

Economic Analysis of Endogenous Growth Model with ICT Capital

Shoji Katagiri, Hugang Han

Abstract—This paper clarifies the role of ICT capital in economic growth. Albeit ICT remarkably contributes to economic growth, there are few studies on ICT capital in ICT sector from theoretical point of view. In this paper, production function of ICT which is used as input of intermediate good in final good and ICT sectors is incorporated into our model. In this setting, we analyze the role of ICT on balance growth path and show the possibility of general equilibrium solutions for this model. Through the simulation of the equilibrium solutions, we find that when ICT impacts on economy and economic growth increases, it is necessary that increases of efficiency at ICT sector and of accumulation of non-ICT and ICT capitals occur simultaneously.

Keywords—Endogenous economic growth, ICT, intensity, capital accumulation.

I. INTRODUCTION

SINCE 1990s, we have thought that IT or ICT which have attracted our attention generally contributes to economic growth. Reference [13] points out that ICT is input to final good production as capital stock and contributes to total factor productivity (TFP) and network effect, and finally enhances economic growth.

There are few theoretical studies relating to ICT, and among others, [7], [10] are theoretical studies, but there ICT is treated as intermediate good in international trade. Therefore these researches do not focus on ICT but international trade. Regarding the empirical studies, there are [3-6], [9], [11], [12], [13], [15], [16]. Using various kind of data, [12], [13] indicate the extents of ICT's spread and contribution to economic growth, and show that from 1995 to 2001 in 14 OECD countries the impact to economic growth caused by ICT capital ranges from 0.1% to 0.9%. Furthermore [6], [11] use Cobb-Douglas production function and so forth as basic model whose inputs are non-ICT asset, ICT asset and labor, and perform growth accounting and dynamic multiple regression analysis. The characteristic of these studies is to divide a conventional physical capital into two assets, non-ICT and ICT assets. Reference [11] shows that the contribution of ICT asset is 1% of average economic growth 4% for business service sector from 1995 to 2005 in Japan so that they confirm some degree of ICT asset-contribution to economic growth. Reference [6] shows that the elasticity of ICT capital for economic growth is about 0.08 and its magnitude is around 20% of general physical capital (its elasticity 0.37), so its

contribution is not so significant in comparison with a general physical capital. However the aforementioned studies do not refer to production function of ICT at all, and they just classify a conventional physical capital into non-ICT and ICT capitals. According to [11], ICT capital belongs to the same category as non-ICT capital, since they mention that capital service flow is composed of ICT and non-ICT capital. Therefore it suggests that production function of ICT capital is able to be taken as similar production function of final good. Also as the characteristic of ICT capital, [6] pointed out that a life of information and communication material is shorter than a conventional physical capital. For example [6] reports 15 years as its life (facsimile)¹, and mentions ICT capital life of about 6 years on average and the depreciation rate of ICT capital ranging from 0.3119 to 0.369². Although ICT capital (product) contributes to all industry as goods and service, it is used as final good such as tablet and smartphone or as intermediate goods such as equipment and apparatus for production, and state-of-the-art ICT products turn into ordinary commodity in a short time. Therefore the characteristic of ICT capital (product) is to be perishable.

Taking into consideration the aboves, in this paper, based on the precedent empirical studies we theoretically develop our model with ICT capital, and confirm and analyze that how ICT capital affects to economy. To do this end, we take ICT capital (products) as stock variable like a conventional physical capital, incorporate it into production functions at final good and ICT sectors, and construct a general equilibrium system. Using the results from the general equilibrium system, we perform simulation based on parameters in the precedent empirical studies, and analyze the role of ICT capital which affects an economy.

The plan of the paper is as follows. Section II presents the model. Section III characterize the steady-state equilibrium. In Section IV, we perform the simulation using the results in Section III and examine the role of ICT capital. Section V concludes the paper and presents some possible extensions to the present research. All the proofs and lengthy computations are in Appendix.

II. THE MODEL

This model being developed here is based on [1], [8], [14].

The agents in this economy are composed of two kinds of producer (final good producer and ICT product producer)

¹See page 16 in [6].

²Reference [1] takes 0.056 as depreciation rate of ICT capital. Reference [17] indicates that the depreciation rate of ICT capital fluctuates at the level of around 0.11 for all industry from 1975 to 1998 in Japan.

S. Katagiri (Professor) is with the Faculty of Management and Information System, Prefectural University of Hiroshima, Hiroshima, 734-8558 Japan (e-mail: katagiri@pu-hiroshima.ac.jp).

H. Han. (Professor) is with the Faculty of Management and Information System, Prefectural University of Hiroshima, Hiroshima, 734-8558 Japan (e-mail: hhan@pu-hiroshima.ac.jp).

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$$\left[1 - (1 - \phi_2^{-(\alpha-\lambda)})s\right] \left[\frac{s + (1-s)\phi_2^{\alpha-\lambda}}{s\phi_3^{\alpha-\lambda} + (1-s)\phi_2^{\alpha-\lambda}} \right]^{-\frac{\epsilon-\beta}{\alpha-\lambda}}$$

$$= \phi_1\phi_2^{1-\alpha}\phi_3^\beta p^{\frac{1}{\alpha-\lambda}} \frac{l}{k} \cdot \left(\frac{x}{l}\right)^{\frac{\epsilon-\beta}{\alpha-\lambda}}. \quad (14)$$

As the initial values (k_0, x_0, p_0) is given (labor l is constant), the share of non-ICT capital for production of final good s is determined. Also we obtain the shares of ICT capital and labor (u, v) as follows.

$$u = \frac{s\phi_3^{\alpha-\lambda}}{s\phi_3^{\alpha-\lambda} + (1-s)\phi_2^{\alpha-\lambda}} < 1, \quad (15)$$

$$v = \frac{s}{s + (1-s)\phi_2^{\alpha-\lambda}} < 1. \quad (16)$$

Accordingly, as s is determined by (14), we insert this s into (15) and (16), then get u, v , and then finally z_1, z_2, z_3 , and z_4 at initial period.

Then the equilibrium growth paths (BGP:Balanced Growth Paths) for consumption, non-ICT capital and ICT capital are given by ⁶

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} \left(\alpha A \left(\frac{sk}{vl} \right)^{\alpha-1} \left(\frac{ux}{vl} \right)^\beta - \rho - \delta \right), \quad (17)$$

$$\frac{\dot{k}}{k} = A \frac{(sk)^\alpha (ux)^\beta (vl)^{1-\alpha-\beta}}{k} - \frac{C}{k} - \delta, \quad (18)$$

$$\frac{\dot{x}}{x} = \frac{\gamma \frac{((1-s)k)^\lambda ((1-u)x)^\epsilon ((1-v)l)^{1-\lambda-\epsilon}}{x}}{-\eta}. \quad (19)$$

Therefore the general equilibrium system comprises dynamic equations, (13), (17), (18), and (19), and equilibrium equations, (14), (15), and (16). Provided that the initial values (k_0, x_0, p_0) are given, by (14), (15), and (16), $s, u,$ and v are determined, and with these shares (s, u, v) , by (1) and (2), final good Y and ICT capital X at initial period are determined. Then from (13), (17), (18), and (19), relative price p , consumption C and two kinds of capital stock (k, x) are determined at next period.

Finally we refer to the total factor productivity (TFP). At first we define GDP as follows ⁷.

$$Q \equiv Y + pX. \quad (20)$$

Here we get the below equation from (37) or (39) in the Appendix A.

$$\frac{\alpha Y}{s} \cdot \frac{1-s}{\lambda X} = \frac{\beta Y}{u} \cdot \frac{1-u}{\epsilon X} = \frac{\mu_2}{\mu_1} = p. \quad (21)$$

Then we insert (21) into (20), and get the equation:

$$Q = A \cdot \frac{s^\alpha u^\beta v^{1-\alpha-\beta}}{u} \cdot \frac{\beta + (\epsilon - \beta)u}{\epsilon} \cdot k^\alpha x^\beta l^{1-\alpha-\beta}$$

$$= A \cdot \frac{s^\alpha u^\beta v^{1-\alpha-\beta}}{s} \cdot \frac{\alpha + (\lambda - \alpha)s}{\lambda} \cdot k^\alpha x^\beta l^{1-\alpha-\beta}. \quad (22)$$

⁶See the Appendix C.

⁷This setting is based on [1], [2].

From (22), total factor productivity (TFP) is

$$A \cdot \frac{s^\alpha u^\beta v^{1-\alpha-\beta}}{u} \cdot \frac{\beta + (\epsilon - \beta)u}{\epsilon}$$

or

$$A \cdot \frac{s^\alpha u^\beta v^{1-\alpha-\beta}}{s} \cdot \frac{\alpha + (\lambda - \alpha)s}{\lambda}.$$

The total factor productivity (TFP) has the following characteristics for intensive parameters.

$$\frac{\partial TFP}{\partial \epsilon} = A \cdot \frac{s^\alpha u^\beta v^{1-\alpha-\beta}}{u} \cdot \frac{\beta(u-1)}{\epsilon^2} < 0, \quad (23)$$

$$\frac{\partial TFP}{\partial \lambda} = A \cdot \frac{s^\alpha u^\beta v^{1-\alpha-\beta}}{u} \cdot \frac{\alpha(s-1)}{\lambda^2} < 0. \quad (24)$$

From the above, we obtain the following proposition.

Proposition 1. Total factor productivity (TFP) is independent of the effectiveness of production for ICT capital γ and in case of increase of the intensive parameters λ and ϵ , total factor productivity (TFP) decreases.

B. Steady State

A steady state equilibrium or BGP is the equilibrium path along which consumption C and state variables k, x grow at a constant rate and the shares of two kinds of capitals and labor allocated to final good production s, u, v and relative price p are constant.

Now from (37) we obtain

$$p = \frac{\mu_2}{\mu_1} = \frac{1-s}{s} \cdot \frac{\alpha Y}{\lambda X}, \quad (25)$$

so that using this equation the following relationship has to hold⁸.

$$\frac{\dot{k}}{k} = \frac{\epsilon - \beta}{\alpha - \lambda} \cdot \frac{\dot{x}}{x}. \quad (26)$$

Also the above equation holds for (14). From (17) as $\left(\frac{sk}{vl}\right)^{\alpha-1} \left(\frac{ux}{vl}\right)^\beta$ is constant at steady state, the following equation holds.

$$\frac{\dot{k}}{k} = \frac{\beta}{1-\alpha} \cdot \frac{\dot{x}}{x}. \quad (27)$$

Therefore from (26) and (27) finally the following equation has to hold.

$$\frac{\beta}{1-\alpha} = \frac{\epsilon - \beta}{\alpha - \lambda}. \quad (28)$$

For (28), we show the some values as an example (when $\alpha = 0.35$ and $\beta = 0.05$) in Table 1⁹.

Table I Example for parameter values

λ	ϵ	λ	ϵ
0.050	0.073	0.330	0.052
0.100	0.069	0.370	0.048
0.150	0.065	0.400	0.046
0.200	0.062	0.450	0.042
0.250	0.058	0.500	0.038
0.300	0.054	0.550	0.035

⁸See Appendix D.

⁹The values in Table I satisfies the Assumption 1 and (28), but there are innumerable values which satisfies (28) except the ones in Table I.

On the simulation we mention later, the values shown in Table I is utilized. Furthermore as (18) grows at a constant rate in steady state, we get

$$g^* = g_C^*, \quad (29)$$

where $g^* \equiv \frac{\dot{k}}{k}$ and $g_C^* \equiv \frac{\dot{C}}{C}$. Then using (1) and (28), the following equation holds.

$$g^* = g_Y^*. \quad (30)$$

From (26) we express

$$g^* = \frac{\epsilon - \beta}{\alpha - \lambda} g_x^*, \quad (31)$$

where $g_x^* \equiv \frac{\dot{x}}{x}$.

The following proposition characterizes the steady state in terms of C , Y , k and x .

Proposition 2. At steady state consumption C , non-ICT capital k and final good Y grow at the same rate g^* . Then the relationship between the growth rate of non-ICT capital and the one of ICT capital is proportional, i.e. $\frac{\epsilon - \beta}{\alpha - \lambda}$.

Next we derive the growth rates of final good, non-ICT capital and consumption at steady state g^* . To do so, we use and manipulate (17) and (18), and get the following equation.

$$g^* = \frac{\alpha}{\alpha - \sigma s^*} \left(\frac{\rho s^*}{\alpha} - \delta \left(1 - \frac{s^*}{\alpha} \right) - \frac{C(0)}{k(0)} \right), \quad (32)$$

where we exclude $s^* = \frac{\alpha}{\sigma}$.

Now we confirm whether the relative price p^* of (13) exists or not at steady state ($\frac{\dot{p}}{p} = 0$) by simulation¹⁰. We show the results of the simulation in Fig. 2 (Case-1) and 3 (Case-2). The right-hand side of (13) is downward sloping in Fig. 2, and upward sloping in Fig. 3. It clearly follows from the both figures that we have the unique intersection between both the curves and the line $\frac{\dot{p}}{p} = 0$ and it indicates the unique existence of relative price at steady state p^* .

Furthermore, we confirm whether s^* of (14) exists or not by simulation.¹¹ We show the result in Fig. 4. From Fig. 4, as s is changed, for Case-1, the left-hand side of (14) monotonously increases, and for Case-2 decreases. Since the right-hand side indicates the horizontal line shown in Fig. 4 for both cases, we confirm that s^* which satisfies (14) uniquely exists.

Proposition 3. At steady state, relative price p^* and share of non-ICT capital allocated to final good production s^* uniquely exist.

¹⁰The application through the simulations is MATLAB. The values of parameters on the simulation are based on the preceding study as much as possible. The parameters being used are as follows. For Case-1, $\alpha = 0.35$, $\beta = 0.05$, $\lambda = 0.33$, $\epsilon = 0.052$. For Case-2, $\alpha = 0.35$, $\beta = 0.05$, $\lambda = 0.37$, $\epsilon = 0.048$. The value of other parameters are $A = 1$, $\gamma = 1$, $l = 1.000$, $x = 1.000$, $k = 107.000$, $\delta = 0.1$, $\eta = 0.32$, and $z_4 = 1000.0$.

¹¹The parameters being used are as follows. The common values for both cases, $A = 1$, $\gamma = 1$, $l = 1.000$, $x = 1.000$, and $k = 4.000$. For Case-1, except $p = 1.030$, the other parameters are same in Fig. 2. For Case-2, except $p = 0.974$. The other parameters are same in Fig. 3.

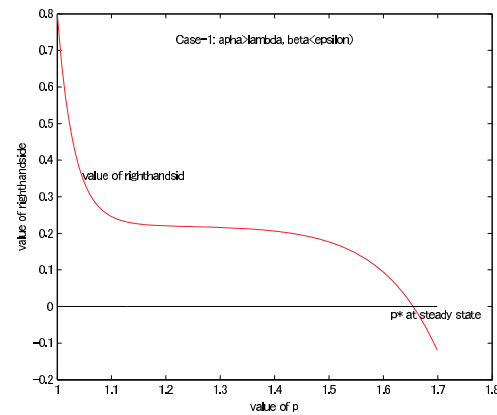


Fig. 2 Existence of relative price p (Case-1)

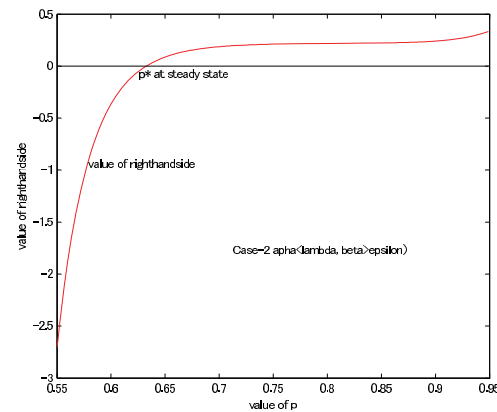


Fig. 3 Existence of relative price p (Case-2)

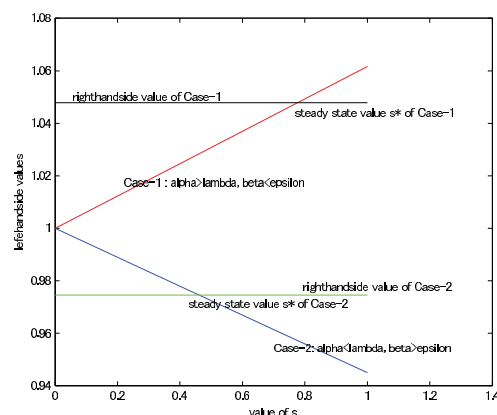


Fig. 4 Existence of share of non-ICT capital s .

The steady state s^* is an endogenous variable in this model, so that s^* is usually not used for parameter on comparative statics. s^* is determined and depends on the parameters in (14), and in chapter IV, we confirm the relationship between s^* and parameters in (14). Therefore we use s^* as parameter for comparative statics mentioned below. Then we get the following characteristic in terms of (32).

$$\frac{\partial g^*}{\partial s^*} = \frac{\alpha(\rho + \delta(1 - \sigma) - \sigma \frac{C(0)}{k(0)})}{(\alpha - \sigma s^*)^2}. \quad (33)$$

For (33), generally $\rho + \delta(1 - \sigma) < 0$, so we reach the following proposition.

Proposition 4. Except $s^* = \frac{\alpha}{\sigma}$, increase of s^* leads to decreases of growth rate of non-ICT capital, consumption and final good g^* .

Based on Proposition 4, we simulate the growth rate g^* , taking the share of non-ICT capital allocated to final good production at steady state s^* as a parameter (except $s^* = \frac{\alpha}{\sigma}$). On this simulation, it is obvious from (32) that without change of α , if we take same parameter-setting in Case-1 and Case-2, we get same figure for both cases. Therefore we change the α 's value from 0.35 to 0.355 for Case 2 as a reference, and show Fig. 5 as a result of simulation¹².

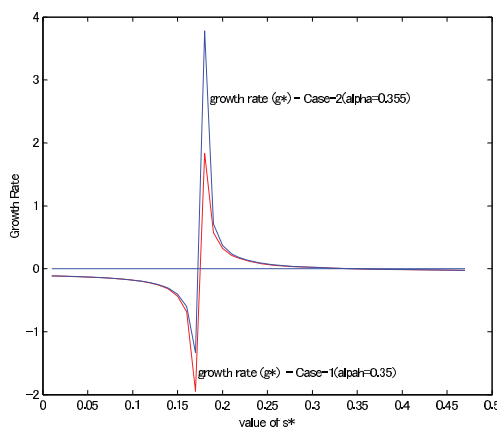


Fig. 5 Simulation of growth rate g^* with s^*

From Fig. 5, on the interval $0 < s^* < \frac{\alpha}{\sigma}$, as g^* is always negative, $\frac{\alpha}{\sigma} < s^* < 1$ might be a suitable interval for the real world. On the interval $\frac{\alpha}{\sigma} < s^* < 1$, we think, if s^* is a large value, production for ICT capital (product) X (flow) decrease due to the small value of $1 - s^*$, the corresponding increment of ICT capital x (stock) for final good production as input decrease, and finally economic growth rate g^* decrease. In other word, small value of s^* (increase of $1 - s^*$) indicates that large amount of non-ICT capital as input is used for

¹²The parameters being used are as follows. For Case-1, $\alpha = 0.35$, $\beta = 0.05$, $\lambda = 0.33$, $\epsilon = 0.052$. For Case-2, $\alpha = 0.355$, $\beta = 0.05$, $\lambda = 0.37$, $\epsilon = 0.048$. The values of other parameters are $A = 1$, $\gamma = 1$, $\sigma = 2$, $\delta = 0.100$, $\rho = 0.012$, $C(0) = 1.000$, and $k(0) = 100.0$.

production of ICT capital (product), then ICT capital X (flow) increases, and the accumulated ICT capital x (stock) is used for production of final good. Even if the share of non-ICT capital s^* is small in final goods sector, increase of ICT capital stock x compensates the small value of s^* , and growth rate of final good ultimately increases.

IV. SIMULATION AND THE ROLE OF ICT CAPITAL

In previous section we obtain the results of economic variables at steady state equilibrium, however it is intractable to get equilibrium solutions explicitly from equilibrium equations and to perform comparative statics by calculation. Instead of comparative statics analysis by calculation, using the results in previous section, we perform simulation based on various kind of parameter, and analyze the role of ICT capital and its influence to economic growth. To do so, from Proposition 2 and 4, as economic growths of two kinds of capital, consumption, and final good g^* depend on the share of non-ICT capital allocated to final good production s^* , hereinafter we confirm the relationship between the share s^* and parameters of ICT capital x .

First of all, we simulate (14) with efficiency of ICT capital (product) in ICT capital production γ . The results of the simulations are shown in Fig. 6 (Case-1) and 7 (Case-2)¹³.

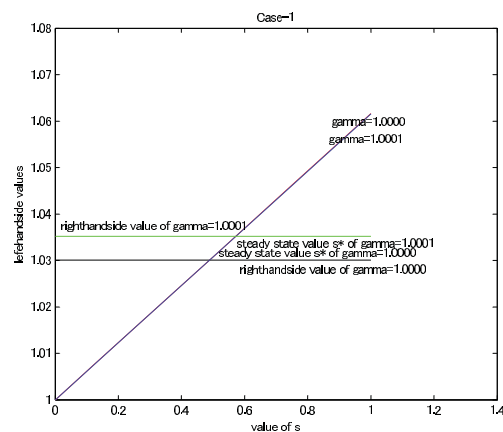
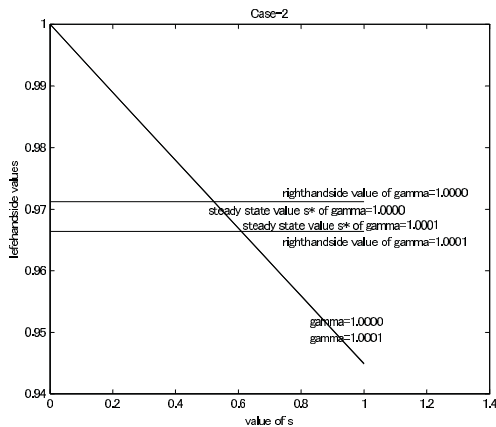


Fig. 6 Simulation of s with γ (Case-1)

For both cases, by subtle change of γ the left-hand side of (14) still remains at same place due to the exclusion of ϕ_1 relating to γ , and its slope is upward for Case-1 and downward for Case-2 due to the difference of non-ICT capital intensity at both sectors (α and λ). The right-hand side of (14) indicates the horizontal line. Since it includes ϕ_1 , for Case-1 the horizontal line shifts upward and for Case-2 downward. When the efficiency parameter in ICT capital production γ slightly increases, the share of non-ICT capital allocated to

¹³The parameters being used are as follows. The common values for both cases, $A = 1$, $l = 1.0000$, $k = 107.0$, and $x = 1.0000$. For Case-1, except $p = 1.0996$, the other parameters are same as ones in Fig. 4. For Case-2, except $\gamma = 1.0001$ and $p = 0.9121$, the other parameters are same as ones in Fig. 5.

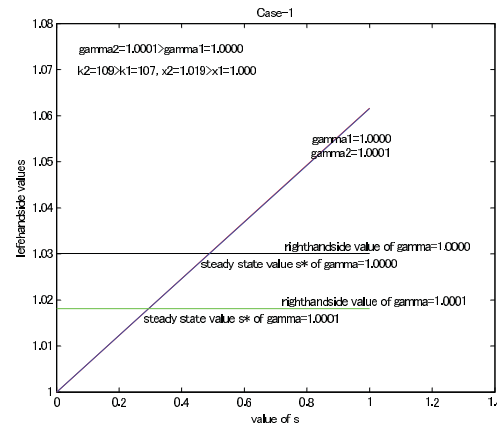
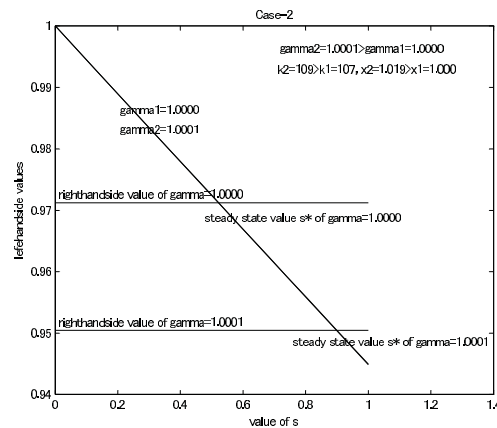
Fig. 7 Simulation of s with γ (Case-2)

final good production s^* increases for both cases. To sum up, from the previous results, the increase of s^* indicates the decrease of growth rates of both capitals and consumption and finally decrease of economic growth rate g^* . From this point of view, the rise of efficiency γ in production of ICT capital leads to decrease of economic growth rate g^* .

As an interpretation for the above description, it is thought that although efficiency γ rises and production of ICT capital temporally rises, γ is constant so that γ does not continually affect the growth rate of ICT capital x (ultimately growth rate of economic growth). Furthermore by the rises of efficiency γ and the share s^* , ICT producer can obtain a same output of product with less input of non-ICT capital, therefore more amount of non-ICT capital can be used into final good sector. Then input of $(1 - s^*)k$ to be used in ICT sector decreases, so its increment of ICT capital x does not so increases, which leads to decrease of increment of ICT capital x as input in final good sector. The corresponding increment of production of final good leads to shrinkage, and finally growth rate g^* decreases in comparison with the growth rate before the rise of efficiency. In spite of the increase of efficiency γ , this result (decrease of growth rate g^*) is unexpected one. As the above simulation is comparative statics, from economic development point of view, it is required that this analysis is modified into dynamic analysis.

With transitional dynamics, we can confirm whether saddle path on BGP uniquely exists or not. However transitional dynamics for this model is very untractable. Therefore, in addition to the above comparative statics, we analyze the case where non-ICT and ICT capitals increase from the previous case. This case seems to be higher economic development stage than previous one. We assume that this case (path) is a candidate for BGP. In this case we proportionally increase the stocks of non-ICT capital k and of ICT capital x on simulation. Taking into consideration the above assumption and setting, we perform simulation¹⁴, and the results are shown in Fig. 8

(Case-1) and 9 (Case-2).

Fig. 8 Simulation of s with γ under increases of k and x (Case-1)Fig. 9 Simulation of s with γ under increases of k and x (Case-2)

In Fig. 8 (Case-1), in case of increase of efficiency γ , the share of non-ICT capital in ICT capital sector s^* decreases, so this enhances economic growth rate. This is the contrast result for the one in Fig. 6. Regarding Fig. 9 (Case-2), it seems that the result has the same tendency in Fig. 7.

As an interpretation for the aforementioned description, in Case-1 ($\alpha > \lambda$ and $\beta < \epsilon$), if efficiency γ increases, inputs of both capitals to ICT sector, $(1 - s^*)k$ and $(1 - u^*)x$, increase¹⁵, so output of ICT X increases. Output of final good also increases due to increases of s^*k and u^*x in spite of decrease of s^* and u^* . Eventually increase of γ enhances growth rate of final good g^* . On the other hand, in Case-2 ($\alpha < \lambda$ and $\beta > \epsilon$), if efficiency γ increases, inputs of both capitals to

¹⁴Except the values on non-ICT capital k and ICT capital x , the values of the other parameters are the same values in Fig. 6 and 7.

¹⁵when γ changes, ϕ_2 and ϕ_2 do not depend on γ , so that u^* in (15) only depends on s^* . Therefore in this case, decrease of s^* indicates decrease of u^* , since $\frac{\partial u^*}{\partial s^*} = \frac{\phi_2^{\alpha-\lambda}}{(s^* \phi_3^{\alpha-\lambda} + (1-s^*) \phi_2^{\alpha-\lambda})^2} > 0$.

ICT sector, $(1 - s^*)k$ and $(1 - u^*)x$, increase, so output of ICT X increase, which is the same as Case-1. However in this Case, the increase of output at ICT sector is not so much in comparison with Case-1 due to intensity of ICT ($\beta > \epsilon$). This affects accumulation of ICT capital x . Its increment of production of final good decreases due to decrease of s^*k attributing to decrease of s^* and intensity of non-ICT ($\alpha < \lambda$). Therefore increase of γ leads to decrease of growth rate of final good.

Result 1. At steady state, in case of increase of efficiency in ICT capital sector γ , growth rates of final good, non-ICT capital, and consumption g^* decrease. If accumulations of non-ICT and ICT capitals (k and x) become a higher stage, and $\alpha > \lambda$ and $\beta < \epsilon$ (Case-1), then growth rates of final good, non-ICT capital, and consumption g^* increase.

Next in ICT capital sector, we confirm the relationship between changes of two kinds of parameter (λ and ϵ) and the share s^* . At first we show the relationship between the parameter of non-ICT capital λ and the share s^* in Fig. 10 (Case-1) and 11 (Case-2).¹⁶

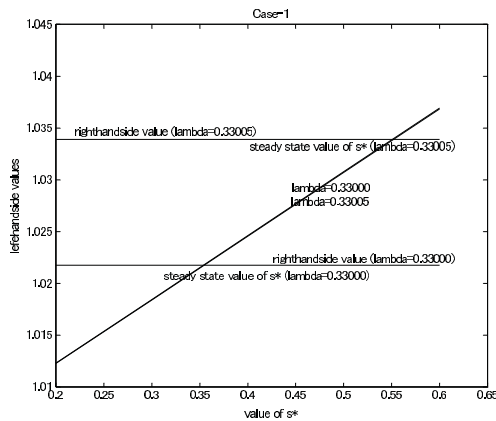


Fig. 10 Simulation of s with λ (Case-1)

From the above figures it follows that in case of increase of non-ICT capital intensity in ICT sector λ , the share of s^* in final good sector increases for both cases. That is to say, the share of non-ICT capital allocated to ICT sector $1 - s^*$ decreases, so that increase of s^* leads to decrease of growth rates of two capitals and consumption, and eventually decrease of economic growth rate. Then we have the following result.

Result 2. At steady state, in case of increase of intensity of non-ICT capital in ICT sector λ , growth rates of final good, two capitals and consumption g^* decreases.

Finally, we confirm the relationship between intensity of ICT capital in ICT sector ϵ and the share of non-ICT capital in

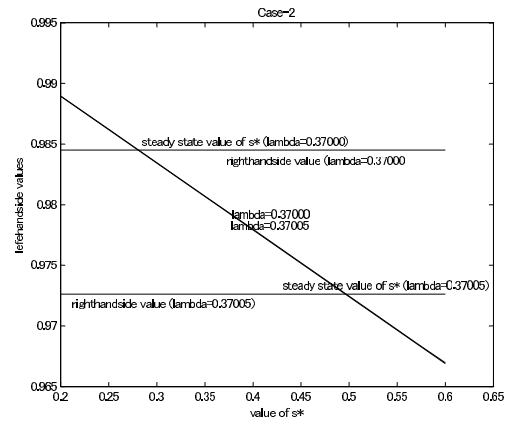


Fig. 11 Simulation of s with λ (Case-2)

final good sector s^* . The results are shown in Fig. 12 (Case-1) and 13 (Case-2).¹⁷

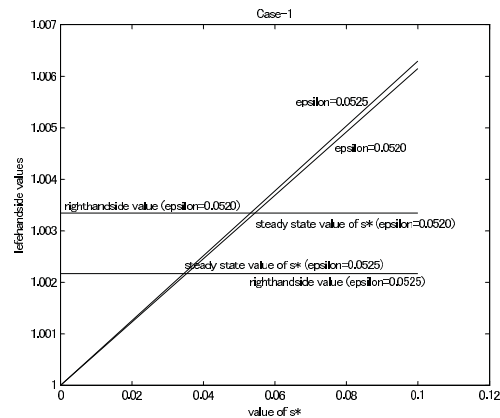


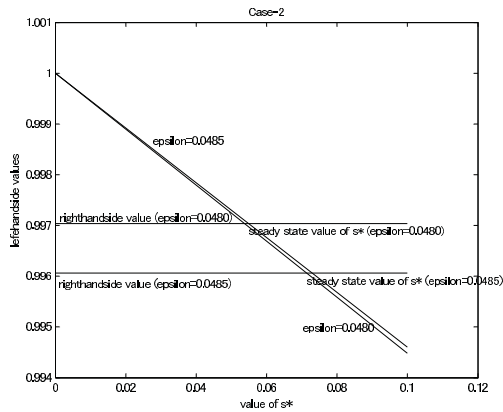
Fig. 12 Simulation of s with ϵ (Case-1)

In Fig. 12 (Case-1), in case of increase of intensity of ICT capital in ICT sector ϵ , the share of non-ICT capital allocated to final good sector s^* decrease. That is to say, the above increase leads to increase of the share of non-ICT capital at ICT sector $1 - s^*$, and then the decrease of s^* leads to increase of growth rates of two kinds of capitals and consumption g^* , and eventually increase of growth rate of final good. On the contrary, in Fig. 13 (Case-2), in spite of increase of ϵ , we confirm the opposite results against Case-1, such as the decrease of growth rate of final good. Then we have the following result.

Result 3. At steady state, in case of increase of intensity of ICT capital in ICT sector ϵ , if $\alpha > \beta$ and $\beta < \epsilon$ (Case-1), then growth rates of final good and consumption g^* increase,

¹⁶The parameters being used are as follows. The common values for both case, $\gamma = 1$, $A = 1$, $l = 1.0000$, $k = 120.0$, and $x = 1.0000$. For Case-1, except λ and $p = 1.1000$, the other parameters are same as ones in Fig. 2. For Case-2, except λ and $p = 0.9082$, the other parameters are same as ones in Fig. 3.

¹⁷The parameters being used are as follows. The common values for both Case, $\gamma = 1$, $A = 1$, $l = 1.0000$, $k = 120.0$, and $x = 1.0000$. For Case-1, except ϵ and $p = 1.0996$, the other parameters are same as ones in Fig. 2. For Case-2, except ϵ and $p = 0.902797$, the other parameters are same as ones in Fig. 3.

Fig. 13 Simulation of s with ϵ (Case-2)

and if $\alpha < \beta$ and $\beta > \epsilon$ (Case-2), then growth rates of final good and consumption g^* decrease.

We summarize the results of the above simulations, shown as Table II ($\frac{\alpha}{\sigma} < s^* < 1$).

Table II Summary of simulations

	Case-1	Case-2	Remarks
$\frac{\partial s^*}{\partial \gamma}$	+	+	k and x are constant
$\frac{\partial s^*}{\partial \lambda}$	-	+	k and x increase
$\frac{\partial s^*}{\partial \epsilon}$	+	+	ϵ is constant
$\frac{\partial s^*}{\partial \lambda}$	-	+	λ is constant

With regard to λ and ϵ in Table I, we note that in steady state the relationship between λ and ϵ is negative. Therefore, with Table I and Table II, for the relationship between growth rate g^* and the concerned parameters, we get the following result in Table III.

Table III Relationship between g^* and $\gamma, \lambda, \epsilon$

	Case-1	Case-2	Remarks
$\frac{\partial g^*}{\partial \gamma}$	-	-	k and x are constant
$\frac{\partial g^*}{\partial \lambda}$	+	-	k and x increase
$\frac{\partial g^*}{\partial \epsilon}$	-	+ or -	ϵ changes.
$\frac{\partial g^*}{\partial \lambda}$	+	+ or -	λ changes.

From Table III, Given that $\alpha > \lambda$ and $\beta < \epsilon$ (Case-1), for contribution of ICT sector to economic growth, it is necessary that accumulations of two kinds of capital become a higher stage and the efficiency in ICT sector γ increases, or the intensity of ICT capital in ICT sector ϵ increases. These results suggests that institutional circumstance and subsidy policy which enhance intensities γ and ϵ and accumulations of k

and x are necessary as industry policy.

V. CONCLUSION

This paper is fundamentally based on [1], [8], [14], however instead of human capital in [8], [14], ICT capital is incorporated into our model. Furthermore, in [1] commodity and consumable goods which are elements for utility are introduced, however the difference between commodity and consumable goods are very vague. In this model, as a way of development of model, we follow [1] and incorporate ICT capital into this model in lieu of consumable goods. Therefore we incorporate two kind of capital (non-ICT and ICT capitals) into our model.

As a result, while we indicate a possibility of existence of general equilibrium solution, we suggest that its possibility depends on values of parameter and initial values, and it means the possibility of no-existence of general equilibrium solution. However through our simulation where parameters of ICT sector are changed at steady state, we show the impact of ICT capital(sector) to economy. In Case-1 in this paper, as efficiency of ICT sector increases and accumulations of two kinds of capital become a higher stage, the share of non-ICT capital allocated to final good sector decreases and economic growth rate increases. We mention the same result in case of increase of the intensity of ICT capital in ICT sector. Conversely, we obtain the result that economic growth rate decreases in case of increase of intensity of non-ICT capital in ICT sector. From the above results, we conclude that ICT sector has impact to economic growth and firmly confirm that increase of efficiency at ICT sector and a higher stage of accumulations of capitals are necessary for economic development.

As further extensions, we conduct empirical research on production function for ICT sector, and ,with this empirical research, estimate values of parameters on production function of ICT. Using these estimated values of parameters, we perform simulation like this paper, analyze accurate role of ICT capital and economy, and show a transitional dynamics on equilibrium path.

APPENDIX A THE EQUILIBRIUM

We set the initial time as $t = 0$ and Hamiltonian as follow.

$$H(C, s, u, v, k, x, t) = e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} + \mu_1 \left(A(sk)^\alpha (ux)^\beta (vl)^{1-\alpha-\beta} - C - \delta k \right) + \mu_2 \left(\gamma((1-s)k)^\lambda ((1-u)x)^\epsilon ((1-v)l)^{1-\lambda-\epsilon} - \eta x \right).$$

Using the above equation, we obtain the first order conditions of the representative household's maximization problem and the related equations as follows.

$$\frac{\partial H}{\partial C} = e^{-\rho t} C^{-\sigma} - u_1 = 0. \quad (34)$$

Using the above equation, we obtain

$$\frac{\dot{\mu}_1}{\mu_1} = -\rho - \sigma \frac{\dot{C}}{C}. \quad (35)$$

$$\frac{\partial H}{\partial s} = \frac{\mu_1 \alpha Y}{s} - \frac{\mu_2 \lambda X}{1-s} = 0. \quad (36)$$

Using the above equation, we obtain

$$\mu_1 \left(\frac{\alpha Y}{s} \right) = \mu_2 \left(\frac{\lambda X}{1-s} \right). \quad (37)$$

$$\frac{\partial H}{\partial u} = \frac{\mu_1 \beta Y}{u} - \frac{\mu_2 \epsilon X}{1-u} = 0. \quad (38)$$

Using the above equation, we obtain

$$\mu_1 \left(\frac{\beta Y}{u} \right) = \mu_2 \left(\frac{\epsilon X}{1-u} \right). \quad (39)$$

$$\frac{\partial H}{\partial v} = \frac{\mu_1 (1-\alpha-\beta)Y}{v} - \frac{\mu_2 (1-\lambda-\epsilon)X}{1-v} = 0. \quad (40)$$

From the above equation, we get

$$\mu_1 \left(\frac{(1-\alpha-\beta)Y}{v} \right) = \mu_2 \left(\frac{(1-\lambda-\epsilon)X}{1-v} \right). \quad (41)$$

$$\frac{\partial H}{\partial k} = -\dot{\mu}_1 \quad (42)$$

From the above equation, we get

$$\frac{\alpha Y}{k} - \delta + \left(\frac{\lambda X}{k} \right) \frac{\mu_2}{\mu_1} = -\frac{\dot{\mu}_1}{\mu_1}. \quad (43)$$

$$\frac{\partial H}{\partial x} = -\dot{\mu}_2 \quad (44)$$

From the above equation, we get

$$\frac{\mu_1 \beta Y}{\mu_2 x} + \frac{\epsilon X}{x} - \eta = -\frac{\dot{\mu}_2}{\mu_2}. \quad (45)$$

Now, Denoting by p the relative shadow price, which is $p \equiv \frac{\mu_2}{\mu_1}$, then we obtain the dynamic equation as follow.

$$\frac{\dot{p}}{p} = \frac{\dot{\mu}_2}{\mu_2} - \frac{\dot{\mu}_1}{\mu_1}. \quad (46)$$

Regarding (46), we use (37), (39), (43), and (45), and then obtain the following equation.

$$\frac{\dot{p}}{p} = \frac{\alpha Y}{sk} - \frac{\epsilon X}{(1-u)x} + \eta - \delta. \quad (47)$$

From (37) we get

$$\frac{\alpha Y}{sk} = \lambda \gamma z_3^{\lambda-1} z_4^\epsilon p. \quad (48)$$

Also the following equation holds.

$$\frac{\epsilon X}{(1-u)x} = \epsilon \gamma z_3^\lambda z_4^{\epsilon-1}. \quad (49)$$

Therefore we substitute (48) and (49) for (47), and get the following equation.

$$\frac{\dot{p}}{p} = -\epsilon \gamma z_3^\lambda z_4^{\epsilon-1} + \lambda \gamma z_3^{\lambda-1} z_4^\epsilon p + \eta - \delta. \quad (50)$$

Next, from (5) to (8), we get

$$z_1 = \frac{s}{1-s} \frac{1-v}{v} z_3, \quad (51)$$

$$z_2 = \frac{1-v}{v} \frac{u}{1-u} z_4. \quad (52)$$

From (37), we get

$$\frac{\alpha Y}{\lambda X} \frac{1-s}{s} = \frac{\alpha A v z_1^\alpha z_2^\beta (1-s)}{\lambda \gamma (1-v) z_3^\lambda z_4^{\epsilon*} s} = \frac{\mu_2}{\mu_1} = p. \quad (53)$$

Furthermore, we substitute (51) and (52) for the above equation and manipulate, then we get

$$z_3^{\alpha-\lambda} = p \cdot \frac{\lambda \gamma}{A \alpha} \left(\frac{s}{1-s} \right)^{1-\alpha} \left(\frac{1-u}{u} \right)^\beta \left(\frac{1-v}{v} \right)^{1-\alpha-\beta} z_4^{\epsilon-\beta}. \quad (54)$$

Using (37), (38) and (39), we obtain the following equations.

$$\frac{\alpha \epsilon}{\beta \lambda} = \frac{s}{1-s} \cdot \frac{1-u}{u}, \quad (55)$$

$$\frac{\alpha(1-\lambda-\epsilon)}{\lambda(1-\alpha-\beta)} = \frac{s}{1-s} \cdot \frac{1-v}{v}. \quad (56)$$

Accordingly from the above equations, we get

$$\frac{\epsilon(1-\alpha-\beta)}{\beta(1-\lambda-\epsilon)} = \frac{1-u}{u} \cdot \frac{v}{1-v}. \quad (57)$$

We solve (56) and (57) for $s/(1-s)$ and $u/(1-u)$ respectively, and substitute these solutions for (54), then obtain the following equation.

$$z_3 = \left(\frac{\lambda \gamma}{A \alpha} \right)^{\frac{1}{\alpha-\lambda}} \left(\frac{\alpha(1-\lambda-\epsilon)}{\lambda(1-\alpha-\beta)} \right)^{\frac{1-\alpha}{\alpha-\lambda}} \left(\frac{\epsilon(1-\alpha-\beta)}{\beta(1-\lambda-\epsilon)} \right)^{\frac{\beta}{\alpha-\lambda}} p^{\frac{1}{\alpha-\lambda}} z_4^{\frac{\epsilon-\beta}{\alpha-\lambda}}. \quad (58)$$

We substitute (58) for (50) and obtain the below equation.

$$\begin{aligned} \frac{\dot{p}}{p} = & -\epsilon \gamma \phi_1^\lambda \phi_2^{\lambda(1-\alpha)} \phi_3^{\beta \lambda} p^{\frac{\lambda}{\alpha-\lambda}} z_4^{\frac{\lambda(1-\beta)-\alpha(1-\epsilon)}{\alpha-\lambda}} \\ & + \gamma \lambda \phi_1^{\lambda-1} \phi_2^{(1-\alpha)(\lambda-1)} \phi_3^{\beta(\lambda-1)} p^{\frac{\alpha-1}{\alpha-\lambda}} z_4^{\frac{\beta(1-\lambda)-\epsilon(1-\alpha)}{\alpha-\lambda}} \\ & + \eta - \delta, \end{aligned} \quad (59)$$

where

$$\phi_1 = \left(\frac{\lambda \gamma}{A \alpha} \right)^{\frac{1}{\alpha-\lambda}},$$

$$\phi_2 = \left(\frac{\alpha(1-\lambda-\epsilon)}{\lambda(1-\alpha-\beta)} \right)^{\frac{1}{\alpha-\lambda}},$$

$$\phi_3 = \left(\frac{\epsilon(1-\alpha-\beta)}{\beta(1-\lambda-\epsilon)} \right)^{\frac{1}{\alpha-\lambda}}.$$

Using these symbols, z_1 , z_2 and z_3 are expressed as follows.

$$z_1 = \phi_2^{\alpha-\lambda} z_3, \quad (60)$$

$$z_2 = \phi_3^{\alpha-\lambda} z_4, \quad (61)$$

$$z_3 = \phi_1 \phi_2^{1-\alpha} \phi_3^\beta p^{\frac{1}{\alpha-\lambda}} z_4^{\frac{\epsilon-\beta}{\alpha-\lambda}}. \quad (62)$$

APPENDIX B

THE SHARE OF NON-ICT AND ICT CAPITAL AND LABOR

Combining (5), (7) and (60), then we get

$$v = \frac{s}{s + (1-s)\phi_2^{\alpha-\lambda}} < 1. \quad (63)$$

Combining (6), (8) and (61), then we get

$$v = \frac{u}{u + (1-u)\phi_3^{\alpha-\lambda}} < 1. \quad (64)$$

On this occasion, (63)=(64), so that we obtain the equation for the share of ICT capital allocated to final good sector as below.

$$u = \frac{s\phi_3^{\alpha-\lambda}}{s\phi_3^{\alpha-\lambda} + (1-s)\phi_2^{\alpha-\lambda}} < 1. \quad (65)$$

Combining (7), (8) and (62), we obtain the following relationship.

$$\frac{(1-s)k}{(1-v)l} = \phi_1\phi_2^{1-\alpha}\phi_3^\beta p^{\frac{1}{\alpha-\lambda}} \left(\frac{(1-u)x}{(1-v)l} \right)^{\frac{\epsilon-\beta}{\alpha-\lambda}}. \quad (66)$$

Next transforming (63) and (65) respectively, we obtain the following equations.

$$1-v = \frac{(1-s)\phi_2^{\alpha-\lambda}}{s + (1-s)\phi_2^{\alpha-\lambda}}, \quad (67)$$

$$1-u = \frac{(1-s)\phi_2^{\alpha-\lambda}}{s\phi_3^{\alpha-\lambda} + (1-s)\phi_2^{\alpha-\lambda}}. \quad (68)$$

We substitute these equation for (66), and then we obtain the equation of the share of non-ICT capital s .

$$\begin{aligned} & \left[1 - (1 - \phi_2^{-(\alpha-\lambda)})s \right] \left[\frac{s + (1-s)\phi_2^{\alpha-\lambda}}{s\phi_3^{\alpha-\lambda} + (1-s)\phi_2^{\alpha-\lambda}} \right]^{-\frac{\epsilon-\beta}{\alpha-\lambda}} \\ &= \phi_1\phi_2^{1-\alpha}\phi_3^\beta p^{\frac{1}{\alpha-\lambda}} \frac{l}{k} \cdot \left(\frac{x}{l} \right)^{\frac{\epsilon-\beta}{\alpha-\lambda}}. \end{aligned} \quad (69)$$

APPENDIX C

THE DYNAMIC EQUILIBRIUM PATH

At first we derive the dynamic equilibrium growth path for consumption. To do so, from (35), we get

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} \left(-\frac{\mu_1}{\mu_1} - \rho \right) \quad (70)$$

and from (37) and (43), we get

$$-\frac{\mu_1}{\mu_1} = \frac{\alpha Y}{sk} - \delta. \quad (71)$$

We substitute the above equation for (70), then we obtain the dynamic equilibrium growth path for consumption.

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} \left(\frac{\alpha Y}{sk} - \delta - \rho \right) \quad (72)$$

$$= \frac{1}{\sigma} \left(\alpha A \left(\frac{sk}{vl} \right)^{\alpha-1} \left(\frac{ux}{vl} \right)^\beta - \rho - \delta \right). \quad (73)$$

Next we derive the dynamic equilibrium growth paths for two kind of capital (k and x). These paths are derived from (3) and (4) as follows.

$$\frac{\dot{k}}{k} = A \frac{(sk)^\alpha (ux)^\beta (vl)^{1-\alpha-\beta}}{k} - \frac{C}{k} - \delta, \quad (74)$$

$$\frac{\dot{x}}{x} = \gamma \frac{((1-s)k)^\lambda ((1-u)x)^\epsilon ((1-v)l)^{1-\lambda-\epsilon}}{x} - \eta. \quad (75)$$

APPENDIX D

THE STEADY STATE

From (37), we get

$$p = \frac{\mu_2}{\mu_1} = \frac{1-s}{s} \cdot \frac{\alpha Y}{\lambda X}. \quad (76)$$

By the definition of steady state the values of p , s , u , and v are constant, so that log-differentiating the both sides of (76) with respect to time leads to the following equation.

$$0 = \alpha \frac{\dot{k}}{k} + \beta \frac{\dot{x}}{x} - \lambda \frac{\dot{k}}{k} - \epsilon \frac{\dot{x}}{x}. \quad (77)$$

Ultimately, at steady state the following relationship holds.

$$\frac{\dot{k}}{k} = \frac{\epsilon - \beta}{\alpha - \lambda} \cdot \frac{\dot{x}}{x}. \quad (78)$$

The above relationship also holds for (69).

Next, from (73), $\left(\frac{sk}{vl} \right)^{\alpha-1} \left(\frac{ux}{vl} \right)^\beta$ is also constant, therefore the following equation have to hold.

$$\frac{\dot{k}}{k} = \frac{\beta}{1-\alpha} \cdot \frac{\dot{x}}{x}. \quad (79)$$

From (78) and (79) eventually the following relationship have to hold.

$$\frac{\beta}{1-\alpha} = \frac{\epsilon - \beta}{\alpha - \lambda}. \quad (80)$$

Also as (74) grows at a constant rate at steady state, we transform the equation as below and confirm that both sides have to be constant.

$$\frac{\dot{k}}{k} + \delta = A \frac{(sk)^\alpha (ux)^\beta (vl)^{1-\alpha-\beta}}{k} - \frac{C}{k}. \quad (81)$$

At steady state, taking into consideration that values of s , u , and v are constant, log-differentiating the both side of (81) with respect to time leads to the following equation.

$$g_C^* \frac{C}{k} + (g^* + \delta)g^* = W k^{\alpha-1} x^\beta g^* (\alpha + \beta \frac{\alpha - \lambda}{\epsilon - \beta}), \quad (82)$$

where $g^* \equiv \frac{\dot{k}}{k}$, $g_C^* \equiv \frac{\dot{C}}{C}$, and $W \equiv A(s/vl)^\alpha (u/vl)^\beta vl$, which are constant, and we make use of $(W k^{\alpha-1} x^\beta - C)/k = g^* + \delta$.

Furthermore log-differentiating the both side of (82) with respect to time, we get

$$\frac{(g_C^*)^2 - g_C^* g^*}{g_C^* + (g^* + \delta)g^* (k/C)} = (\alpha - 1)g^* + \beta g_x^*. \quad (83)$$

As both sides of (83) is constant, we obtain

$$g^* = g_C^*. \quad (84)$$

Finally, we derive the growth rate of final good, non-ICT capital and consumption g^* . From (73) we get

$$\alpha A z_1^{\alpha-1} z_2^\beta = \sigma \frac{\dot{C}}{C} + \rho + \delta. \quad (85)$$

And from (74) we get

$$\alpha A z_1^{\alpha-1} z_2^\beta = \frac{\alpha}{s} \left(\frac{\dot{k}}{k} + \frac{C}{k} + \delta \right). \quad (86)$$

Accordingly from (85) = (86), using $g^* = g_C^*$, we get

$$g^* = \frac{\alpha}{\alpha - \sigma s^*} \left(\frac{\rho s^*}{\alpha} - \delta \left(1 - \frac{s^*}{\alpha} \right) - \frac{C(0)}{k(0)} \right). \quad (87)$$

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