp. 2434-40.
[7] Y L Chow and C Charalambous, "Static-field computation by method of optimized simulated images", Proc. IEE, Vol.126, No.1, 1979, pp.123-125.
[8] M R Iravani, M R Raghuveer, "Accurate Field Solution in the Entire Inter electrode Space of A Pod-Plane Gap Using Optimized Charge Simulation", IEEE Transactions on EI Vol.EI-17 No.4, August 1982, pp. 333-337.
[9] M M Abouelsaad, M M El Bahy: "Accurate Field Computation of Needle-Plane Gaps using an Optimized Charge Simulation Method", Conference on Electrical Insulation and Dielectric Phenomena, 2000, pp506-509.
[10] Ryo Nishimura, Katsumi Nishimori, Naganori Ishihara, "Automatic arrangement of fictitious charges and contour points in charge simulation method for two spherical electrodes", Journal of electrostatics 57, 2003, pp. 337-346.
[11] M.Th.El-Mohandes, H Okubo, "Error analysis based on the interaction between simulating charges in the CSM for the Electric-Field Calculation of HV Apparatus", *European Transactions on Electric Power, ETEP,* Vol.4, No.6, November/December 1994, pp565-570.
[12] H. McL Ryan and C.A. Welley, "Field Auxiliary Factors for simple electrode geometries". Proc IEE Vol 114, no.10, Oct, 1967, pp 1529-1536.
[13] Y. Qui, "Simple expression of field nonuniformity factor for hemisphereically capped rod-plane gap", IEEE Trans. Electrical Insulation, Vol.21, No.4, 1986, pp. 673-675.
[14] Christopher R Houck, Jeffery A. Joines and Michael G.Kay, A genetic algorithm for function optimization: A Matlab implementation, North Carolina State University, available at http://www.ie.ncsu.edu/mirage/GAToolBox/gaot//papers/gaotv5.ps.

Gururaj S Punekar received his B.E (Electrical Engg.) from Karnataka University Dharwad, Karnataka, India. He received the MSc (Engg.) degree from High Voltage Engineering department, IISc, Bangalore in 1991. He is presently pursuing his doctoral research having registered with Department of Electrical Engineering, IIT Kharagpur, West Benga, India.
Currently, he is with department of Electrical Engineering, National Institute of Technology, Karnataka, India. He was with Tata Electric Co., Mumbai, in the year 1991-92. From 1992 he is in academics. His areas of interest include HV testing, electric field computation, optimization and GIS.
Mr. Punekar is a member of Institution of Engineers (India), Life member of Indian Society for Technical Education, Life Member of Systems Society.
N K Kishore (SM'96) obtained B.E.(Electrical Engg.) from Osmania University, Hyderabad in 1983, M.E. (Electrical Engg.) from IISc Bangalore in 1985 and Ph. D from IISc. Bangalore in 1991. He worked as a Scientific Officer with IISc Bangalore from 1987 to 1991. He joined on the faculty of Electrical Engg. at IIT Kharagpur from 1991.
Currently, he is a Full Professor there. His areas of interest include High Voltage Engineering, Power Systems, Lightning, EMI/EMC, Condition Monitoring of Power Apparatus and Industrial Applications of High Voltages.
Dr Kishore is a Senior Member of IEEE, a Fellow of Institution of Engineers (India) and a life member of System Society of India.
H. S. Y. Shastry received BE degree in electrical engineering from Bangalore University, Karnataka, India, (1969), M.E. degree in power systems from Bangalore University, Karnataka, India, (1971), and the Ph.D degree from IIT Kanpur (1984).
He joined department of electrical engineering K R E C Surathkal (presently NITK), Karnataka , India as lecturer in the year 1972, became Assitant Professor in the year 1984. He became full Professor in the year 1994. His areas of interests include power system dynamics and numerical computations.
Dr Shastry is a member of Indian Society for Technical Education (ISTE).