

Effect of Linear Thermal Gradient on Steady-State Creep Behavior of Isotropic Rotating Disc

Minto Rattan, Tania Bose, Neeraj Chamoli

Abstract—The present paper investigates the effect of linear thermal gradient on the steady-state creep behavior of rotating isotropic disc using threshold stress based Sherby's creep law. The composite discs made of aluminum matrix reinforced with silicon carbide particulate has been taken for analysis. The stress and strain rate distributions have been calculated for discs rotating at linear thermal gradation using von Mises' yield criterion. The material parameters have been estimated by regression fit of the available experimental data. The results are displayed and compared graphically in designer friendly format for the above said temperature profile with the disc operating under uniform temperature profile. It is observed that radial and tangential stresses show minor variation and the strain rates vary significantly in the presence of thermal gradation as compared to disc having uniform temperature.

Keywords—Creep, isotropic, steady-state, thermal gradient.

I. INTRODUCTION

UNDER many service conditions, materials are required to sustain steady loads for long period of time under different temperature conditions. In such conditions, material may continue to creep until its usefulness is seriously impaired. Creep can take place and lead to fracture at static stresses much lower than those which will break the specimen when loaded quickly [Journal of Iron and Steel Institute [9]]. Discs of gas turbines, jet engines, automotive and aerospace braking systems are usually operated at relatively higher angular velocities and are subjected to elevated temperatures or thermal gradations. However, the enhanced creep of aluminium and its alloys may be a big hindrance in such applications Gupta et al. [8] Wahl et al. [18] conducted the creep tests for rotating discs made of 12 percent chrome steel and simulated the results theoretically using von Mises and Tresca yield criteria where they observed that, by using Norton's creep law, the creep deformation in a rotating disc based on Mises criterion yield slightly lower values as compared to the experimental values; however, the theoretical results based on maximum-shear theory, was found to be in a better agreement with the test values. Masaru [10] described analytical method for steady-state creep of a rotating disk with variable thickness and nonuniform temperature distribution. Creep tests of rotating solid disks of forged 18-8 Mo stainless steel were carried out in a hot spin tester and specimens were cut out from the various parts of the disks and were put to creep tests and other material tests. The analytical results were

compared with experimental ones and it was observed that Mises criterion showed better agreement with the test data. Bhatnagar et al. [2] carried out creep analysis of orthotropic rotating disks with variable thickness for secondary stage of creep. General expressions for stresses and strain rates in the disks were derived using Norton's power law of creep and have been utilized to find stress and strain rate distributions for disks with uniform thickness, linearly and hyperbolically varying thickness. The numerical computations to study the effect of anisotropy and the profile of the disk on stresses and strains have been carried out using the method of successive approximations taking five different cases of anisotropy. Singh and Ray [17] estimated steady-state creep response using Norton's power law in an isotropic FGM rotating disc of aluminum silicon carbide particulate composites assuming linearly decreasing variation of silicon carbide particles from the inner to the outer radius of the disc. Singh and Rattan [16] carried out the analysis of steady-state creep in a rotating disc made of $Al - SiC_p$ using isotropic Hoffman yield criterion and the results were compared with those using von Mises criterion ignoring difference in yield stresses and observed that the stress distribution was not much affected due to the presence of phase specific thermal residual stress. But its presence significantly affected the tangential strain rates in an isotropic rotating disc. Rattan et al. [12] investigated the creep response for isotropic rotating disc made of particle-reinforced FGM where the result obtained for non linear variation of particle distribution along the radial distance of the disc were compared with the disc containing the same amount of particle, distributed uniformly or linearly along the radial distance. Chamoli et al. [4] investigated the creep behavior of a thick walled hollow circular cylinder, made of aluminium silicon carbide particulate composite material. The effect of pressure on the stresses and strain rates in the cylinder were investigated and it was observed that with the increase of the internal pressure in the cylinder the strain rate increases. Further, Rattan et al. [13] studied the creep behavior of an anisotropic rotating disc of functionally graded material (FGM) using Hill's yield criteria and the creep behavior was assumed to follow Sherby's constitutive model and observed that the anisotropy of the material had a significant effect on the creep behavior of the FGM disc. It was also observed that the FGM disc showed better creep behavior than the non-FGM disc. Garg et al. [6] analyzed the steady-state creep in a rotating disc having linearly varying thickness in the presence of linear thermal gradient and observed that the tangential stress increased near the inner radius but decreased towards the outer radius and the radial stress increased throughout when

Minto Rattan is Assistant Professor in the Department of Applied Science, University Institute of Engineering and Technology, Panjab University, Chandigarh, India (e-mail: minto@pu.ac.in).

N. Chamoli is Assistant Professor in the Department of Mathematics, D.A.V. College, Chandigarh, India (e-mail: neeraj_chamoli@yahoo.com).

the FGM disc was assumed to operate under a linear thermal gradient.

Thus literature reveals that a number of studies (Singh and Ray [17], Gupta et al. [7], Gupta et al. [8], Rattan et al. [12]) have been conducted to investigate the creep behavior of rotating discs operating at uniform temperature. Rattan et al. [14] investigated the steady-state creep behavior of thermally graded isotropic disc rotating at elevated temperature taking into account the phase-specific thermal residual stress. It was observed that there is a significant change in the stress distribution due to the presence of thermal residual stress. The creep analysis was carried out using isotropic Hoffman yield criterion. Bose and Rattan [3] made an attempt to study the effect of thermal gradation on steady-state creep for rotating disc made of linearly varying functionally graded material. The stress and strain rate distributions have been calculated for the discs rotating at linearly and parabolically decreasing temperatures. The analysis indicated that stress in composite disc operating under thermal gradient slightly increases as compared to disc operating at constant temperature.

Thus this paper throws light on the effect of linear thermal gradation on steady-state creep for isotropic rotating composite disc. The results will be compared graphically for discs operating at linear temperature profile with disc operating at uniform temperature.

II. MATHEMATICAL ANALYSIS

A. Disc Profile

Let us consider an annular disc of aluminium, Al , matrix reinforced with silicon carbide particles, SiC_p , having inner radius, ($a = 0.03175mm$), and outer radius, ($b = 0.1524mm$), as taken by [18]. The volume fraction, V , of the reinforcement is considered to be 10 vol. % fraction of the constituent material $Al - SiC_p$. Using the law of mixtures, the density variation in the composite is expressed as

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \frac{V}{100} \quad (1)$$

where $\rho_m = 2713kg/m^3$ and $\rho_d = 3210kg/m^3$ are the densities of the matrix alloy and of the dispersed silicon carbide particles, respectively [5].

For the present study, the discs with following temperature profiles have been considered:-

- Disc D_1 operating at *uniform temperature* of 623K along the radial distance.
- Discs D_2 , D_3 and D_4 operating at *linearly decreasing* thermal gradient have temperature distribution, T_r given as:

$$T_r = A - Br, \quad a \leq r \leq b \quad (2)$$

where

$$A = \frac{bT_a - aT_b}{b - a} \quad (3)$$

and

$$B = \frac{T_a - T_b}{b - a} \quad (4)$$

TABLE I
DESCRIPTION OF DISCS INVESTIGATED

Disc	Profile	Temperature (K)			Temperature Gradient (K)
		T_a	T_b	T_{avg}	$T_b - T_a$
D_1	uniform	623	623	623	0
D_2	linear	638	613.37	623	24.63
D_3		653	603.75	623	49.25
D_4		668	594.13	623	73.87

Here T_a and T_b are the imposed temperatures at the inner radius a and outer radius b respectively. The values of T_a and T_b for discs D_2 , D_3 and D_4 are given in Table I.

B. Creep Law and Material Parameters

The steady-state creep response of the $Al - SiC_p$ composite has been described by Sherbys' law [15] given by:

$$\dot{\epsilon} = [M(\bar{\sigma} - \sigma_0)]^8 \quad (5)$$

where the symbol $\dot{\epsilon}$, $\bar{\sigma}$, σ_0 , denote the effective strain rate under biaxial stress, the effective stress under biaxial stress, threshold stress respectively, and the creep parameter M is given by:

$$M = \frac{1}{E} \left[\frac{AD_L \lambda^3}{|b_r|^5} \right]^{1/8} \quad (6)$$

where the symbols A , D_L , λ , E , $|b_r|$, denote respectively, constant sensitive to microstructure, lattice diffusivity, subgrain size, Young's modulus of elasticity and magnitude of burgers vector.

The creep parameters M and σ_0 as given by (6) depend on the type of material and are also affected by the operating temperature T . In a composite material, the particle size (p) and the particle content (V) are the primary material variables influencing these parameters. The value of M and σ_0 have been obtained from the creep results reported for $Al - SiC_p$ composite by [11] and are shown in Table II.

A regression analysis has been performed using Data Fit software. In the analysis, variables $M(r)$ and $\sigma_0(r)$ are depending on p , V and $T(r)$ and the developed regression equations are given as:

$$\ln(M(r)) = 0.211 \ln(p) + 4.89 \ln(T(r)) - 0.59 \ln(V) - 34.91 \quad (7)$$

$$\sigma_0(r) = -0.0205(p) + 0.037(T(r)) + 1.033(V) - 4.969 \quad (8)$$

C. Mathematical Modeling

From symmetry considerations, principal stresses are in the radial, tangential and axial directions for the disc rotating with angular velocity ω . For the purpose of modeling, the following assumptions are made:

- Steady state condition of stress.
- Elastic deformations neglected as compared to the creep deformations.
- Biaxial state of stress exists at each point of the disc.

TABLE II
MATERIAL PARAMETERS OBTAINED FROM EXPERIMENTAL DATA OF PANDEY ET AL. [11]

Particle size	Temperature	Particle content	Creep parameters	
p (μm)	T (K)	V (vol.%)	M ($s^{-1/8} MPa$)	σ_0 (MPa)
1.7	623	10	0.00963	15.24
14.5			0.01444	11.46
45.9			0.01897	13.65
1.7	623	10	0.00963	15.24
		20	0.00594	24.83
		30	0.00518	34.32
1.7	623	20	0.00594	24.83
	673		0.00897	24.74
	723		0.01295	25.72

(d) The disc is fitted on a splined shaft where small axial movement is permitted.

Taking reference frame along the directions r , θ and z , the generalized constitutive equations for creep in an isotropic rotating disc are given by:

$$\dot{\epsilon}_r = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_r - (\sigma_\theta + \sigma_z)] \quad (9)$$

$$\dot{\epsilon}_\theta = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_\theta - (\sigma_z + \sigma_r)] \quad (10)$$

$$\dot{\epsilon}_z = \frac{\dot{\bar{\epsilon}}}{2\bar{\sigma}} [2\sigma_z - (\sigma_r + \sigma_\theta)] \quad (11)$$

where the effective stress using von Mises' yield criterion is given by:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} [(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_r - \sigma_z)^2]^{1/2} \quad (12)$$

and $\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$, $\dot{\epsilon}_z$ and σ_r , σ_θ , σ_z are the strain rates and stresses respectively in the directions indicated by the subscripts and $\dot{\bar{\epsilon}}$ is the effective strain rate. For biaxial state of stress ($\sigma_z = 0$) and the constitutive equations are

$$\dot{\epsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{[M(r)(\bar{\sigma} - \sigma_0(r))]^8 (2x - 1)}{2(x^2 - x + 1)^{1/2}} \quad (13)$$

$$\dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{[M(r)(\bar{\sigma} - \sigma_0(r))]^8 (2 - x)}{2(x^2 - x + 1)^{1/2}} \quad (14)$$

$$\dot{\epsilon}_z = -(\dot{\epsilon}_r + \dot{\epsilon}_\theta) \quad (15)$$

where $x = \frac{\sigma_r(r)}{\sigma_\theta(r)}$ is the ratio of radial and tangential stress at any radius r and u_r is the radial displacement.

The equation of motion for a rotating disc of uniform thickness h may be obtained by considering the equilibrium of an element in the composite disc confined between radial distances r and $r + dr$ and an interval of angle between θ and $\theta + d\theta$. The equilibrium of forces in the radial direction of the element implies that

$$\frac{d}{dr} [r\sigma_r(r)] - \sigma_\theta(r) + \rho(r)\omega^2 r^2 = 0 \quad (16)$$

where ρ is the density of the composite.

Equations (13) and (14) can be solved to obtain $\sigma_\theta(r)$ as given below:

$$\sigma_\theta(r) = \frac{(\dot{u}_a)^{1/8}}{M(r)} \psi_1(r) + \psi_2(r) \quad (17)$$

where

$$\psi_1(r) = \frac{\psi(r)}{(x^2 - x + 1)^{1/2}}, \quad (18)$$

$$\psi_2(r) = \frac{\sigma_0(r)}{(x^2 - x + 1)^{1/2}}, \quad (19)$$

$$\psi(r) = \left[\frac{2(x^2 - x + 1)^{1/2}}{r(2 - x)} \exp \int_a^r \frac{\phi(r)}{r} dr \right]^{1/8}, \quad (20)$$

and

$$\phi(r) = \frac{(2x - 1)}{(2 - x)}. \quad (21)$$

Knowing the tangential stress distribution $\sigma_\theta(r)$, values of $\sigma_r(r)$ can be obtained from (16) as follows:

$$\sigma_r(r) = \frac{1}{r} \int_a^r \sigma_\theta(r) dr - \rho \frac{\omega^2}{3r} (r^3 - a^3). \quad (22)$$

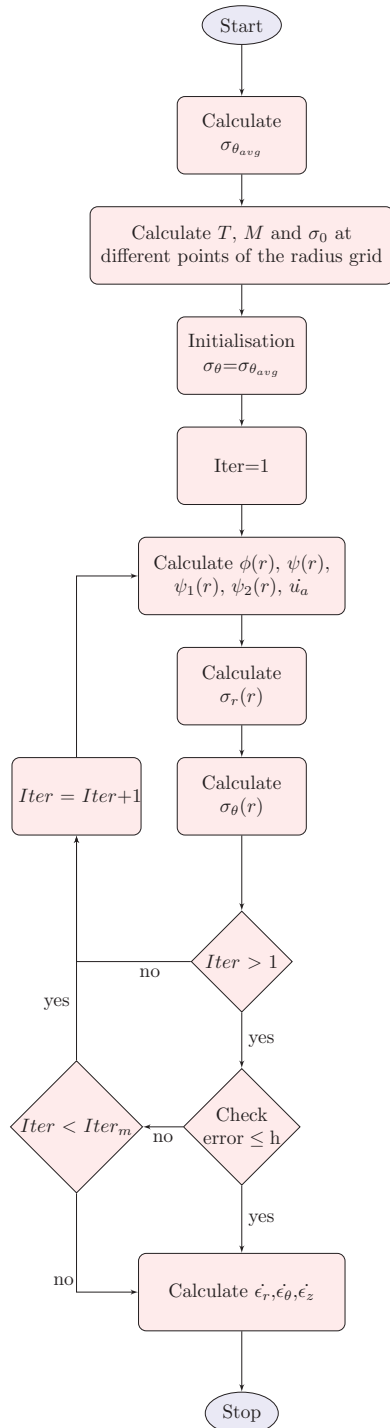
As the tangential stress, σ_θ , and the radial stress, σ_r , are determined by equations (17) and (22) at any point within the composite disc, the strain rates $\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$ and $\dot{\epsilon}_z$ are calculated from equations (13)-(15) respectively.

D. Numerical Computation

The stress distribution is evaluated from the above analysis by iterative numerical scheme of computations as given in Fig. 1.

The iterations are continued till the process converges, yielding the values of stresses at different points of the radius grid. An iterative solution technique can be employed until the boundary conditions $\sigma_r(a) = 0$ and $\sigma_r(b) = 0$ are satisfied (Arnold et al. [1]). For rapid convergence, as Gupta et al. [8] has done, 75% of the value of $\sigma_\theta(r)$ obtained in the current iteration has been mixed with 25% of the value of $\sigma_\theta(r)$ obtained in the last iteration for the use in the next iteration, i.e.

$$\sigma_{\theta_{next}} = 0.25 \sigma_{\theta_{previous}} + 0.75 \sigma_{\theta_{current}}. \quad (23)$$

**LEGENDS:**

$Iter$ = Iteration no.

$$ERR = \frac{[\sigma_{\theta}(r)]_{iter} - [\sigma_{\theta}(r)]_{iter-1}}{[\sigma_{\theta}(r)]_{iter-1}}$$

h = Limiting value of ERR (=0.01)

$iter_m$ = Maximum no. of iteration

Fig. 1 Numerical scheme of computation

TABLE III

VALUES USED BY WAHL ET AL. [18] DURING EXPERIMENT

Parameters for steel disc:-		
Density of disc material,	ρ	7823.18 kg/m ³
Inner radius of the disc,	a	0.03175 m
Outer radius of the disc,	b	0.1524 m
Creep parameter,	M	$4.72 \times 10^{-4} (s^{-\frac{1}{5}} / MPa)$
Creep parameter,	σ_0	-54.05 MPa
Operating conditions:-		
Angular velocity,	ω	15000 r/min
Operating Temperature,	T	810.78 K
Creep duration,	t	180 hr

Based on the above mathematical formulation, a computer code in MATLAB has been generated to calculate the steady-state creep behavior of the composite disc subjected to thermal gradient. Numerical calculation has been done to obtain the steady-state creep response of the discs having particle size $p = 1.7\mu m$ and particle content $V = 10vol\%$.

E. Validation of the Developed Computer Program

In order to validate the analysis and the developed computer program, the results for a rotating steel disc were obtained and compared with the available experimental results of Wahl et al. [?] for the same type of disc and operating under same conditions as mentioned in Table III.

The comparison of present theoretical study and experimental results of Wahl et al. [18] for strain rates versus radial distance is shown graphically in Fig. 2.

This graph depicts that there is a good agreement between the present theoretical and Wahl's experimental results for radial strain rates versus radial distance. The tangential strain rates also follow the same trend for present as well as experimental results of Wahl throughout the radial distance as one moves from inner to outer radii. A good agreement and similar trends observed between the present theoretical and the Wahl's experimental strain rates inspires the confidence in the computer program developed.

III. RESULTS AND DISCUSSION

A. Radial and Tangential Stresses

The effect of imposing linear thermal gradation on the creep behavior of the composite disc is discussed here by determining the radial and tangential stress.

The effect of linear thermal gradient on radial stress developed in rotating discs D_1 , D_2 , D_3 and D_4 is shown in Fig. 3. With the increase in linear thermal gradient, the radial stress is observed to increase over the radial distance. The maximum value of radial stress for discs D_4 , D_3 , D_2 and D_1 are 38.09MPa, 37.93MPa, 37.77MPa and 37.61MPa, respectively, near the middle of the disc at radius 0.08001m. Thus, value of radial stress for disc D_1 is less in comparison to discs operating under linear thermal gradient.

Effect of linear thermal gradient on tangential stress is shown in Fig. 4. The graph shows that the tangential stress attains values 80.57MPa, 80.94MPa, 81.32MPa and

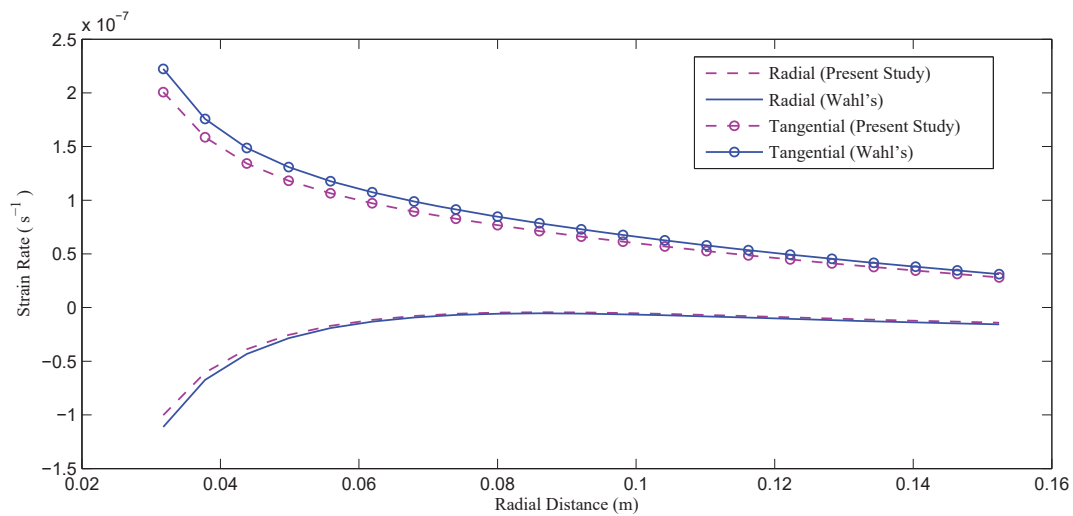
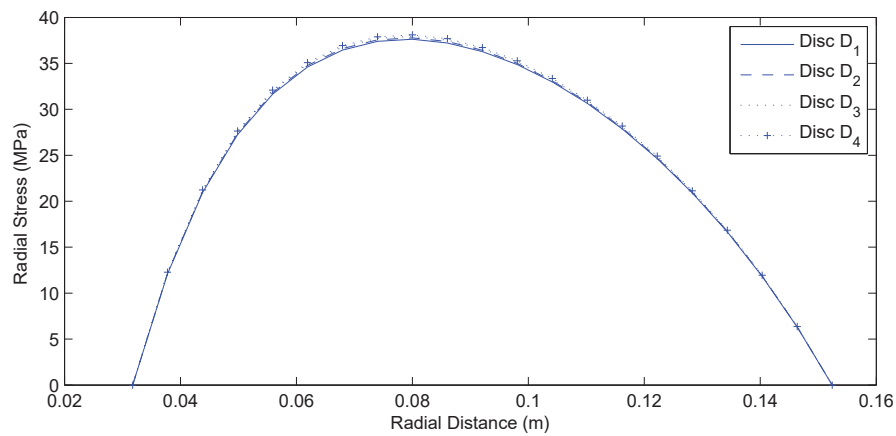
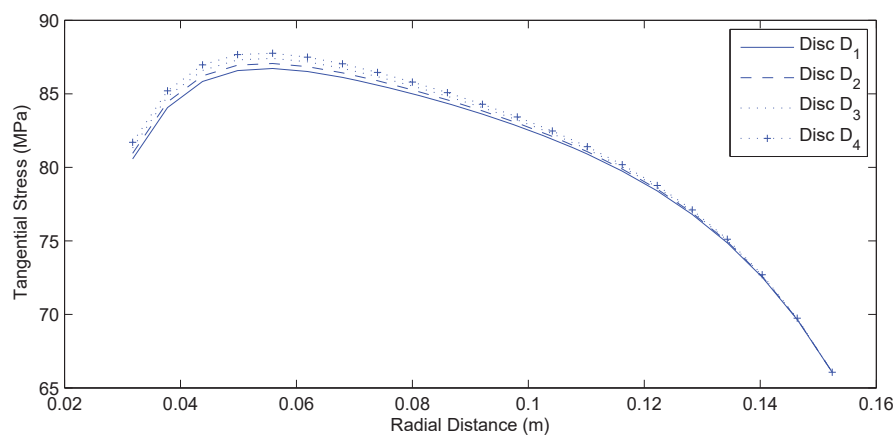


Fig. 2 Theoretical results vs experimental results of Wahl et al. [18]

Fig. 3 Variation of radial stress along radial distance in discs D_1 , D_2 , D_3 and D_4 operating under linear thermal gradientFig. 4 Variation of tangential stress along radial distance in discs D_1 , D_2 , D_3 and D_4 operating under linear thermal gradient

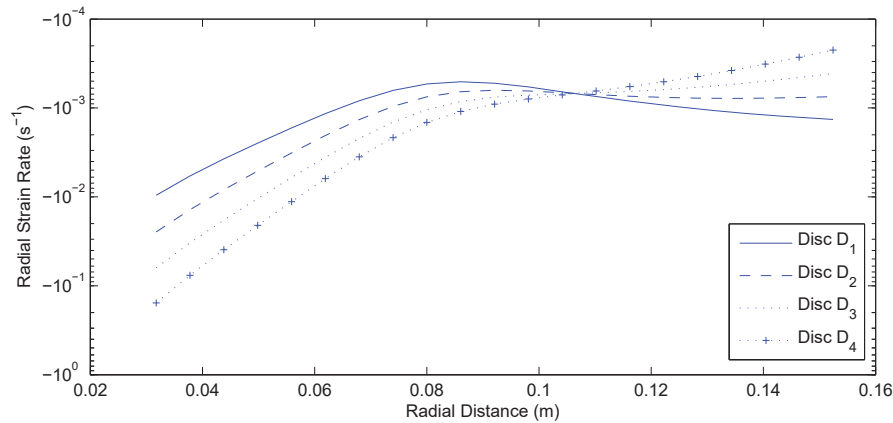


Fig. 5 Variation of radial strain rate along radial distance in discs D_1 , D_2 , D_3 and D_4 operating under linear thermal gradient

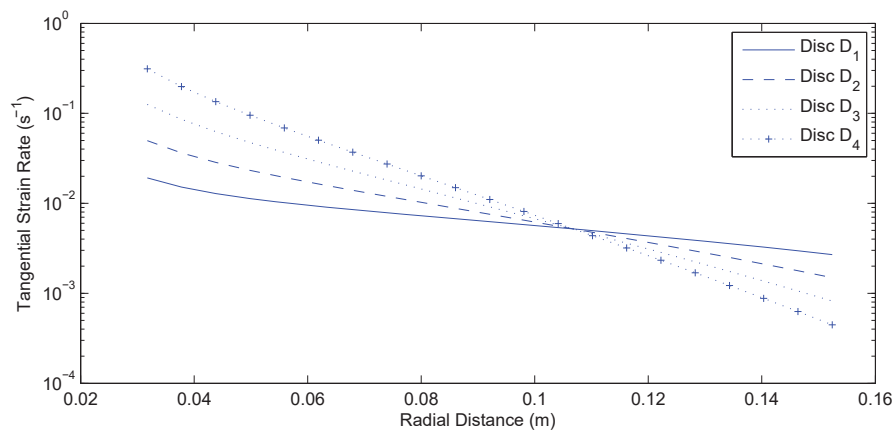


Fig. 6 Variation of tangential strain rate along radial distance in discs D_1 , D_2 , D_3 and D_4 operating under linear thermal gradient

81.69MPa at the inner radius then increases to 86.71MPa, 87.06MPa, 87.40MPa and 87.75MPa at radius 0.0559m for discs D_1 , D_2 , D_3 and D_4 , respectively, and then decreases to 66.05MPa as one approaches outer radius. Thus, as the thermal gradient increases, the tangential stress also increases.

B. Strain Rates

The effect of imposing linear thermal gradation on creep behavior of the composite disc is discussed here by determining the radial and tangential strain rates.

Fig. 5 shows the effect of thermal gradation on radial strain rate along radial distance developed in rotating discs D_1 , D_2 , D_3 and D_4 . The radial strain rates are compressive in nature throughout the radial distance. The radial strain rate decreases as one moves from inner radius, slightly increases and attains $-1.34 \times 10^{-3} s^{-1}$, $-7.46 \times 10^{-4} s^{-1}$, $-4.10 \times 10^{-4} s^{-1}$ and $-2.23 \times 10^{-4} s^{-1}$ at the outer radius for discs D_1 , D_2 , D_3 and D_4 respectively. Thus, at the inner radius, with increase in thermal gradient, radial strain rate increases however, at the outer radius, radial strain rate decreases.

The variation of tangential strain rate along radial distance for discs D_1 , D_2 , D_3 and D_4 under the effect of linear thermal gradient is shown in Fig. 6. At the inner radius, disc D_1 has lowest tangential strain rate $1.91 \times 10^{-2} s^{-1}$ and disc D_4 has

highest strain rate $0.312 s^{-1}$. At the outer radius, disc D_1 has highest tangential strain rate $2.69 \times 10^{-3} s^{-1}$ and disc D_4 has lowest strain rate of $4.46 \times 10^{-4} s^{-1}$. The graph clearly shows that the tangential strain rate decreases for the discs along the radial distance as the thermal gradation increases.

IV. CONCLUSION

The study reported here provides a framework for finding the stress and strain rate distributions in a composite disc operating under linear thermal gradient. The radial stress in the rotating composite disc increases throughout in the presence of linear thermal gradient. The maximum variation in radial stress is observed near the middle of the disc. The increase in linear thermal gradient in the composite disc leads to increase the tangential stress near the inner radius but decreases towards the outer radius. The radial strain rate is compressive everywhere over the radius of the disc. The strain rate decreases everywhere in the disc under the presence of thermal gradient. At the inner radius, with increase in thermal gradient, tangential strain rate increases in comparison to disc operating under uniform temperature however, at the outer radius, opposite trend is observed. Therefore extension of the domain of thermal gradation to the design of rotating discs will be beneficial for material engineers and designers.

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Dr. Neeraj Chamoli is Assistant Professor in the Department of Mathematics, D.A.V. College, Chandigarh, India. E-mail: neeraj_chamoli@yahoo.com



Dr. Minto Rattan is Assistant Professor in the Department of Applied Sciences, University Institute of Engineering and Technology, Panjab University, Chandigarh, India. E-mail: mintor@pu.ac.in



Tania Bose is Senior Research Fellow in the Center of Advanced Study in Mathematics, Panjab University, Chandigarh, India. E-mail: justtania12@gmail.com