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Effect of a Multiple Stenosis on Blood Flow through a Tube

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Abstract**—**The development of double stenosis in an artery can have serious consequences and can disrupt the normal functioning of the circulatory system. It has been realized that various hydrodynamics effects (i.e. wall shear, pressure distribution etc.) play important role in the development of this disease. Generally in the literature, the cross-section of the artery is assumed to be uniform with a single stenosis. However, in real situation the multiple stenosis develops in series along the length of artery whose cross-section varies slowly. Therefore, the flow of blood is laminar through a small diameter artery with axisymmetric identical double stenosis in series.

Keywords**—**Wall shear, multiple stenosis, artery.

I. Introduction

STENOSIS refers to localized narrowing of an artery and is caused mainly due to intravascular atherosclerotic plaque which develops at the arterial wall and protrudes into the lumen of vessel. Inflammation plays an important role in atherosclerosis and the genesis of acute coronary syndromes i.e. athermanous plaque disruption. Neturophil count and C-reactive protein (CRP) levels are makers of ongoing inflammation and predictors of cardiovascular risk.. We sought to assess whether these inflammatory markers are associated with the presence of multiple stenosis in patients which chronic stable angina. The heart may be the most important organ in the body for the preservation of life in individual organs and tissues of just as the heart can never rest, so the arteries are under the continuous strain of the pressure of blood. Post-stenotic dilatation of the coronary arteries can occur at high flow rates [4].

Although studies of Bingham flow [1] through vessels with post-stenotic dilatation have been conducted; it is clear that the clinical complications of the atherosclerosis are primarily thromboembolic in its origin in coronary and cerebral circulations.

The stenosis is formed by the deposition of certain lipids on the vessel wall. A stenosis may initiate and lead to the blockage of the vessel lumen in some cases and therefore poses a serious medical problem. Reference [5] observed that a sequence of multiple stenosis is functionally equivalent to a single equivalent stenosis of greater functionally severity. This finding allows the term ‘critical stenosis’ to be defined precisely in terms of the functional effect rather than the anatomical appearance, particularly where disease is multi-focal. Reference [2] investigated that fully developed casson flow through a stenosis with multiple anomalies in arterial radius. Reference [3] have also considered the effect of Reynolds’s number on the flow of blood through a tube identically symmetrical multiple stenosis and reported that the pressure recovery behind the proximal stenosis is hindered by presence of the second stenosis.

Therefore, in this paper, to analyze the characteristics of blood flow through an artery with axisymmetric multiple stenosis in series. It has been recognized recently that multiple stenosis may also develop in series in a tube, instead of a single stenosis, leading to complete blockage of the lumen.

II. Analysis

The geometry of the model for this study is depicted in Fig. 1. We shall consider the steady flow of a Newtonian fluid an axially-symmetric whose surface is given by

$$R_1 = R_o - \frac{\delta_1}{2}\left(1+\cos\frac{\pi(z-z_o)}{z_o}\right), \quad -2z_o \le (z-z_o) \le 0 \tag{1}$$

and

$$R_2 = R_o - \frac{\delta_2}{2}\left(1+\cos\frac{\pi(z-z_o)}{z_o}\right), \quad 0 \le (z+z_o) \le 2z_o \tag{2}$$

The radius of two obstructed regions due to the multiple stenosis are described by R_1 and R_2. And δ_1, δ_2 are maximum heights of two stenosis in series. L_1 and L_2 are the lengths of stenosis.

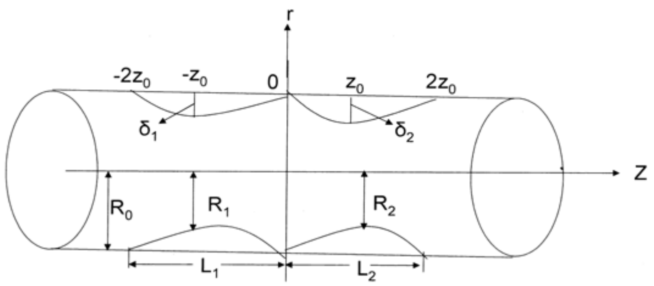

Fig. 1 Idealized model for the multiple stenoses

The basic equation of motion in cylindrical polar coordinates, it is shown that the radial velocity can be

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neglected in relation to axial velocity v which is determined by

$$-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial r}\right)=0 \quad (3)$$

$$-\frac{\partial p}{\partial z}=0 \quad (4)$$

$$-P(z)=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \quad (5)$$

The no-slip condition on the stenosis surface gives

$$v=0 \ at \ r=R(z) \ ,-2z_0 \le z \le 2z_0$$
$$v=0 \ at \ r=R_0, \ |z|\ge 2z_0 \quad (6)$$

Thus for a mild stenosis, the main difference from the usual Poiseuille flow is that the pressure gradient and axial velocity are function of Z also. However, for a stenosis in stage II and III, the radial velocity can be significant and turbulence may have to be considered. Obviously then, the analysis is more complicated.

Integrating (5) we get,

$$r\frac{\partial v}{\partial r}=-P(z)\frac{r^2}{2\mu}+A(z) \quad (7)$$

but $A(z)=0$, since $\frac{\partial v}{\partial r}=0$ on the axis.

Again integrating and using (6) we get,

$$v=\frac{-P(z)}{4\mu}\left[r^2-R^2(z)\right] \quad (8)$$

if Q is the flux through the tube, then

$$Q=\int_o^{R(z)}V\,2\pi r dr=\frac{\pi P(z)}{8\mu}R^2(z) \quad (9)$$

Q is constant for all sections of the tube, the pressure gradient varies inversely as the fourth power of the surface distance of the stenosis from the axis of the artery so that it is minimum at the middle of the stenosis and is maximum at the ends. So that the pressure drop across the length of first stenosis is

$$\Delta p=\frac{8\mu Q}{\pi R^2(z)}\int_{-2z_o}^{o}\frac{du}{a-b\cos u} \quad (10)$$

where

$$a=1-\frac{\delta_1}{2R_o},\ b=\frac{\delta_1}{2R_o}$$

$$\int_{-3\pi}^{\pi}\frac{du}{a-b\cos u}=2\pi(a^2-b^2)^{-1/2} \quad (11)$$

Similarly differentiating (11) thrice partially w.r.t. ‘a’ we get

$$\int_{-3\pi}^{\pi}\frac{du}{(a-b\cos u)^2}=2\pi\left(1-\frac{\delta_1}{2R_o}\right)\left(1-\frac{\delta_1}{2R_o}\right)^{-3/2} \quad (12)$$

$$\int_{-3\pi}^{\pi}\frac{du}{(a-b\cos u)^3}=2\pi\left(1-\frac{\delta_1}{R_o}+\frac{3\delta_1^2}{8R_o^2}\right)\left(1-\frac{\delta_1}{2R_o}\right)^{-5/2} \quad (13)$$

$$\int_{-3\pi}^{\pi}\frac{du}{(a-b\cos u)^4}=2\pi\left(1-\frac{\delta_1}{2R_o}\right)\left(1-\frac{\delta_1}{R_o}+\frac{5\delta_1^2}{8R_o^2}\right)\left(1-\frac{\delta_1}{R_o}\right)^{-7/2} \quad (14)$$

$$=2\pi f\left(\frac{\delta_1}{R_o}\right) \quad \text{(say)} \quad (15)$$

Similarly for second stenosis we get

$$=-2\pi f\left(\frac{\delta_2}{R_o}\right) \quad (16)$$

So, for first stenosis

$$\Delta P=\frac{32\mu Q z_o}{\pi R_o^4}f\left(\frac{\delta_1}{R_o}\right) \quad (17)$$

where there is no stenosis, $\delta_1=0$ and $f\left(\frac{\delta_1}{R_o}\right)=1$, then pressure drop across the stenosis length is given by

$$(\Delta P)_p=\frac{32\mu Q z_o}{\pi R_o^4} \quad (18)$$

where the subscript P denotes Poisseuille flow from (17) and (18) we get

$$k_1=\frac{(\Delta P}{(\Delta P)_p}=2f\left(\frac{\delta_1}{R_o}\right)=2\left(1-\frac{\delta_1}{2R_o}\right)\left(1-\frac{\delta_1}{R_o}+\frac{5}{8}\frac{\delta_1^2}{R_o^2}\right)\left(1-\frac{\delta_1}{R_o}\right)^{-7/2} \quad (19)$$

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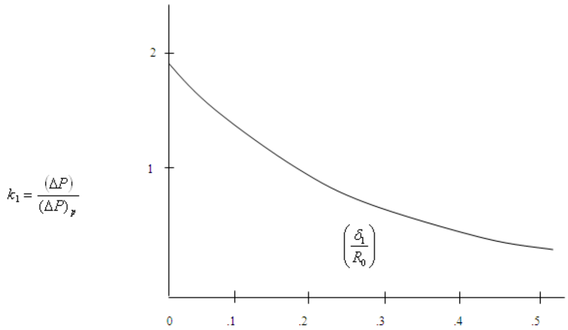

Fig. 2 Pressure drop across first stenosis length

Similarly for second stenosis

$$k_2 = \frac{(\Delta P)}{(\Delta P)_p} = -2f\left(\frac{\delta_2}{R_o}\right) = -2\left(1-\frac{\delta_2}{2R_o}\right)\left(1-\frac{\delta_2}{R_o}+\frac{5\delta_2^2}{8R_o^2}\right)\left(1-\frac{\delta_2}{R_o}\right)^{-7/2} \quad (20)$$

$$k_3 = \frac{\Delta P}{(\Delta P)_p} = 1 + \frac{z_o}{L}\left[2f\left(\frac{\delta}{R_o}\right) - 1\right] \quad (21)$$

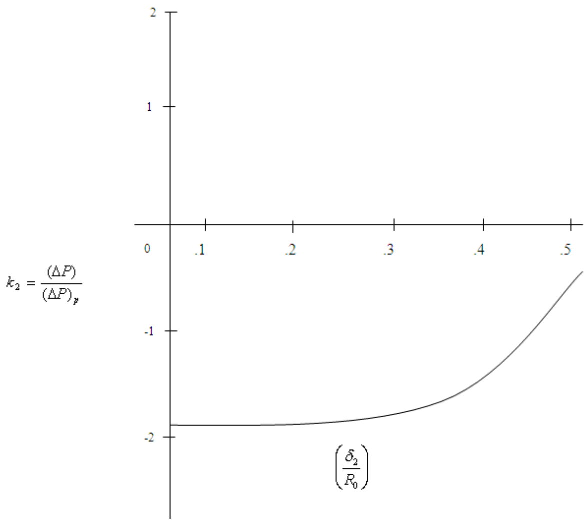

Fig. 3 Pressure drop across second stenosis length

Now, resistive impedance denote by ^ so that

$$\frac{\wedge}{\wedge p} = \frac{(\Delta p)}{(\Delta p)_p} = 1 + \frac{z_o}{L}\left[2f\left(\frac{\delta}{R_o}\right) - 1\right] \quad (22)$$

Shear stress on the stenosis surface is given by

$$\tau = \left(-\mu\frac{\partial v}{\partial r}\right)_{r=R_1(z)} = \frac{1}{2}P(z)R_1(z) = \frac{4\mu Q}{\pi R_1^3(z)}$$

so that

$$\frac{\tau}{\tau_p} = \left(\frac{R_1(z)}{R_o}\right)^{-3} \quad (23)$$

$$\frac{\tau}{\tau_p} = \left(\frac{R_2(z)}{R_o}\right)^{-3} \quad (24)$$

so the maximum value of shear stress and minimum value of shear stress is given by

$$\frac{\tau_{\max}}{\tau_{\min}} = \left(1-\frac{\delta_1}{R_o}\right)^{-3} = \frac{1}{1-3\frac{\delta_1}{R_o}+3\frac{\delta_1^2}{R_o^2}-\frac{\delta_1^3}{R_o^3}} \quad (25)$$

The ratio increase rapidly as the stenosis in size

$\frac{\tau_{\max}}{\tau_{\min}} =$	$\frac{1000}{735}$	$\frac{1000}{520}$	$\frac{1000}{400}$	$\frac{1000}{230}$	$\frac{1000}{130}$
$\frac{\delta_1}{R_0} =$	.1	.2	.3	.4	.5

$$\frac{\tau_{\max}}{\tau_{\min}} = \left(1-\frac{\delta_2}{R_o}\right)^{-3} = \frac{1}{1-3\frac{\delta_2}{R_o}+3\frac{\delta_2^2}{R_o^2}-\frac{\delta_2^3}{R_o^3}} \quad (26)$$

$\frac{\tau_{\max}}{\tau_{\min}} =$	$\frac{1000}{710}$	$\frac{1000}{500}$	$\frac{1000}{390}$	$\frac{1000}{200}$	$\frac{1000}{100}$
$\frac{\delta_2}{R_0} =$	.1	.2	.3	.4	.5

It is clear that when $\left(\frac{\delta_1}{R_o}\right) and \left(\frac{\delta_2}{R_o}\right)$ is .5 the stress in the middle.

III. Results and Discussions

Figs. 2-3 illustrate the numerical results obtained. Fig. 2 shows that if $\left(\frac{\delta_1}{R_o}\right)$ increases then k_1 decreases. We can see this result from (19) itself since $\left(\frac{\delta_1}{R_o}\right)$ increases, the integrand decreases and therefore k_1 decreases. In Fig. 3 if $\left(\frac{\delta_2}{R_o}\right)$ increases, then k_2 also increases. In the absence of stenosis, the ratio of an artery length to pressure drop which increases as $\left(\frac{z_o}{l}\right)$ increases. It is clear that maximum value of shear stress in the middle of first and second stenosis and minimum at the ends of stenosis.

IV. Conclusion

This study was to test the hypothesis that the model for single stenosis is also valid for multiple stenosis arranged in series and to test the hypothesis that the effect of the stenosis can be predicted from measurement of the individual stenosis components. The condition of critical stenosis also depends on the inflow rates. And the present theory holds only for small values of $\left(\frac{\delta}{R_o}\right)$.

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