Dynamic Voltage Stability Estimation using Particle Filter

Osea Zebua, Norikazu Ikoma, Hiroshi Maeda

Abstract—Estimation of voltage stability based on optimal filtering method is presented. PV curve is used as a tool for voltage stability analysis. Dynamic voltage stability estimation is done by using particle filter method. Optimum value (nose point) of PV curve can be estimated by estimating parameter of PV curve equation. This optimal value represents critical voltage and maximum loading condition at specified point of measurement. Voltage stability is then estimated by analyzing loading margin condition dynamically.

Keywords—normalized PV curve, optimal filtering method, particle filter, voltage stability.

I. INTRODUCTION

GOOD quality and high reliability of electrical energy to supply consumer is ideal operation of an electrical power system. Voltage instability is one of major concerns in power system planning and operation. Load growth and disturbances had caused power systems are operated closer to their voltage stability boundaries. Several voltage instability or voltage collapse incidents lead to blackout of power system [1].

When the loading of power system approaches their maximum power, voltage magnitude of a particular bus or area decreases rapidly. However, the magnitude of the voltage itself is not a good index for determining the voltage stability [2].

The conventional P-V or Q-V curve is usually used as a tool for assessing voltage stability and hence for finding voltage collapse point or voltage stability limit and maximum loading condition. This curve shows the characteristic relationship between power and voltage at specified bus. This curve is usually generated from the results of repetitive load flow simulation and thus involved significant computation.

In [3]-[5], the proximity to voltage collapse is estimated by calculation of system equivalent parameters seen from a bus. These parameters are calculated using local bus voltage and current phasor measurement. The main disadvantage of this method is that required parameters are not constant and must be calculated repeatedly. Otherwise, analysis in this method is done in static way.

With the advent of phasor measurement unit (PMU), it is possible to get online time varying measurement data.

The recent developments of phasor measurement unit makes dynamic supervision for voltage stability become visible. In [6], author proposed simple method to calculate voltage stability margin dynamically by using PV curve.

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Online analysis for estimating maximum permissible loading and voltage stability margin is done by using locally measured quantities, such as bus voltage magnitude and load power [7]. It is assumed that any change in network topology or operating point usually modifies the system voltage profile. The disadvantage of this method is the estimation of parameters using least-square curve fitting technique requires two or more sets of data. Therefore, provided estimation will be less dynamic, if used data is very large.

Optimal filtering method, such as Kalman filter and Particle filter can estimate current value only by using previous value and hence, these methods can provide real-time estimation [8],[9].

This paper proposes a method to estimate parameters of PV curve equation using constrained optimal filtering method. Particle filter with constraint is employed to estimate parameter of PV curve equation and also provide voltage stability analysis information dynamically.

II. PROBLEM STATEMENT

If the load at a bus increases, the reactive power loss in transmission line also increases. If there is no adequate reactive power to control this increasing load, then voltage at bus starts to decrease until its critical value. Further increase of load can make voltage value exceeds their critical value and voltage starts to become unstable.

The relationship between voltage and power at bus is described with PV curve as shown in Fig. 1.
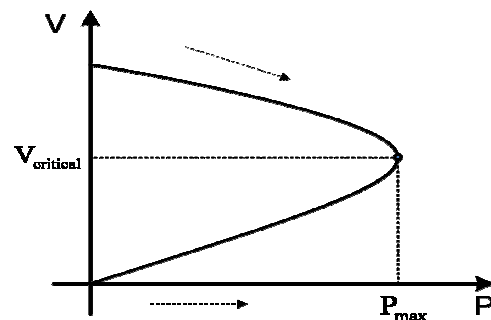

Fig. 1 PV curve

To simplify the analysis, two-bus power system is used as target system. The system consists of one generator (generating system), transmission line reactance and one load as shown in Fig. 2. Transmission line resistance is negligible.

In this research, generating system (generator) is assumed can supply reactive power with any changes of load. Therefore, any increase in load has no effect on the generating system.
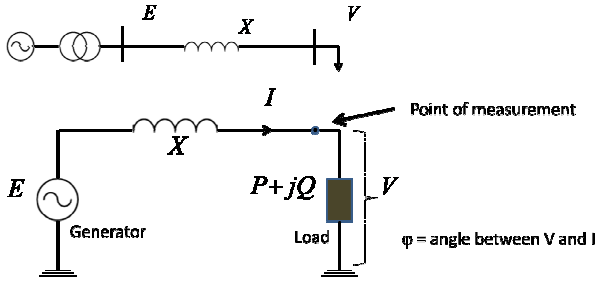
Fig. 2 Two-bus power system

At point of measurement, the relationship between power (P) and voltage (V) at any time is defined as [10]:

Equation (1) and (2) are called as PV curve equations.

$$P = -\frac{V^2}{X}\sin\varphi\cos\varphi + \frac{V}{X}\cos\varphi\sqrt{E^2 - V^2\cos^2\varphi} \qquad (1)$$

or,

$$V = \sqrt{\frac{E^2 - 2PX\tan\varphi \pm \sqrt{E^4 - 4PX\tan\varphi - P^2X^2}}{2}} \qquad (2)$$

For two-bus system, drawing the curve is done by gradually increasing the load on (2) until the load reaches the critical value. This curve is called *a normalized PV curve*, because the increased load is assumed to have no effect on the generation system.

Sometimes, equipment provides limited online measurement data in term of electrical quantity. In this research, we assume only the data of power (P) and voltages (V) are available. Thus, the problem becomes to estimate the value of an unknown quantities or unknown parameters in (1), i.e., E, X and φ at all times. Non-linear least squares method can also be used to solve this problem. This method requires a large of data to obtain a better estimation, the least amount of required data should be greater than the number of estimated parameters [11]. Thus, due to the large number of used data, changes in parameter values are not taken into account, and consequently the estimation information is less dynamic. Also, with redundant parameters and square root part in (1) and (2), it is possible to obtain inaccurate result or the method may fail to find a solution. State-space approach with probability is more reliable to overcome this problem. Particle filter method can deal with time-varying data measurement and provides estimation value at real-time. However, changes in parameter values should be considered before making the estimation process. For example, if V is much larger than E, then the solution of (1) cannot be achieved, because it will produce the square root of negative number.

III. ESTIMATION AND SOLUTION

A. Particle Filter

Particle filter is a sophisticated method of estimation based on simulation. It is sequential estimation of value at time k, x_k, only by using previous value at time k-1, x_{k-1} and observation

data at time k, y_k. Estimation value based on approximation of probability distribution using set of particles [12],[13].

Algorithm of particle filter method for prediction or estimation of non-linear system model is mainly consists of three steps:

1) Draw particles and update:

$$\hat{x}_k^{(l)} \sim f\left(x_k | x_{k-1}^{(l)}, y_k\right) \qquad l = 1,2,\dots M. \qquad (3)$$

2) Likelihood computation:

$$\hat{w}_k^{(l)} = h\left(y_k | \hat{x}_k^{(l)}\right) \quad l = 1,2,\dots M. \qquad (4)$$

where weight is normalized according to

$$w_k^{(l)} = \frac{\hat{w}_k^{(l)}}{\sum_{j=1}^{M} \hat{w}_k^{(l)}} \qquad l = 1,2,\dots M. \qquad (5)$$

3) Resampling: $x_k^{(l)} \sim \begin{cases} \hat{x}_k^{(1)} \text{ with probability } w_k^{(1)} \\ \hat{x}_k^{(2)} \text{ with probability } w_k^{(2)} \\ \vdots \\ \hat{x}_k^{(M)} \text{ with probability } w_k^{(M)} \end{cases}$ (6)

$$l = 1,2,\dots M.$$

where l is number of particle.

Based on the algorithm, suitable system model of (1) for usage with particle filter method is made. Parameters to be estimated are source voltage E, transmission reactance X and power factor φ. Time varying data of power (P) and voltage (V) is used as observation data. If noise with normal (Gaussian) distribution is added to each parameter, then state-space model can be written as follow:

$$\theta_k = F\theta_{k-1} + w_k^{\theta} \qquad (7)$$

or,

$$\begin{bmatrix} E_k \\ X_k \\ \varphi_k \end{bmatrix} = F \begin{bmatrix} E_{k-1} \\ X_{k-1} \\ \varphi_{k-1} \end{bmatrix} + \begin{bmatrix} w_k^E \\ w_k^X \\ w_k^{\varphi} \end{bmatrix} \qquad \begin{matrix} w_k^E \sim N(0,\sigma_E^2) \\ w_k^X \sim N(0,\sigma_X^2) \\ w_k^{\varphi} \sim N(0,\sigma_E^2) \end{matrix} \qquad (8)$$

where x = θ and θ = {E,X,φ}.

Observation model with adding Gaussian noise to the measurement data can be written as follow:

$$\begin{bmatrix} P_k^o \\ V_k^o \end{bmatrix} = \begin{bmatrix} f(V_k; \theta_k) \\ V_k \end{bmatrix} + \begin{bmatrix} e_k^P \\ e_k^V \end{bmatrix} \qquad \begin{matrix} e_k^P \sim N(0,\sigma_P^2) \\ e_k^V \sim N(0,\sigma_V^2) \end{matrix} \qquad (9)$$

where : θ_k is parameter to be estimated at time k (E_k,X_k,φ_k).
θ_{k-1} is parameter at time (k-1)
w_k^{θ} is added noise of each parameter at time k.
y_k is observation at time k.
F is state transition process.

P_k^o is observation of P at time k.
V_k^o is observation of V at time k.
e_k^P is observation noise of P at time k.
e_k^V is observation noise of V at time k.

B. Constraint

Constraint of PV curve equation is applied in estimation using particle filter method. Several constraints for parameter and observation data must be set before estimation start. These constraints are needed to achieve good estimation and are set based on mathematical analysis of (1) and electrical quantities behavior on actual operation of electrical power system.

Refer to (1), applied constraints are:

1. P>0, active power should be greater than zero.
2. $V_{min}<V<V_{max}$, voltage should not greater than specified minimum and maximum value.
3. $E_{min}<E<E_{max}$, source voltage must not greater than specified minimum and maximum value.
4. $-90^0<\varphi<90^0$, cosine of φ must have positive value.
5. E>V, source voltage must greater than voltage at load point. This constraint also fulfills equation (1) with range of φ at point 4.
6. $X = 2\pi fL$, $X_{min}\leq X\leq X_{max}$, transmission reactance must have values within specified range. This reactance value depends on the given value of frequency, and inductance, L.

C. Voltage Stability Assessment

Dynamic voltage stability assessment is analyzed using the difference of value between actual voltage and critical voltage as written in (10) and loading margin condition as written in (11) [1]. Loading margin is defined by the difference of value between maximum power and actual power.

If the difference between the actual and critical voltage is larger, it is said that the voltage value will be more stable. If loading margin is bigger, then the voltage stability of the system will be more stable and vice versa.

$$V_a^k = V_{actual}^k - V_{critical}^k \qquad (10)$$

where V_a^k is voltage difference at time k.

$$P_l^k = P_{max}^k - P_{actual}^k \qquad (11)$$

where P_l^k is loading margin at time k.

In each time k, parameter of PV curve equation is estimated and the curve is drawn based on (2). Nose point is computed and voltage difference and loading margin can be defined.

IV. SIMULATION EXPERIMENT

This research uses synthetic data which is generated in order to mimic PMU data. Generating data uses three scenarios and each scenario has scheduled parameter value of E, X, and φ. Scenarios with scheduled parameter are shown in Table I. By using the scaled random generation, and sampling time of 3.33 ms, 100 data of power and voltage are generated for each scenario. As a result, there are 300 time-varying data as shown in Fig. 3 and Fig. 4.

TABLE I
SCENARIOS WITH SCHEDULED PARAMETER

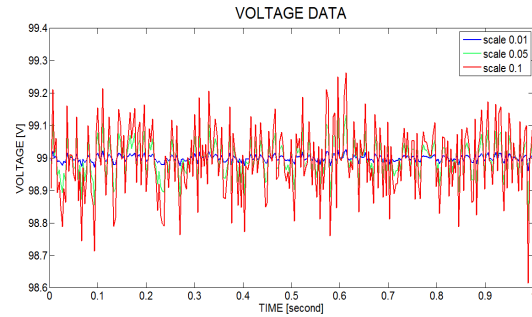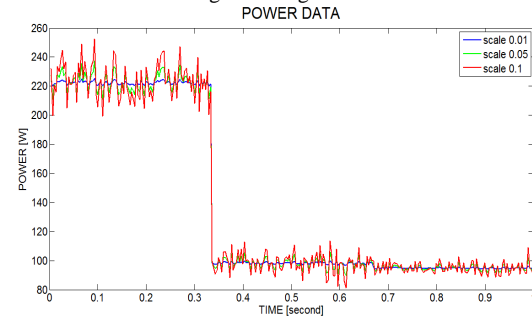| Scenario	Parameter		
	E [V]	X [OHM]	φ [DEGREE]
I	101.0	1.5	30.0
II	100.5	1.5	45.0
III	101.5	1.5	60.0

Fig. 3 Voltage data

Fig. 4 Power data

In each time step k, parameter of PV curve equation is estimated using constrained particle filter. Using same variance value of 0.01 for all parameter's noise distributions and for observation noise distribution, the result of the estimated value compared to the true value of each parameter are shown at Fig. 5, Fig. 6, and Fig. 7, respectively.
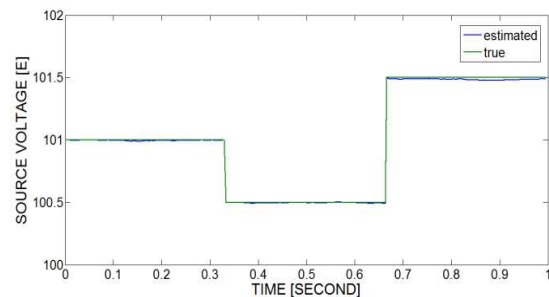

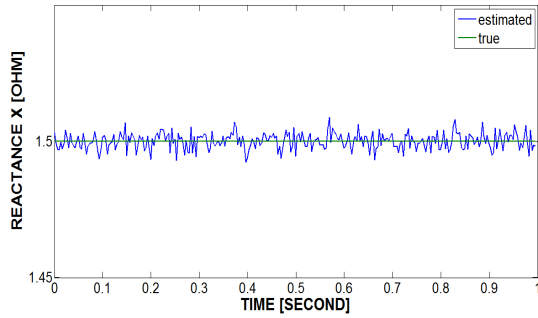Fig. 5 Comparison between estimated and true value of parameter E

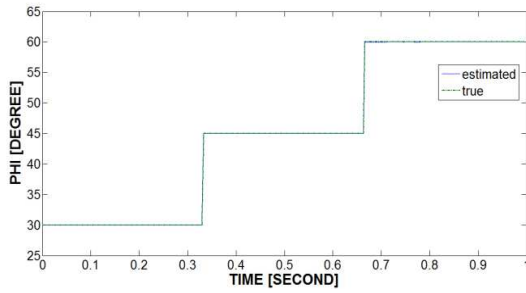Fig. 6 Comparison between estimated and true value of parameter X

Fig. 7 Comparison between estimated and true value of parameter φ

PV curve is drawn for each time k, find nose point value, compute dynamic voltage difference, and loading margin simultaneously. The results of dynamic voltage difference and loading margin are shown in Fig. 8 and Fig. 9.
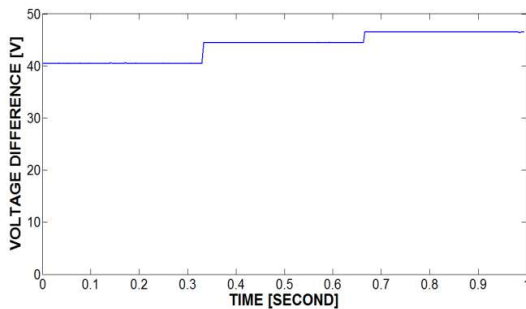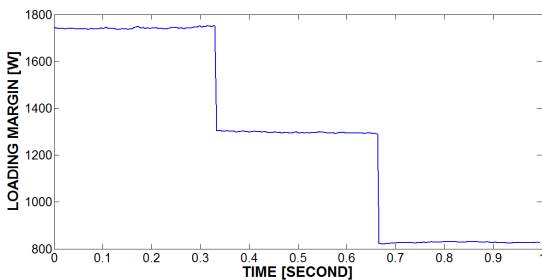

Fig. 8 Voltage difference

Fig. 9 Loading margin condition

From Fig.8 and Fig.9, it is shown that voltage value is more stable with the parameter of scenarios III, but the voltage stability of whole system is more stable at parameter of scenario I. This fact also informs that voltage value at specified point is not good indicator to measure voltage stability of the

system. Changes in estimated PV curve shape are not significant when the parameter is still the same, but the shape will change significantly when the value of the parameter change. This is described in Fig. 10, PV curve are drawn at time k = 0.333 second, k = 0.336 second and k=0.666 second. These chosen time describe the changes in scheduled parameter of measurement data, respectively. The values of nose point are, at time k = 0.333 second is 1973.3W, 58.5V, at time k = 0.336 second is 1402.9W, 54.5V, and at time k = 0.666 second is 1389.4W, 54.5V, respectively.
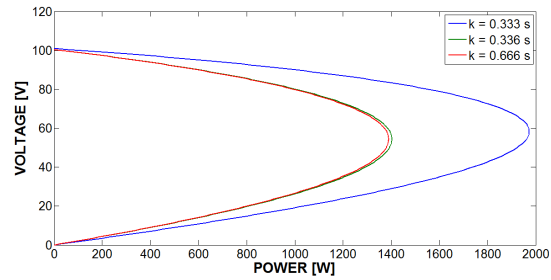

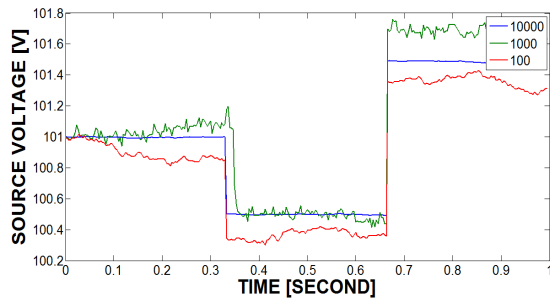Fig. 10 Estimated PV curve at specified time k

Fig. 11 Estimation with different number of particle

Estimation process is also performed using different number of particle and the results are shown in Fig. 11. Process is done by using 10^2 particles, 10^3 particles, and 10^4 particles, respectively. From the figure, it is shown that estimation by using a larger number of particles has better estimation results. Estimation using 10^4 particles can predict parameter values closer to the true value.

This research used synthetic data for estimation and proposed method can successfully estimate parameter values of PV curve equation and simultaneously provide information about voltage stability dynamically. Load change is described by change in parameter value. Constraint is used in achieving better estimation result and avoiding this method to be failed.

V. CONCLUSION AND FUTURE WORKS

This paper proposed a method to estimate voltage stability dynamically by using particle filter method. In the case of limited data, especially limitation in electrical quantity, this method can be used as dynamic estimator tool for estimating

parameter of PV curve equation. Knowledge of constraint, however, is needed in order to achieve better estimation.

For system with more than two buses, however, this method needs more information in order to develop system model. In large system, this method is also possible and more suitable to estimate voltage stability at adjacent buses, but still with assumption that while drawing the curve, condition of other system is assumed not changed. This work is considered in the future task.

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