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Dynamic Model of a Buck Converter with a Sliding Mode Control

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***Abstract*—**This paper presents the averaging model of a buck converter derived from the generalized state-space averaging method. The sliding mode control is used to regulate the output voltage of the converter and taken into account in the model. The proposed model requires the fast computational time compared with those of the full topology model. The intensive time-domain simulations via the exact topology model are used as the comparable model. The results show that a good agreement between the proposed model and the switching model is achieved in both transient and steady-state responses. The reported model is suitable for the optimal controller design by using the artificial intelligence techniques.

***Keywords*—**Generalized state-space averaging method, buck converter, sliding mode control, modeling, simulation.

I. Introduction

PRESENTLY, power electronic converters are widely used in many industrial applications. The mathematical models of these converters are very important for engineers to study the system dynamic behavior. However, the power converter models are normally time-varying due to the switching actions. The analysis of the system with time-varying models is very complicated. Moreover, such model consumes the vast simulation time because of the switching devices. Hence, there are many techniques that are used to eliminate the converter switching behavior to achieve the time-invariant model suitable for the system analysis and design. Besides, this model requires the faster computational time compared with that of the switching model from the software packages. In the paper, the generalized state-space averaging (GSSA) method is selected to analyze the buck converter in which this method is well known for analyzing the DC/DC converter [1]-[3]. In addition, the sliding mode control (SMC) [4] is applied to regulate the output voltage of the system. This is because the SMC can provide a good dynamic response, robustness, and simple implementation [5]-[7]. The aim of the paper is to derive the mathematic model of a buck converter with SMC. The intensive time-domain simulation is considerably used as the comparator with the proposed model in terms of accuracy and computational time. The averaged models of power converters can be used for studies of dynamic behavior of complex power systems. The results will show later that simulations using the proposed model drastically reduce simulation time in comparison with the available software package. Therefore, the proposed model is suitable for the optimal controller design via artificial intelligence (AI) techniques in which the repeating calculations are needed for searching the best solution.

The paper is structured as follows. In Section II, the considered system definition is explained. Deriving the dynamic model including the SMC is described in Section III. In Section IV, the simulation results for the model comparison between the proposed model and the full switching model are illustrated. Finally, Section V concludes and discusses the advantages of the proposed model for studies the system dynamic behavior and optimal SMC design.

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II. Power System Considered

The studied power system in the paper is depicted in Fig. 1. It consists of a DC voltage source, a buck converter feeding a resistive load, and SMC to keep the output voltage constant, here regulated to the V_{ref}. For model derivation, the buck converter is assumed to operate under the continuous conduction mode (CCM).

For the open-loop control, the switching signal of the switching device is shown in Fig. 2. When the power converter is controlled by SMC to regulate the output voltage, the switching signal $u(t)$ becomes u_{eq} as depicted in Fig.1. The u_{eq} is from the control signal via the SMC depending on the V_{ref}. In the previous publication [1]-[3], the buck converter model is derived by using the averaging technique, namely the GSSA method. The averaged model is normally analyzed under the open-loop operation. In this paper, the SMC parameters are taken into account and will be occurred into the dynamic model.

III. Deriving Dynamic Model

As mentioned in Section II, the buck converter model can be derived by using the GSSA method. The GSSA method is an alternative method to eliminate the time-varying switching function to achieve a time-invariant power converter model. The approach uses the time-dependent coefficients of the complex Fourier series as the state variables. The overview of this approach [1]-[3] is as follows:

In general, a periodic waveform with period T can be represented by the complex Fourier series of the form

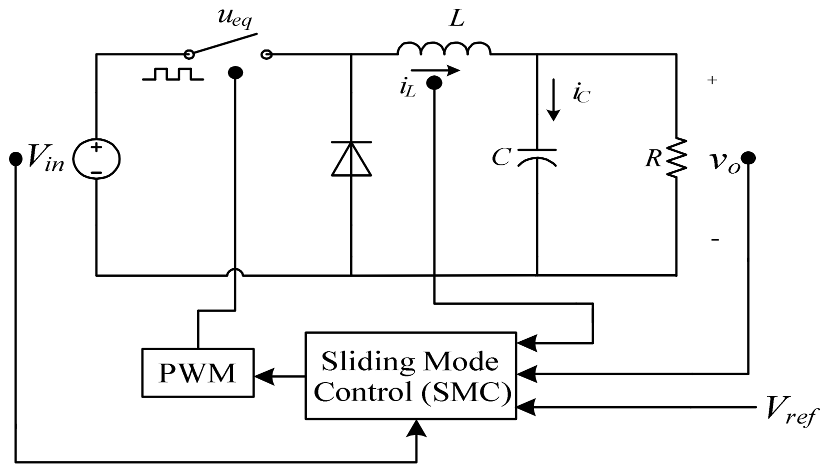

Fig. 1 Power system considered with SMC

$$f(t) = \sum_{k=-\infty}^{\infty} \langle x \rangle_k (t) e^{jk\omega_s t} \tag{1}$$

where $\omega_s = 2\pi/T$ and $\langle x \rangle_k (t)$ is the complex Fourier coefficients.

The GSSA approach uses the $\langle x \rangle_k (t)$ of the waveform as the state variables of the system. These coefficients can be determined by

$$\langle x \rangle_k (t) = \frac{1}{T} \int_{t-T}^{t} f(t) e^{-jk\omega_s t} dt \tag{2}$$

The necessary properties of the $\langle x \rangle_k (t)$ for modeling the power system using the GSSA technique are as follows:

- differentiation with respect to time:

$$\frac{d\langle x \rangle_k}{dt} = \left\langle \frac{dx}{dt} \right\rangle_k - jk\omega_s \langle x \rangle_k \tag{3}$$

- the convolution relationship:

$$\langle xy \rangle_k = \sum_i \langle x \rangle_{k-i} \langle y \rangle_i \tag{4}$$

- if $f(t)$ is real (real-value periodic waveform),

$$\langle x \rangle_{-k} = \overline{\langle x \rangle_k} = \langle x \rangle_k^* \tag{5}$$

In equations (1) and (2), the value of k depends on the accuracy level. Theoretically, if k approaches infinity, the approximation error approaches zero. If the waveform can be assumed to have no ripple, it can be set to $k = 0$ called zero-order approximation [1]-[3]. On the other hand, if the waveform is similar to a sinusoidal signal, k can normally be set to -1, 1. This particular case is referred to as the first harmonic approximation [2]. In this section, the details of how to derive the zero-order approximation model of the considered system by using the GSSA method are fully described.

Applying KVL and KCL to the circuit in Fig. 1 without considering the SMC, the time-varying differential equations can be written by:

$$\begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L}\left[V_{in}u(t) - v_o\right] \\ \dfrac{dv_o}{dt} = \dfrac{1}{C}\left[i_L - \dfrac{v_o}{R}\right] \end{cases} \tag{6}$$

According to (2) with the switching signal of the buck converter as depicted in Fig. 2, the zero-order of the $u(t)$ can be expressed as:

$$\langle u \rangle_0 = \frac{1}{T_s}\int_0^{dT_s} 1e^0 dt = \frac{1}{T_s}\left[t\right]_{t=0}^{t=dT_s} = d \tag{7}$$

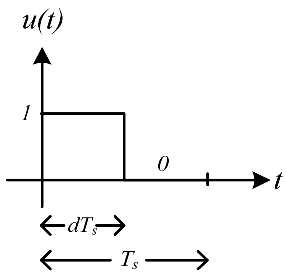

Fig. 2 The switching signal for the open-loop operation

For the voltage source V_{in}, due to the DC voltage source, the zero-order of V_{in} can be given in

$$\langle V_{in} \rangle_0 = V_{in} \tag{8}$$

From (1), the zero-order approximations of the actual states ($k = 0$) are denoted as:

$$\begin{cases} i_L = \langle i_L \rangle_0 \\ v_0 = \langle v_0 \rangle_0 \end{cases} \tag{9}$$

The Fourier expansion leads to 2 state variables denoted by x_k as follows:

$$\begin{cases} x_1 = \langle i_L \rangle_0 = i_L \\ x_2 = \langle v_0 \rangle_0 = v_0 \end{cases} \tag{10}$$

Applying the property (3) into (6) for $k = 0$, the $\frac{d\langle i_L \rangle_0}{dt}$ and $\frac{d\langle v_0 \rangle_0}{dt}$ can be expressed as:

$$\begin{cases} \frac{d\langle i_L \rangle_0}{dt} = \frac{1}{L}\left[\langle V_{in}u(t)\rangle_0 - \langle v_0 \rangle_0\right] \\ \frac{d\langle v_0 \rangle_0}{dt} = \frac{1}{C}\left[\langle i_L \rangle_0 - \frac{\langle v_o \rangle}{R}\right] \end{cases} \tag{11}$$

Using (4), (7), and (8), $\langle V_{in}u(t)\rangle_0$ in (11) can be expressed as:

$$\langle V_{in}u(t)\rangle_0 = \langle V_{in}\rangle_0 \langle u \rangle_0 = dV_{in} \tag{12}$$

Therefore, (11) can be rewritten in terms of the state variables defined in (10) as:

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_o \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_o \end{bmatrix} + \begin{bmatrix} \frac{dV_{in}}{L} \\ 0 \end{bmatrix} \tag{13}$$

where i_L and v_o are the circuit state-variables and d is the duty cycle of the buck converter. Note that before applying the GSSA method, the dynamic model of the considered system is a time-varying as given in (6). Using the GSSA method, (6) becomes (13) that is the averaging model of the buck converter under the open-loop operation with the zero-order approximation.

For SMC, the sliding surface can be set as the linear combination of the state-variables because it is simple for implementation. The sliding surface equation can be expressed as:

$$S = J^T \mathbf{x} = ax_1 + bx_2 + mx_3 = 0 \tag{14}$$

Note that a, b, and m are the coefficients of SMC, while x_1, x_2, and x_3 are defined as:

$$\begin{cases} x_1 = i_{ref} - i_L \\ x_2 = V_{ref} - v_o \\ x_3 = \int \left[x_1 + x_2\right] dt \end{cases} \tag{15}$$

where $i_{ref} = K\left[V_{ref} - v_o\right]$ and K is the gain to amplify the voltage error.

Applying (15) into (14), the sliding surface equation becomes

$$\begin{aligned} S &= a\left(i_{ref} - i_L\right) + b\left(V_{ref} - v_o\right) \\ &\quad + m\int \left(K(V_{ref} - v_o) - i_L + V_{ref} - v_o\right) dt = 0 \end{aligned} \tag{16}$$

Then,

$$\begin{aligned} \frac{dS}{dt} &= \left[-a \quad -b\right]\frac{d}{dt}\begin{bmatrix} i_L \\ v_o \end{bmatrix} - \frac{aKi_L}{C} + \frac{aKv_o}{RC} \\ &\quad + m\left[(K+1)(V_{ref} - v_o) - i_L\right] = 0 \end{aligned} \tag{17}$$

Substituting $\frac{d}{dt}\begin{bmatrix} i_L \\ v_o \end{bmatrix}$ from (13) into (17) and replacing d by u_{eq} yields:

$$\begin{aligned} \frac{dS}{dt} &= \frac{av_o}{L} - \frac{aV_{in}u_{eq}}{L} - \frac{bi_L}{C} + \frac{bv_o}{RC} \\ &\quad - \frac{aKi_L}{C} + \frac{aKv_o}{RC} + m\left[(K+1)(V_{ref} - v_o) - i_L\right] = 0 \end{aligned} \tag{18}$$

From (18), u_{eq} of SMC can be calculated by:

$$\begin{aligned} u_{eq} &= \frac{aRCv_o - bL(Ri_L - v_o) - aLK(Ri_L - v_o)}{aRCV_{in}} \\ &\quad + \frac{mRLC\left[(K+1)(V_{ref} - v_o) - i_L\right]}{aRCV_{in}} \end{aligned} \tag{19}$$

where u_{eq} is continuous and limited from 0 to 1.

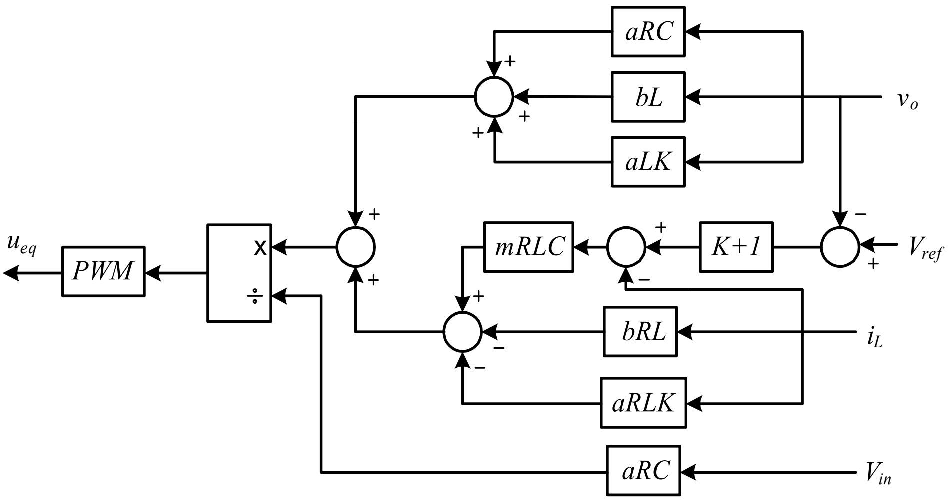

Fig. 3 The block diagram of u_{eq} calculation

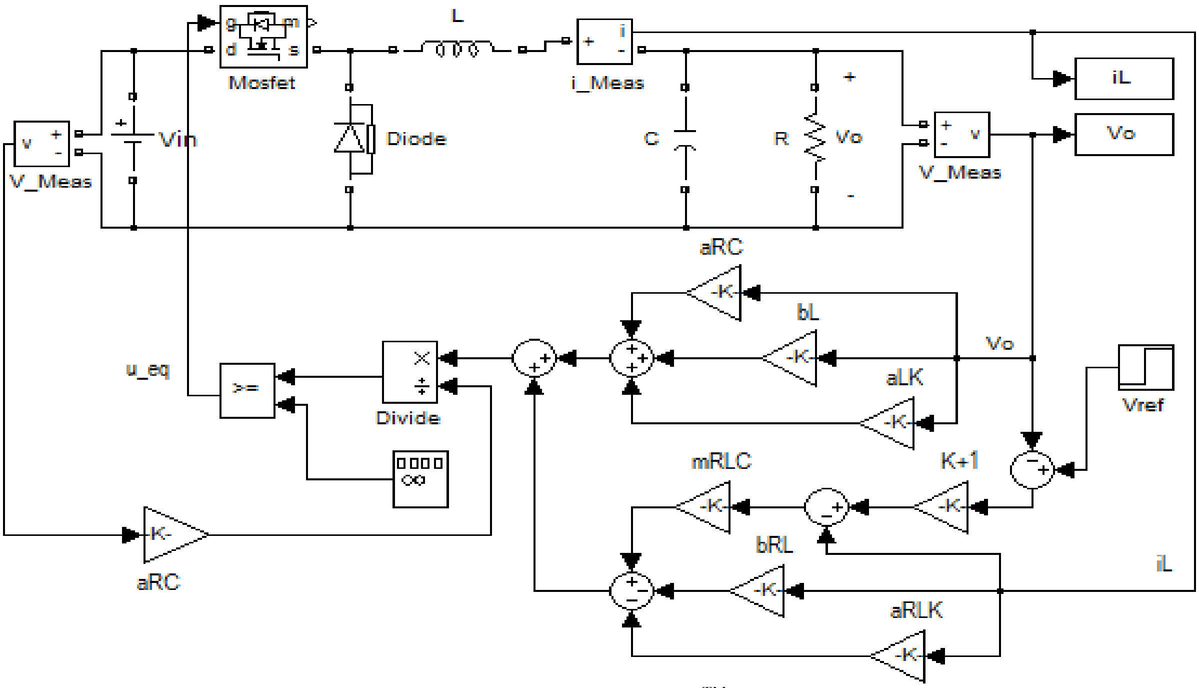

Fig. 4 The exact topology model of SPS^{TM} in SIMULINK

The block diagram for u_{eq} calculation based on (19) is depicted in Fig. 3. For deriving the dynamic model of the power system including the controllers of buck converter, the schematic of the SMC as shown in Fig. 3 is considered. It can be seen that the SMC parameters are represented by *a*, *b*, *m*, and *K*. In addition, when the buck converter is regulated by SMC, the *d* in (13) becomes u_{eq} as given in (19). Therefore, the dynamic model of the system on sliding surface can be expressed as:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} = \mathbf{Cx} + \mathbf{Du} \end{cases} \tag{20}$$

where state variables: $\mathbf{x} = [i_L \ v_o]^T$, input: $\mathbf{u}$ =[V_{ref}], output: $\mathbf{y} = [i_L \ v_o]^T$.

The details of **A**, **B**, **C**, and **D** in (20) are as follows:

$$\begin{cases} \mathbf{A} = \begin{bmatrix} -\left(\dfrac{b+aK+mC}{aC}\right) & \dfrac{b+aK-mRC(K+1)}{aRC} \\ \dfrac{1}{C} & -\dfrac{1}{RC} \end{bmatrix} \\ \mathbf{B} = \begin{bmatrix} \dfrac{m(K+1)}{a} \\ 0 \end{bmatrix} \\ \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \mathbf{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{cases} \quad (21)$$

In (20) and (21), the SMC parameters are occurred in the dynamic model. Therefore, the proposed averaging model can represent the dynamic behavior of the system under the SMC.

IV. Simulation Results

The linear time-invariant (LTI) model in (20) is compared by using the exact topology model in SimPowerSystem™ (SPS™) of SIMULINK as depicted in Fig. 4. The set of parameters for the system in Fig.1 is given as follows: V_{in} = 60 V, R = 30 Ω, L = 15 mH, C = 125 μF, T_s = 0.1 ms., a = 3, b = 25, m = 2500, K = 2000. The system in Fig.1 was simulated to a step change of V_{ref} from 10 V to 15 V that occurs at t = 0.03 second. The output voltage and inductor current responses from the proposed model of (20) compared with the result from SPS™ are shown in Fig. 5 and Fig. 6, respectively. Similarly, the voltage and current responses for changing the V_{ref} from 10 V to 20 V are also depicted in Fig. 7 and Fig. 8.

For changing the V_{ref} from 10 V to 15 V, the simulation time when the system was simulated via the proposed model coding in MATLAB requires 35 second, while the full topology model as shown in Fig.4 consumes 735 second. The comparison of simulation time demanded from both models can be illustrated in terms of computational saving time that can be defined by:

$$\%t_{saving} = \frac{t_{fs} - t_{av}}{t_{fs}} \times 100\% \quad (22)$$

where t_{fs} and t_{av} are the simulation times of full topology model and the proposed averaging model, respectively.

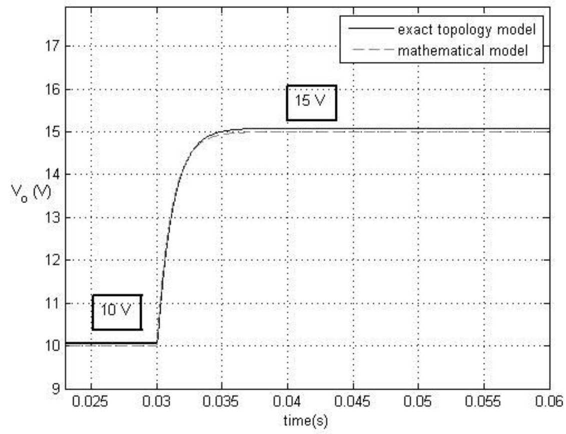

Fig. 5 The output voltage response for changing V_{ref} from 10 V to 15 V

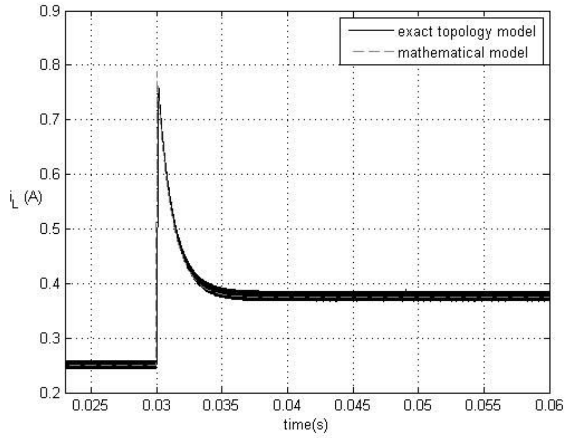

Fig. 6 The inductor current response for changing V_{ref} from 10 V to 15 V

Therefore, according to (22), the $\%t_{saving}$ = 95.24% is obtained when the proposed model is applied to simulate the system. Similarly, for changing the V_{ref} from 10 V to 20 V, $\%t_{saving}$ equal to 96.51% is achieved. The results indicate that simulation using the proposed model of (20) requires the faster computational time compared with those of SPS™.

Moreover, an excellent agreement between the averaging model based on SMC and the full switching model is obtained. It confirms that the reported model in the paper can provide a high accuracy representation of the real system. Therefore, the dynamic model from the paper can be used for the simulation with faster computational time. The proposed model is also suitable for the optimal controller design via the AI techniques because the fast simulation time can be achieved.

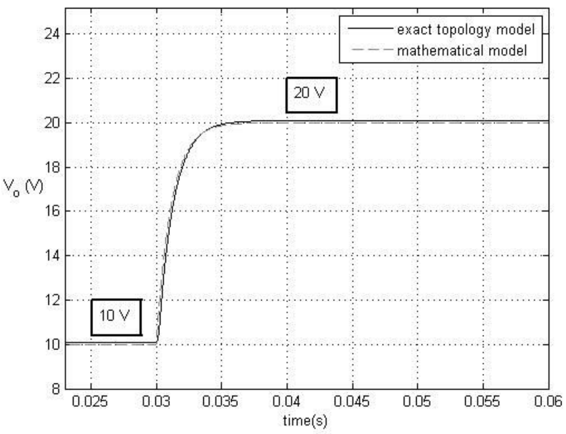

Fig. 7 The output voltage response for changing V_{ref} from 10 V to 20 V

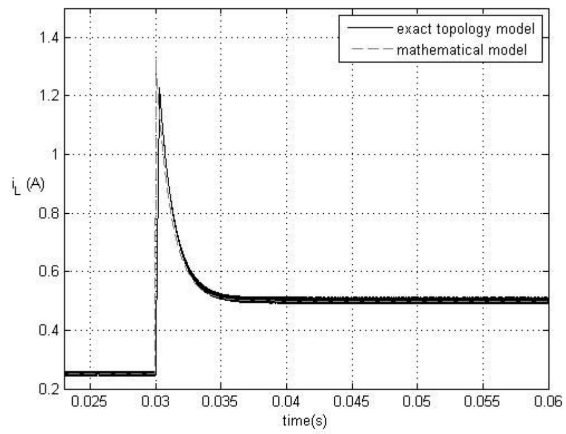

Fig. 8 The inductor current response for changing V_{ref} from 10 V to 20 V

V. Conclusion

This paper has described how to derive the dynamic model of buck converter with SMC by using the GSSA method. The results obtained from the proposed model show excellent agreement with the commercial software package, namely SPS™ of SIMULINK. The simulation time consumed by the approximation model coding in MATALB is much less than that consumed by the comparable switching model of simulation package. Therefore, the proposed model with SMC derived from the GSSA method is suitable for transient simulation of the system with the power converters in which the computational time can be considerably reduced. According to the advantages of the proposed model, it can be applied for the optimal controller design via the AI technique in which the repeating calculations are needed for searching the best solution.

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