International Journal of Earth, Energy and Environmental Sciences
ISSN: 2517-942X
Vol:9, No:7, 2015

Dynamic Active Earth Pressure on Flexible Cantilever Retaining Wall

Snehal R. Pathak, Sachin S. Munnoli

***Abstract*—**Evaluation of dynamic earth pressure on retaining wall is a topic of primary importance. In present paper, dynamic active earth pressure and displacement of flexible cantilever retaining wall has been evaluated analytically using 2-DOF mass-spring-dashpot model by incorporating both wall and backfill properties. The effect of wall flexibility on dynamic active earth pressure and wall displacement are studied and presented in graphical form. The obtained results are then compared with the various conventional methods, experimental analysis and also with PLAXIS analysis. It is observed that the dynamic active earth pressure decreases with increase in the wall flexibility while wall displacement increases linearly with flexibility of the wall. The results obtained by proposed 2-DOF analytical model are found to be more realistic and economical.

***Keywords*—**Earth pressure, earthquake, 2-DOF model, plaxis, wall movement, retaining walls.

I. Introduction

RETAINING walls are the structures designed to retain soil to unnatural slopes. Accurate estimation of earth pressure is very important for safe and economical design of retaining wall especially when it is subjected to ground motion. After the great Kanto earthquake in 1923, due to disastrous effect on retaining wall, it became necessary to know the earth pressure which acts on the wall during earthquake. The nature, magnitude and distribution of the dynamic earth pressure is highly different from that of static earth pressure case. Thus, with the increasing development in design criteria of retaining wall, the evaluation of the earth pressure and displacements of earth retaining structures under seismic conditions is a topic of considerable interest.

II. Literature Review

The seismic response of retaining structure depends on the response of the foundation soil, and the backfill, the inertial and flexural responses of the wall itself, and the nature of the input motions. In engineering practice, the current state of the art is primarily based on the pioneering work carried out in Japan by [1], [2] which is the pseudo-static force-based analysis considering only the maximum amplitude of ground motion and not the frequency. Further, [3] developed a new method to determine the dynamic earth pressure on rigid retaining wall using pseudo-dynamic approach in which they considered the effect of both amplitude and frequency by assuming finite shear and primary wave velocity. [4] First proposed a simplified SDOF mathematical model to evaluate the seismic-active earth pressure acting on rigid retaining wall. This model was suitable for small translational modes of wall movement. However, this approach does not take into account the wall properties and assumes total dependency on the stiffness of the springs between the far-field and the wall. Several researchers [5], [6] have later modified this model; most of which are too complex to use for engineering practices.

The existing approaches, for seismic case, are limited to non-deflecting rigid walls only and do not in general explain the important effect of wall flexibility. As in practice, most of the retaining walls are flexible in nature, evaluation of dynamic earth pressure and wall displacement of such wall is of prime importance. In present study, an attempt has been made to determine the dynamic active earth pressure on cantilever flexible retaining wall and displacement of such wall by modeling it as 2-DOF mass-spring-dashpot system. As both the wall and backfill are idealized by incorporating their properties, the solution for the 2 DOF systems is expected to have much more accuracy in its results when compared with other approaches. The effect of wall flexibility on the dynamic earth thrust and wall displacement has also been studied using this 2-DOF MSD model.

III. Analytical Model

A. Idealization of Retaining Wall

The soil–wall system in which the semi-infinite, homogeneous, cohesion less soil is considered to be retained by a vertical flexible retaining wall of height H along one of its vertical boundaries as shown in Fig. 1. The base of the soil layer is excited by a harmonic excitation to simulate earthquake conditions. The material of the wall and the soil layer is defined by the mass density ρ, shear modulus G, Poisson's ratio μ, and the damping factor ζ of the wall material and backfill sand.

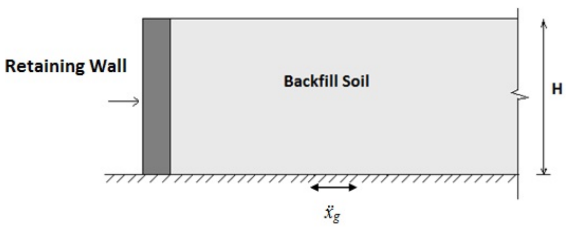

Fig. 1 Soil-Wall System

Snehal R. Pathak and Sachin S. Munnoli are with Department of Civil Engineering, College of Engineering, Pune, India (e-mail: srp.civil@coep.ac.in, sach.ssm23@gmail.com).

This soil-wall system is modeled by a simple two-degree freedom (2-DOF) mass–spring–dashpot model, having mass M_1, damping constant c_1, and spring stiffness k_1 for the backfill soil and that for the wall as M_2, c_2 and k_2 respectively as shown in Fig. 2. The backfill soil layer of mass M_1 is excited by ground acceleration with amplitude $\ddot{x}_g$. The external forces applied on the backfill soil and retaining wall are $f_1(t)$ and $f_2(t)$ respectively.

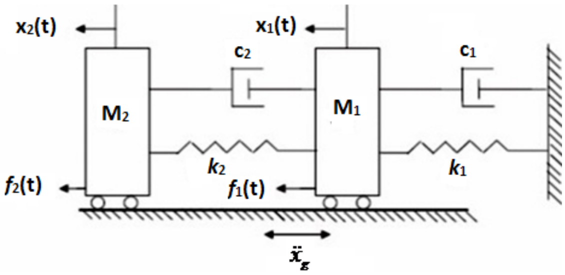

Fig. 2 Proposed 2-DOF Model

B. Formulation

The idealized soil-wall system is analyzed from free body diagram of both backfill soil and wall. Considering the dynamic equilibrium of these two masses by using D'Alembert's principle, basic dynamic equation of motion has been obtained with inertia force as $M_1 \ddot{x}_1$ for backfill soil and $M_2 \ddot{x}_2$ for wall.

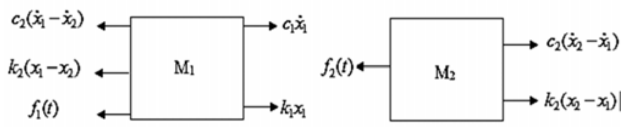

Fig. 3 Free-Body Diagram of Wall and Backfill-soil

From the free-body diagram portrayed in Fig. 3, the following equation of motion is evident:

$$M_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1x_1 + k_2(x_1 - x_2) = f_1(t) \quad (1)$$

where, x_1 and x_2 are the displacements of masses M_1 and M_2 respectively; $\dot{x}_1$ and $\dot{x}_2$ are the velocities of masses M_1 and M_2 respectively; $\ddot{x}_1$ is the acceleration of mass M_1 and $f_1(t)$ is the applied external force on mass M_1 active up to a finite time, the duration of an earthquake. In similar fashion, Fig. 3 further yields:

$$M_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = f_2(t) \quad (2)$$

where, $\ddot{x}_2$ is the acceleration of mass M_2. As it is assumed that the earthquake acceleration of amplitude $\ddot{x}_g$ is applied only to the backfill soil and not on retaining wall, the applied external force on mass M_2 i.e. the wall retaining the backfill $f_2(t)$ is equal to zero.

Using (1) and (2), the basic dynamic equation for 2-DOF system in matrix form can be written as:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (C_1+C_2) & -C_2 \\ -C_2 & C_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix} \quad (3)$$

As an Ansatz, one prescribes the solution to this equation as $x = Xe^{i\omega t}$ in accord with $f(t) = f_0e^{i\omega t}$. Substitution of these forms in the matrix equation (3) the following matrix equation is engendered:

$$\{-\omega^2[M] + i\omega[c] + [k]\}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix} \quad (4)$$

The solution to (4) yields the displacement, velocity and acceleration responses for both soil and the wall. Due to applied ground motion to backfill soil, retaining wall moves away from, or towards the backfill. Hence the earth thrust (Q_b) is generated behind retaining wall. In order to evaluate this thrust, forces acting on the retaining wall considered are as depicted in Fig. 4.

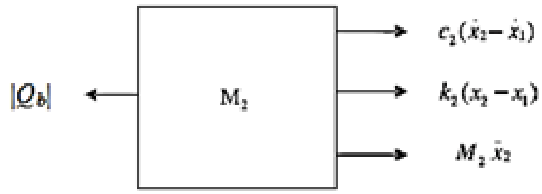

Fig. 4 Magnitude of Dynamic Earth Thrust

$$|Q_b| = M_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) \quad (5)$$

Thus, the wall displacement and dynamic earth thrust now can be evaluated by using (4) and (5).

C. Design Parameters

To obtain a solution, the related design parameters, like stiffness of the wall (k_2), stiffness of backfill (k_1), and damping constant c of both wall and backfill are calculated assuming they are independent of the frequency of excitation. The stiffness value for backfill soil (k_1) is determined such that the undamped natural frequency of the model equals the fundamental natural frequency of the medium idealized as a series of vertical shear-beams [6]. Thus, the stiffness of backfill is given by,

$$k_1 = M_1\left(\frac{\pi^2}{4H^2}\right)\frac{G}{\rho_s} \quad (6)$$

The mass of the backfill (M_1) is calculated by assuming the length of the backfill 10H, where H is the height of the wall which can be taken as the minimum horizontal distance of influence from the face of the wall. Further increase of this influence zone does not contribute much to the resulting earth thrust [7]. The stiffness of the wall (k_2) is calculated by using slope deflection properties of cantilever wall. The expression for the stiffness is obtained by considering the wall fixed at bottom and thus resulting into cantilever action. Thus stiffness of wall (k_2) is given by:

$$k_2 = \frac{30E_wI_w}{H_w^4} \quad (7)$$

where E_w is the modulus of elasticity of the retaining wall material, I_w is moment of inertia of the wall. The mass of the wall *(M_2)* is obtained by the wall section volume and the density of the wall material. The viscous damping *(c)*, for both wall and backfill, is obtained from basic fundamental expression:

$$c = 2\zeta\sqrt{kM} \tag{8}$$

IV. Results and Discussion

A typical concrete retaining wall of height 4.5m and thickness 0.45m retaining cohesion-less medium dense backfill soil is considered. The analysis is carried out typically for acceleration amplitude of 0.3g and frequency 2Hz. The wall and backfill properties used are as below.

TABLE I
WALL-BACKFILL PROPERTIES

Backfill Properties		**Wall Properties**	
Type of soil	Cohesionless soil	Unit weight of concrete	25 kN/m^3
Unit weight (γ)	15.71 kN/m^3	Height (H)	4.5m
Poisson's ratio (μ_s)	0.3	Thickness (t)	0.45m
Damping ratio (ζ_1)	20%	Poisson's ratio (μ_w)	0.17
Friction angle (ϕ)	32°	Damping ratio (ζ_2)	5%

The magnitude of dynamic active earth thrust and wall displacement values are obtained using the proposed model. For assumed wall-backfill properties, the magnitude of dynamic earth thrust and wall displacement by present 2-DOF model is 50.61 kN and 5.07mm respectively. The displacement of the wall is less than 0.2% of wall height. Similar observation is reported by [4] 1-DOF analysis, here also these results compare well with the assumption of the present model, which is valid for small displacements only.

The above obtained results are then compared with those obtained by conventional methods and also with experimental analysis carried out by various researchers.

A. Comparison of Earth Thrust and Wall Displacement

1. Conventional Methods

Dynamic earth thrust and wall displacement obtained from present 2-DOF model is compared with various conventional methods for the same data of RW and backfill soil and presented in Table II.

TABLE II
COMPARISON OF RESULTS WITH CONVENTIONAL METHODS

Comparison of Wall Displacement		Comparison of Dynamic Earth Thrust	
Method	Wall Displacement	Method	Earth Thrust
Present Analysis	5.07 mm	Present Analysis	50.61 kN
Scott's Analysis	6.86 mm	Scott's Analysis	49.05 kN
Saran-Reddy-Viladkar Method	6.75 mm	Veletsos & Younan (2000) [9]	51.33 kN
		MO method	67.60 kN

From Table II, it is observed that the displacement and earth thrust obtained by proposed 2-DOF model agrees well with the results obtained by conventional methods. However, it is observed from the results that the widely accepted MO theory significantly overestimate the dynamic earth pressure value. Interestingly, it is also observed that, dynamic earth thrust obtained using 65% of the peak ground acceleration with the MO method is in good agreement with the obtained earth thrust using proposed 2-DOF model.

2. Experimental Analysis

The results obtained by present analysis are also compared with the experimental work carried out by [8]. The centrifuge tests were carried out by them on two retaining walls with thicknesses 0.25m and 0.4m and having height equals to 5.67m retaining dry medium dense sand. Experiments were carried out for various earthquake acceleration amplitudes and frequencies. The dynamic active earth pressure has been obtained for this experimental data using the present 2-DOF model. The comparison of the results as shown in Table III. From Table III, it is observed that dynamic earth thrust values calculated by 2-DOF method are in line with those obtained from centrifuge experimental results. It is also observed that, experimental results are slightly on the conservative side as compared with proposed 2-DOF analysis.

V. Effect of Wall Flexibility

The effect of wall flexibility on the dynamic earth pressure and wall displacement is studied by varying the thickness of the wall (t) from 0.3m to 1m, while the height of the wall (H) is kept constant. The analysis is carried out for constant base acceleration amplitude of 0.3 g and frequency of 2 Hz.

Wall flexibilities are evaluated in terms of relative flexibility *(d_w)* as given by [9]:

$$d_w = 12(1-\mu^2)\frac{G}{E}\left(\frac{H}{t}\right)^3 \tag{9}$$

where, G-Shear modulus of backfill soil, (N/m2), E-Young's modulus of wall, (N/m2), t-Thickness of wall, (m), H-Height of wall (m), μ– Poisson's ratio of wall material.

For 4.5 m wall height and varying thickness as mentioned above, the displacement of the wall and earth thrust generated behind wall have been calculated and the results are shown in graphical form by plotting dynamic earth thrust values against wall flexibility *(dw)* (Fig. 5).

TABLE III
COMPARISON OF 2-DOF RESULTS WITH EXPERIMENTAL DATA VALUES

Earthquake	PGA	Frequency	Thickness of wall	Earth Pressure (KN)	
				Expt Method	2 DOF method
Loma Prieta SC2	0.49g	12.08	0.4	136.83	131.7
Kocail YPT060-2	0.15g	22.44	0.4	68.41	52.75
Loma prieta SC1	0.66g	13.09	0.25	141.75	123.72

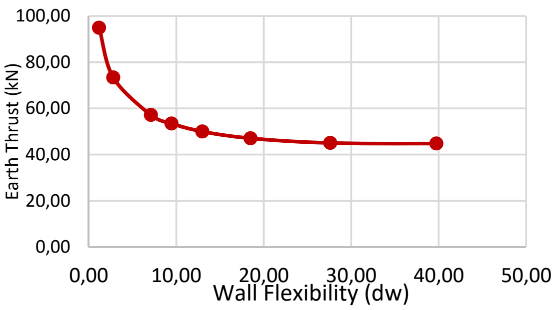

Fig. 5 Effect of wall flexibility on earth pressure by present 2-DOF model

It is observed that dynamic earth thrust decreases and approaches to a constant value with the increase in flexibility of the wall. As $d_w \rightarrow 0$ i.e. as wall tends to act as a rigid wall, the earth pressure increases rapidly. The earth thrust acting on flexible wall is approximately equal to one-half of that obtained for, rigid wall ($d_w \rightarrow 0$). The same results are also reported by [9] in their study. Similarly, the variation of the wall displacement with wall flexibility in terms of d_w is studied (Fig. 6).

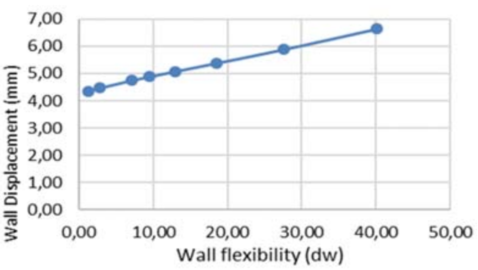

Fig. 6 Effect of wall flexibility on wall displacement

The wall displacement increases linearly with increase in the wall flexibility. The maximum wall displacement of the cantilever walls of realistic flexibilities is found to be approximately 0.1 to 0.2 percent of the wall height.

Thus, the dynamic active earth thrust decreases with the increase in the wall flexibility while the wall displacement increases linearly with the flexibility. Hence for flexible retaining wall, the analysis carried out by proposed mathematical model proves to be economical compared to other conventional methods.

B. Comparison with Earlier Research Work

The results obtained by the 2-DOF model are compared with the analysis carried out by [9]. For the same wall-backfill properties the variation of the earth thrust with wall flexibility is presented in Fig. 7.

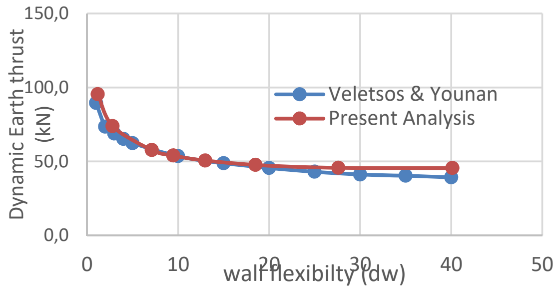

Fig. 7 Comparison of results with Veletsos and Younan analysis

The results obtained by the proposed 2-DOF model are in good agreement with those obtained by [9].

The effect of wall flexibility on dynamic earth thrust obtained from present analysis is also compared with the PLAXIS analysis carried out by [10]. The flexibility of the wall is varied by varying the thickness of the wall from 0.35 to 1m, while height of the wall is kept constant, 8m. The analysis was carried out at constant base acceleration amplitude of 0.1g and frequency of 5 Hz. Fig. 8 shows the comparison between the effect of wall flexibility on dynamic earth thrust obtained by present model and PLAXIS analysis.

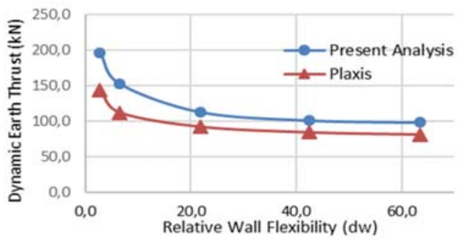

Fig. 8 Effect of wall flexibility on dynamic earth thrust: comparison with PLAXIS analysis

From Fig. 8, it is observed that the trend of variation of dynamic earth thrust with wall flexibility as obtained from present model is same as that reported by researcher using PLAXIS analysis. However, results obtained from 2-DOF analysis are on conservative side as compared to PLAXIS. The reason could be, PLAXIS analysis considers the wall-soil interface angle for calculation of earth thrust which is not taken into account in the present 2-DOF model, and thus the values obtained by 2-DOF model are on conservative side.

VI. Conclusion

Based on the analytical work carried out in the present study, the following conclusions are drawn:

1. As both wall and backfill properties are incorporated in the analysis, the wall displacement and dynamic earth thrust obtained by proposed 2-DOF mass-spring-dashpot system for the flexible cantilever retaining wall are more realistic. In flexible wall, displacement of both wall and backfill is significant. Thus idealization of wall-backfill as 2-DOF system proves to be advantageous. Further, the results obtained by the proposed analysis agree well with the same obtained from conventional methods and experimental analysis.
2. The effect of wall flexibility on dynamic earth thrust as well as wall displacement can be studied by proposed model. The magnitudes of the wall displacements and earth thrust induced by horizontal ground shaking are quite sensitive to the flexibility of the wall. The total Earth Pressure on flexible wall ($d_w > 40$) approximately equals to one-half of that obtained for, rigid wall ($dw \rightarrow 0$).

References

[1] Mononobe N, Matsuo H 1929 On the determination of earth pressures during earthquakes. Proc. World Engineering Conference 9: 176.

[2] Okabe S 1926 General theory of earth pressure. J. Jap. Soc Civil Eng. (Tokyo) 12(1).

[3] Steedman, R. S. and Zeng, X. (1990) "The influence of phase on the calculation of pseudo-static earth pressure on a retaining wall," Geotechnique, Vol. 40(1), 103–112..

[4] Scott R.F., (1973), "Earthquake induced Earth Pressures on Retaining wall", Proc. 5thWorld Conf. on Earthquake Engineering (Tokyo: Int. Assoc. Earthquake Eng.) 2: 1611–1620..

[5] Wood J. H. (1973), "Earthquake-induced pressures on retaining walls" Rep. EERL 73-05, Earthquake Eng. Res. Lab., California Institute of Technology, Pasadena, CA.

[6] Veletsos A. S., Younan A. H. (1994b), "Dynamic soil pressures on rigid vertical walls", Earthquake Eng. Struct. Dyn. 23: 275–301.

[7] Choudhury D., S. Chatterjee, (2006), "Dynamic active earth pressure on retaining structures." Sadhana 31(6) 721-730.

[8] Linda Al Atik, Nicholas Sitar, (2010), "Seismic Earth Pressures on Cantilever Retaining Structures." Journal of Geotechnical and Geo-environmental Engineering, ASCE, 136(10) 1324-1333.

[9] Younan A. H., Veletsos A. S and K. Bandyopadhyay, (2000), "Dynamic response of flexible Retaining walls", Journal of Geotechnical Engineering, ASCE, Vol. 120(12)-275-301.

[10] Ozgur L. Ertugrul, Selman Saglam, M. Yener Ozkan, (2008), "Excitation and Flexibility Effects on Dynamic Response of Cantilever Retaining Walls" Geotechnical Earthquake Engineering and Soil Dynamics IV, ASCE pp 1-11.